Photoluminescence of Tetrahedral Quantum-Dot Quantum Wells

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Taking into account the tetrahedral shape of a quantum-dot quantum well (QDQW) when describing excitonic states, phonon modes, and the exciton-phonon interaction in the structure, we obtain within a nonadiabatic approach a quantitative interpretation of the photoluminescence spectrum of a single CdS/HgS QDQW. We find that the exciton ground state in a tetrahedral QDQW is bright, in contrast to the dark ground state for a spherical QDQW. The position of the phonon peaks in the photoluminescence spectrum is attributed to interface optical phonons. We also show that the experimental value of the Huang-Rhys parameter can be obtained only within the nonadiabatic theory of phonon-assisted transitions.

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Multilayer quantum dots (QD’s) are of broad interest not only due to the striking tunability of their physical properties, but also because of the intensively investigated prospects for applications in optoelectronics, microscopy, and biology [1,2]. A quantum-dot quantum well (QDQW) has been fabricated [3] by including a layer of HgS into a nanocrystal of CdS, which has a band gap larger than that of HgS. The excitons in a QDQW are mainly confined to the HgS well, which results in a high quantum yield and an improved photochemical stability. A detailed experimental investigation of the crystalline structure, the shape, and the optical spectra can be carried out on a single QD [4]. High-resolution transmission electron microscopy has revealed that QDQW’s are preferentially tetrahedral particles [see Fig. 1(a)] with a zinc blende crystal lattice [5]. Using confocal optical microscopy, photoluminescence (PL) from a single QDQW has been measured [6]. Only the lowest excitonic transitions in spherical QDQW’s have been studied theoretically [7–12] until now. In this Letter, we report the first quantitative interpretation of the PL spectrum of a QDQW.

To obtain electron and hole energy spectra in spherical QDQW’s, k-p [7–9] and tight-binding [10–12] calculations have been carried out. Multiband k-p studies for spherical QDQW’s yield s-type electron and p-type hole ground states [7,8], leading to the dark exciton ground state [9]. It has been shown that both multiband k-p and tight-binding calculations provide a similar qualitative physical description for the lowest states in QDQW’s [12]. Recently, we used Burt’s envelope-function representation to develop an eight-band theory which is particularly suitable for QD heterostructures with thin shells, consisting of materials with notably different effective-mass parameters and narrow band gaps [13]. This multiband theory was successfully applied to spherical QDQW’s [8], and it is used in this Letter to study tetrahedral QDQW’s.

Experiments show that the PL spectra of single QDQW’s do not change appreciably for different temperatures (1.4–50 K), excitation wavelengths (442–633 nm), and intensities (0.02–50 kW/cm²) [6]. Therefore, the spectrum corresponds to the equilibrium PL from the exciton ground state β0. In the low-temperature limit, and using the dipole approximation, the intensities of the phonon lines in the PL spectrum of QD’s are given by [14]

\[ I(\Omega) \propto \sum_{K=0}^{\infty} \sum_{\lambda_1,\ldots,\lambda_K} F^{(\lambda_1,\ldots,\lambda_K)}_{K,\beta_0} \delta \left( \Omega_{\beta_0} - \sum_{j=1}^{K} \omega_{\lambda_j} - \Omega \right), \]

where K is the number of a phonon sideband and \( \lambda_j \) labels the phonon modes with frequencies \( \omega_{\lambda_j} \). The general expression for the amplitudes \( F \) of the phonon lines is given in Ref. [14]. For the zero-phonon line (\( K = 0 \)) and for the one-phonon lines (\( K = 1 \)), these amplitudes are given by

\[ F^{(0)}_{\beta_0,0} = |d_{\beta_0}|^2, \]

\[ F^{(1)}_{\beta_0,1} = \sum_{\beta_1,\beta_2} \frac{\hbar^{-2} d_{\beta}^2 d_{\beta_1} \langle \gamma_{\lambda_1} | \beta_1 | \beta_0 \rangle \langle \beta_0 | \gamma_{\lambda_1}^* | \beta_2 \rangle}{\Omega_{\beta_0} - \Omega_{\beta_1} - \omega_{\lambda_1} - \Omega_{\beta_2} - \omega_{\lambda_1}}, \]

where \( \gamma_{\lambda_j} \) are the amplitudes of the exciton-phonon interaction. The dipole matrix element \( d_{\beta} \) is given by

\[ d_{\beta} \propto \int \delta(\mathbf{r}_e - \mathbf{r}_h)(\mathbf{e}, \hat{\mathbf{p}}) \Psi_{\text{exc}}^{(\beta)}(\mathbf{r}_e, \mathbf{r}_h) d\mathbf{r}_e d\mathbf{r}_h, \]

where \( \mathbf{e} \) denotes the light polarization vector and \( \hat{\mathbf{p}} \) is the...
The exciton energies and wave functions have been derived numerically from the Hamiltonian:

\[ \hat{H}_{\text{exc}} = [\hat{H}_e + V_{s-a}(\mathbf{r}_e)] - [\hat{H}_h - V_{s-a}(\mathbf{r}_h)] + V_{\text{int}}(\mathbf{r}_e, \mathbf{r}_h), \]

where \( \hat{H}_e \) is the two-band electron Hamiltonian and \( \hat{H}_h \) is the six-band hole Hamiltonian, which we derived from a general eight-band Hamiltonian for heterostructures [13].

The dielectric mismatch between QDQW shells leads to the electron and hole self-interaction potential \( V_{s-a} \) and significantly changes the potential of the electron-hole interaction \( V_{\text{int}} \) [16]. The effective-mass parameters for CdS are taken from Ref. [16], the dielectric constants from Ref. [17], and all other required parameters from Ref. [8]. The effective-mass parameters for HgS entering the Hamiltonians \( \hat{H}_e \) and \( \hat{H}_h \) are calculated as a function of the energies of the electron and the hole ground states. The numerical calculation is carried out using a finite-difference method on the same cubic mesh, both inside and outside the QDQW, as for tetrahedral CdS QD’s [16]. A mesh length of 0.05 \( \sqrt{3} \) nm ensures a relative error for the exciton ground state energy less than 1%.

HgS is a quantum well for both electrons and holes in the QDQW. While only a few electron states lie below the CdS bulk conduction band edge and are localized near the HgS shell, many hole states are trapped in the HgS shell and lie above the CdS bulk valence band edge. It is seen from Figs. 1(b) and 1(c) that the electron in the ground state is localized near the edges of the tetrahedral HgS shell. Figure 1(d) shows that, unlike in the ground state, the hole in the first excited state lies above the CdS bulk valence band edge. It is seen from Figs. 1(b) and 1(c) that the electron in the ground state is localized near the edges of the tetrahedral HgS shell. Figure 1(d) shows that, unlike in the ground state, the hole in the first excited state lies above the CdS bulk valence band edge. It is seen from Figs. 1(b) and 1(c) that the electron in the ground state is localized near the edges of the tetrahedral HgS shell. Figure 1(d) shows that, unlike in the ground state, the hole in the first excited state lies above the CdS bulk valence band edge.
excited state substantially penetrates the region of facets of the HgS shell, where the electron density in the ground state is high.

In the QDQW’s optical absorption spectrum, the intensity of a transition to the exciton level $\beta$ is $|d_{\beta}|^2$. Figure 2 shows that the intensity of the second exciton level is $\sim 8.5$ times larger than that of the ground state level. Since the second exciton state is absorbing and the exciton ground state is luminescing, a Stokes shift should be observed between the frequencies of the excitation and the zero-phonon equilibrium PL. In addition to the fact that our calculated exciton ground state energy of 1857 meV is in excellent agreement with the experimental data of Refs. [5,6], the obtained excitation energy of 18 meV is very close to the value 19 meV of the Stokes shift found in the fluorescence line narrowing spectrum [5]. The inset of Fig. 2 shows the result of an analogous finite-difference calculation of the exciton states for a spherical QDQW [8] with the same thickness of the HgS shell as that for the tetrahedral QDQW. The exciton ground state is eightfold degenerate for both spherical and tetrahedral QDQW’s. The remarkable effect of the QDQW’s shape is that the exciton in the ground state is bright for the tetrahedral QDQW, while it is dark for the spherical QDQW. We find that the ratio of the intensity of the exciton ground state to the intensity of the first excited state is sensitive to the shape of the QDQW’s also for their PL spectra.

The amplitudes of the exciton-phonon interaction are $\gamma_{\lambda}(r_n, r_h) = V_\lambda(r_n) - V_\lambda(r_h)$. In the dielectric continuum approach, the amplitudes $V_\lambda(r_n)$ of the electron-phonon interaction, $V_\lambda(r_h)$ of the hole-phonon interaction, and the phonon frequency $\omega_\lambda$ are solutions of the eigenvalue problem for the equation

$$\nabla(\varepsilon(\omega_\lambda) \nabla V_\lambda) = 0 \quad \text{with} \quad \varepsilon(\omega) = \varepsilon_0 \frac{\omega^2 - \omega_{LO}^2}{\omega^2 - \omega_{TO}^2},$$

where, for each shell, $\omega_{LO}$ and $\omega_{TO}$ are the frequencies of the bulk longitudinal optical (LO) and the transverse optical (TO) phonons, and $\varepsilon_0$ and $\varepsilon_\infty$ are the static and the high-frequency dielectric constants, respectively. There exist two kinds of solutions of Eq. (6). Solutions of the first kind $[\varepsilon(\omega_\lambda) = 0]$ describe bulklike phonon modes [18]. In the $k$th shell they have the frequency $\omega_{LO}^k$ and their amplitude is chosen to satisfy the equation $\nabla^2 V_\lambda = -q_k^2 V_\lambda$, with boundary conditions $V_\lambda[S_0, S_1, \cdots, S_k] = 0$. The number of bulklike phonon modes is restricted by the condition $q_k \leq \pi/a$, where $a$ is the lattice constant. Solutions of the second kind $[\varepsilon(\omega_\lambda) \neq 0]$ describe interface phonon modes. To the best of our knowledge, the phonons for the tetrahedral QDQW are theoretically derived here for the first time. We propose for the tetrahedral QDQW an approximation with a spatial behavior close to that for spherical QD’s, i.e., $s$-like, $p$-like, etc. This means that we seek interface phonon modes as a product of a function $\Phi(\xi)$, which is constant on the tetrahedral surface $S(\xi)$, and a function $\varphi(r)$, which satisfies the equation $\nabla^2 \varphi = -q^2 \varphi$ and which is zero at infinity. If this method is applied to a spherical QDQW, where $\xi = r$, $\Phi(\xi)$ turns out to be a radial function and $\varphi(r)$ explicitly contains the spherical harmonics that define the spatial symmetry. Substituting $V_\lambda = \Phi(\xi) \varphi(r)$ into Eq. (6), multiplying by $\varphi^*_Q$ and integrating over the tetrahedral surface $S(\xi)$, we obtain a one-dimensional problem with eigenfrequencies $\omega_{Q,n}$ and eigenfunctions $\Phi_{Q,n}(\xi)$. We used the frequencies of the LO phonons found in Ref. [5]. For the numerical calculation of the bulklike and interface phonon modes we have used the same finite-difference scheme as described above. For any spatial symmetry of $\varphi_0$ we found five different functions $\Phi_{Q,n}$ that correspond to five eigenfrequencies $\omega_{Q,n}(n = 1, \ldots, 5)$ of the interface phonons. Figure 3 shows the amplitudes of the interface phonon modes $V_{Q,n}$ with $p$-like spatial symmetry, averaged over the tetrahedral surface $S(\xi)$. In contrast to the amplitudes of the bulklike phonon modes that vanish at the boundaries, the amplitudes of the interface phonon modes have their extremal values at some of the boundaries.

In Fig. 4 the PL spectrum calculated using Eq. (1) for a single tetrahedral QDQW is compared with two experimental spectra, measured at different excitation wavelengths and intensities [6,19]. Most likely spectral diffusion, which has been explained recently by a reorganization of local electric fields in or around the particles [20], is responsible for the relatively broad lines in the PL spectra of a single QDQW. The experimentally determined Huang-Rhys parameter is $0.25 \pm 0.05$ [6]. Because of the localization of the exciton in the HgS well, the amplitudes of the bulklike phonon modes are much smaller than those of the interface phonon modes. The dominant contribution to the intensity of the

![FIG. 3 (color). Amplitudes of several interface phonon modes averaged as $V_{Q,n}(\xi) = \Phi_{Q,n}(\xi) \int_{S(\xi)} |\varphi_Q|^2 dS / \int_{S(\xi)} dS$ (for denotations see text). Vertical dashed lines show the boundaries of the tetrahedral shells. Isosurfaces $V_{p,5}(r) = 10$ meV (red) and $V_{p,5}(r) = -10$ meV (blue) in the HgS shell are shown. The amplitudes $V_{p,4}$ and $V_{p,5}$ give the highest one-phonon peaks in the PL spectrum in Fig. 4.](image-url)
In summary, we have shown a strong difference between the optical spectra of spherical and tetrahedral QDQW's. Only the simultaneous consideration of the tetrahedral shape of a QDQW, interface optical phonons, and nonadiabatic phonon-assisted transitions allows for profound understanding of the optical response of a QDQW. In particular, this analysis allowed us to interpret the observed PL spectra of a CdS/HgS/CdS QDQW.

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FIG. 4 (color). PL spectrum of a tetrahedral QDQW normalized to the zero-phonon line's intensity. Calculated spectral lines (black curve) are broadened by a Gaussian to facilitate visualization. Experimental spectra are measured at $T = 10$ K with excitation wavelength 633 nm and intensity 15 kW/cm$^2$ (green curve) [6] and with excitation wavelength 442 nm and intensity 5 kW/cm$^2$ (red curve) [19]. The inset gives one- and two-phonon bands (magnified by factors of 40 and 4000, respectively) as calculated within the adiabatic approximation.

one-phonon lines in the calculated PL spectrum is due to $p$-like interface phonon modes. The two most intense one-phonon peaks correspond to the modes with amplitudes $V_{p,4}$ and $V_{p,5}$ in Fig. 3 and energies 34.3 and 36.7 meV, respectively. The average energy, weighted with the intensities of the two peaks, is 35.5 meV, in good agreement with the experimentally determined value of 35.3 ± 0.6 meV [6]. The intensity of the calculated two-phonon lines cannot directly be compared with that in the experimental spectra because of the large background in this spectral region. The inset of Fig. 4 shows the one- and two-phonon bands in the PL spectrum calculated within the adiabatic approximation. In this approximation, the intensities of the one-phonon peaks are defined by Eq. (3), however, without the summation over the states $\beta_1$ and $\beta_2$, which are replaced by $\beta_0$. Since the exciton ground state $\beta_0$ is eightfold degenerate and its intensity $|d_{\beta_0}|^2$ is small as compared to the intensities of the higher lying exciton states (see Fig. 2), the intensities of the phonon peaks in the adiabatic approximation are dramatically lower than those calculated within the nonadiabatic theory [14].