# Rare Events: Limiting Their Damage Through Advances in Modeling

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**PREVIEW** *Rare events such as tsunamis and avalanches often result in severe losses, but such "acts of God" have been beyond the predictive ability of our forecasting models. Advances are being made, however, in forecasting rare* economic events. *As Gloria tells us, the key is to account for* system connectedness *to single out the fragile side of an economic network, to quantify the cross-linkages among financial and other institutions, and to perform stress tests that, when credible, will be able to reduce the uncertainty associated with a potentially catastrophic event.*

## INTRODUCTION

The financial crisis of 2008, with its enormous consequences for the world economy, has brought a renewed interest in forecasting rare events. The crisis has triggered questions about the validity and forecasting ability of economic models when confronted with unfamiliar circumstances.

One of the aftershocks has been a crisis of confidence in forecasters and their methods. However, the academic community has responded with unusual energy in exploring new venues of research, revisiting old methods, and opening new paths to include methods from other disciplines. The 9th Workshop of the International Institute of Forecasters, jointly organized with the Federal Reserve Bank of San Francisco and held in September 2012, showcased the latest advances on predicting rare events and modeling systemic and idiosyncratic risk. For a summary, see González-Rivera and colleagues (2012) in *The Oracle*.

While rare events, as the name indicates, occur very infrequently, their consequences tend to be catastrophic. Natural disasters like earthquakes, tsunamis, floods, and avalanches fit this category. In general, they are isolated events and classified as "acts of God." However, when we consider an economic system, rare events are defined as low-probability, high-magnitude episodes that carry devastating losses, do not for the most part happen spontaneously or in isolation, and are man-made. Precisely because of these last two characteristics – lack of spontaneity, human causation – there is hope for forecasting economic rare events.

The hope rests on a multidimensional approach to the modeling and monitoring of the many risks in an economic system, which is increasingly hyper-connected. The key notion is *system connectedness*. Episodes that seem rare and idiosyncratic at the outset, like the Lehman bankruptcy in 2008, can quickly become systemic events that jeopardize the stability and functioning of entire economies.

## EXTREME VALUE DISTRIBUTIONS AND QUANTILES

The objective of conventional time series models, e.g., simple and exponential-smoothing mechanisms, seasonal and trend filters, ARIMA models, etc., is to construct a forecast as the expected value (that is, the average) of a future random variable. But with rare events, focusing our forecast on the average outcomes is useless; rather, we need to focus on the low-probability sections of the distribution of possible outcomes. But here is where we find our first problem. Precisely because they are low-probability events, the data is very sparse. In addition, we are forced to contemplate the rare event beyond the range of the available data. In the absence of long historical records, we need mathematical and statistical theories to extrapolate from the available data.

***Extreme Value Theory*** (EVT) is a standard tool in the insurance industry as well as in many engineering sectors (Embrechts and colleagues, 1997). These industries are concerned with the modeling of extreme events. For instance, insurance firms need to set up their premiums such that, when the big event happens, the payments of insurance claims do not make the insurers insolvent.

At the core of EVT, we have extreme-value distributions like Fréchet, Weibull, and Gumbel, which are distributions that characterize the *maximum* outcomes – for instance, the modeling of the *largest* claims in an insurer portfolio. These functions model exclusively the low-probability area (location of rare events) of the variable of interest, disregarding the most frequent events. Figure 1 shows an example of a standard Fréchet distribution.

Figure 1. An Extreme Value Distribution

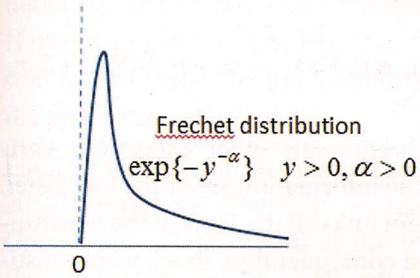
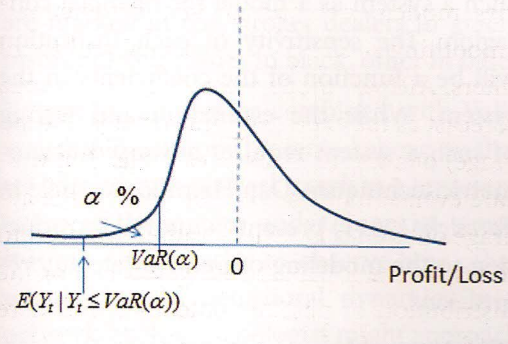



Figure 2. Quantile Estimation




In EVT, it is also of interest to compute the *mean excess function* beyond some threshold $u$ set by the researcher. For instance, within the insurance industry, we could compute what the average claim will be once the claims exceed a threshold of \$1 million.

## Key Points

- Man-made rare events are defined as low-probability, high-magnitude episodes that carry devastating losses, but for the most part do not happen spontaneously or in isolation. For forecasting rare events, the key notion is *system connectedness*. Episodes that seem rare and idiosyncratic at the outset, like the Lehman bankruptcy in 2008, could quickly become systemic events that jeopardize the stability and functioning of entire economies.
- The objective of conventional time-series models is to construct a forecast as the expected value (that is, the average) of a future random variable. But with rare events, focusing our forecast on the average outcomes is useless; rather, we need to focus on the low-probability sections of the distribution of possible outcomes. Use of *extreme-value distributions* and *quantile estimation* techniques are proving to be valuable.
- Modeling of systemic risk must be *multivariate*; it may start by assessing individual risk functions, but in the aggregate we need a notion of interconnectedness able to single out the fragile sides of the network. *Network modeling* is useful to find the critical time just before the network is ready to collapse.
- Though forecasting the rare event is not a straightforward exercise, the consequences of a rare event can be simulated and stress testing performed to reduce the uncertainty associated with the potentially catastrophic event.

***Quantile Estimation*** is another approach to model low-probability events. In this case, we consider the full distribution of possible values for the variable to be predicted – that is, the cumulative distribution function – and we focus on the lower or upper quantiles. The low α % quantile is defined as a value in the tail of Figure 2 such that the probability associated with that value is α %. Since our interest is in low-probability events, customary levels for α are 5%, 1%, or 0.5%.

A standard tool for risk management in the financial industry, required as well by financial regulators, is the *Value-at-Risk* (VaR) of the market portfolios of the banking institutions. VaR is the 5% or 1% quantile of the profit/loss function of the market portfolio (Jorion, 2001). As in EVT, it is also useful to compute the expected shortfall, which is the average loss of the portfolio once the losses exceed the VaR value.

In summary, forecasting rare events will require building time-series models for maximum (or minimum) observations based on EVT and/or time-series models for low-level quantiles, and computing their corresponding average excesses, and shortfalls.

## NEW RESEARCH

Three major themes permeated the contributions of the September 2012 IIF workshop mentioned above:

- Multivariate analyses of risks
- Understanding of risk networks
- Stress testing to analyze the consequences of rare events

At the micro level, companies may shield from risk or manage a rare event by "diversifying" it away. At the macro level, however, there is a limit on how much risk can be transferred or how many institutions are willing to carry out the other side of a trade. Any macro modeling of systemic risk must be *multivariate*; it may start by assessing individual risk functions, but in the aggregate we need a notion of interconnectedness able to single out the fragile sides of the network. *Network modeling* has been embraced in other disciplines and is making its way into economic modeling. By nature, the rare event may be outside the range of available data; to assess the consequences of the event, we will have to resort to *stress testing*, creating scenarios that are rare and then, under these circumstances, forecasting the reaction of the economy.

### *Multivariate Analysis of Risk*

From a system perspective, it is crucial to understand that the risk faced by an individual institution may have spillovers on other institutions. This means that a rare event for an institution can become a systemic event through the linkages in the system.

From a macroeconomic point of view, it is important under situations of financial distress to understand the risk sensitivity of financial institutions to market shocks. For instance, suppose that there are n institutions in the system. The macroeconomic analyst would like to estimate a multivariate system of VaRs (Values-at-Risk) as the following (for the sake of simplicity, the lag structure is very short – just one lag – but we could include further lags in each equation):

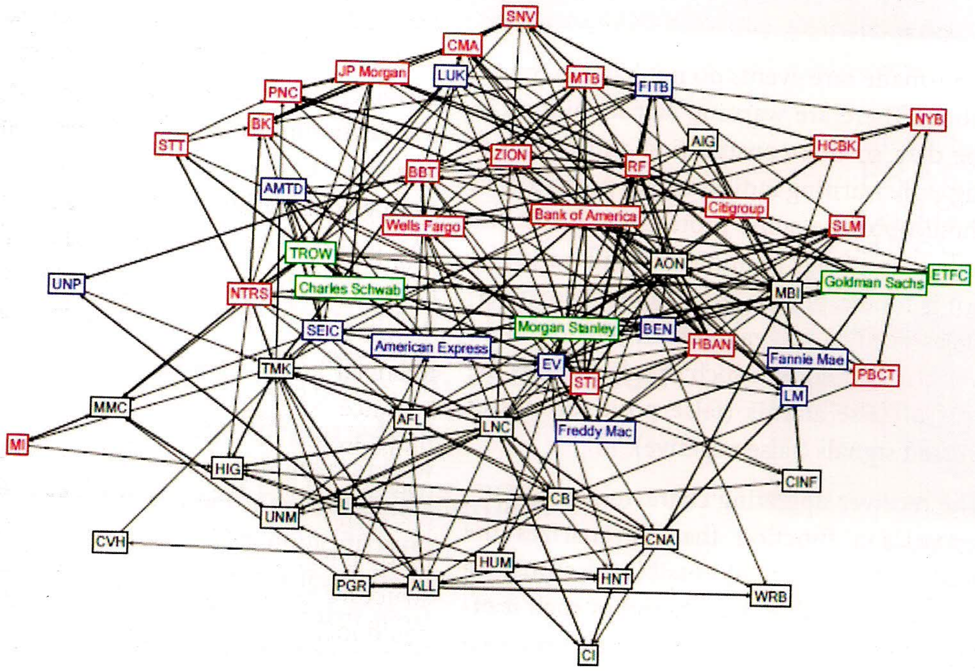
$$
\begin{aligned}
VaR_{1t} &= \beta_{11}VaR_{1,t-1} + \beta_{12}VaR_{2,t-1} + \ldots.. + \beta_{1n}VaR_{n,t-1} + X_t^{'}\gamma_1 \\
VaR_{2t} &= \beta_{21}VaR_{1,t-1} + \beta_{22}VaR_{2,t-1} + \ldots.. + \beta_{2n}VaR_{n,t-1} + X_t^{'}\gamma_2 \\
\ldots.. & \\
VaR_{nt} &= \beta_{n1}VaR_{1,t-1} + \beta_{n2}VaR_{2,t-1} + \ldots.. + \beta_{nn}VaR_{n,t-1} + X_t^{'}\gamma_n
\end{aligned}
$$

Here, $X_t$ represents other exogenous variables in the information set of the forecaster. This system links all the *VaR*s of the *n* institutions, and consequently a shock in any institution would work its way to the rest through the dynamics of the system. We could view such a system as a model for financial contagion. The sensitivity of each institution will be a function of the coefficients in the system. While the estimation and testing of such a system requires advanced econometric techniques, Dan Hamilton's article in *Foresight* (2011) presents a tutorial introduction to the modeling of multivariate autoregressions.

### *Risk Networks*

The analysis of networks is very prevalent in other disciplines – computer science, to give one example. It consists of quantifying the cross-linkages (nodes, number of connections, distances between nodes, etc.) among financial and other institutions, such that

Figure 3. Risk Network of the U.S. Financial System




those firms that are more systemically relevant are identified. For an introduction to networks, see Barabási and Frangos (2002). Network graphs, such as Figure 3, offer a visual tool to assess the interconnectedness of the system of institutions.

Figure 3 (taken from Hautsch and colleagues, 2012, "Financial Network Systemic Risk Contributions," presented at the 9th Workshop of the IIF) shows a risk network of the U.S. financial system highlighting key companies in the system in 2000-8. Depositories are marked in red, broker dealers in green, insurance companies in black, other in blue.

Networks can also be combined with VaRs systems, like the one presented above, so that we obtain a *risk network* graph as a representation of the system under financial stress. It is very informative to observe how other sciences model behavioral dynamics in a network, such as a zoologist might approach the social collapse of the hierarchy within a group of monkeys. The methodological objective is to find the critical time just before the network is ready to collapse. These methods also have direct applications to the modeling of financial networks and monitoring of systemic risk.

## *Stress Testing*

Though forecasting the rare event is not a straightforward exercise, the consequences can be simulated. Stress testing is a practice that, when credible, has the capability to reduce the uncertainty associated with the potentially catastrophic event. The objective of stress testing is to convert uncertainty into a risk assessment by mapping extreme but probable macro scenarios to micro outcomes, e.g. losses in loan portfolios, shortfalls in provisions/capital, lower income streams, etc. (González-Rivera, 2003). Once the stressful macro scenarios are defined, stress testing is the ultimate exercise in forecasting, including dynamic projections of revenues and expenses, evolution of the balance sheet of the institution, projections of liquidity and capital ratios, probability of defaults, bankruptcy points, and so on. Stress testing can also be understood as a protection mechanism for the solvency of institutions, because it will allow implementing corrective measures in anticipation of potentially devastating shocks to the system. Stress testing can also be incorporated into a risk network to analyze the reaction across institutions and possible feedback mechanisms