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Critical Assessments of Leading Philosophers

Edited by Michael Beaney and Erich H. Reck

Volume I
Frege’s philosophy in context

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PREFACE

In preparing this collection, we have consulted as widely as possible with Frege scholars across the world. The first full draft of the list of contents was drawn up in July 2003 when we met in Erlangen, Germany; we are indebted to Christian Thiel for his help both then and subsequently. We have also benefited from the advice of many of the contributors, who sent us copies of their papers and made suggestions as to what best represented their work. We are particularly grateful to those who made revisions to their original papers, contributed new material or made or checked through translations of their work (in the case of papers originally written in German). Our aim was to produce the definitive collection of papers on Frege, covering all aspects of his philosophy, in the period from 1986 to 2004. Whether or not we have succeeded in doing so, however, we are convinced that it demonstrates the vigour of work in this area and the profound importance of Frege’s philosophy.

We would also like to thank the staff at Routledge who have guided the project through its various phases; in particular, to Natalie Foster and to the three Development Editors, Jennifer Lovel, Zoe Botterill and Jessica Spencer. Michael Beaney would like to acknowledge the support provided by the Alexander von Humboldt Foundation and the Institut für Philosophie of the University of Erlangen-Nürnberg during his stay in Germany in the summer of 2003 and the Open University in allowing him periods of research leave at certain key stages. Erich Reck is grateful to the University of California, Riverside, for granting him a term of research leave while working on this project; he is also indebted to Leonard Linsky, for awakening his interest in Frege originally, and to Thomas Ricketts for deepening that interest. We would both like to thank Gottfried Gabriel for turning the philosophy department at the University of Jena, Germany, into an inviting place for Frege scholars from around the world and for the help that he has given on this project. Last, but not least, we are grateful to our families, for putting up with email activities out of normal working hours as we endeavoured to communicate across an ocean and a continent.

Michael Beaney and Erich H. Reck
February 2005