2009 Fall Stat 160A hw4 solution

Book Exercises

3.4, 3.14, 3.21, 3.33, 3.38, 3.56, 3.57, 3.67, 3.80

Problem 1 - Cereal

Children (and some adults) are frequently enticed to buy breakfast cereal in an effort to "collect all six action figures." Assume that there are six action figures and each cereal box contains exactly one of the six with each figure being equally likely. Find the expected number of boxes needed to collect all six action figures. Also, find the standard deviation (by finding the variance) of the number of boxes needed to collect all six action figures.

Solution to Problem 1 – Cereal

Define the following events:

E_1: Collect 1 action figure.

E_2: Collect 2 distinct action figures.

E_3: Collect 3 distinct action figures.

E_4: Collect 4 distinct action figures.

E_5: Collect 5 distinct action figures.

E_6: Collect 6 distinct action figures.

Let X_1 = the number of boxes needed in E_1, then $X_1 \sim Geo(1)$.

X_2 = the number of boxes needed in E_2 after E_1, then $X_2 \sim Geo(5/6)$.

X_3 = the number of boxes needed in E_3 after E_2, then $X_3 \sim Geo(4/6)$.

X_4 = the number of boxes needed in E_4 after E_3, then $X_4 \sim Geo(3/6)$.

X_5 = the number of boxes needed in E_5 after E_4, then $X_5 \sim Geo(2/6)$.

X_6 = the number of boxes needed in E_6 after E_5, then $X_6 \sim Geo(1/6)$.

Y = the number of boxes needed to collect all six action figures, then

$Y = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$.

So $E(Y) = E(X_1 + X_2 + X_3 + X_4 + X_5 + X_6) = E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5) + E(X_6)$

$= 1 + 6/5 + 6/4 + 6/3 + 6/2 + 6/1 = 14.7$.

3.4 Define the events: A: value 1 fails B: valve 2 fails C: valve 3 fails

$P(Y=2) = P(\overline{A} \cap \overline{B} \cap \overline{C}) = .8^3 = 0.512$

$P(Y=0) = P(A \cap (B \cup C)) = P(A)P(B \cup C) = .2(.2 + .2 - .2^2) = 0.072.$

Thus, $P(Y = 1) = 1 - .512 - .072 = 0.416$.

3.14 **a.** $\mu = E(Y) = 3(.03) + 4(.05) + 5(.07) + \ldots + 13(.01) = 7.9$

b. $\sigma^2 = V(Y) = E(Y^2) - [E(Y)]^2 = 3^2(.03) + 4^2(.05) + 5^2(.07) + \ldots + 13^2(.01) - 7.9^2 = 67.14 - 62.41 = 4.73$. So, $\sigma = 2.17$.

c. $(\mu - 2\sigma, \mu + 2\sigma) = (3.56, 12.24)$. So, $P(3.56 < Y < 12.24) = P(4 \leq Y \leq 12) = .05 + .07 + .10 + .14 + .20 + .18 + .12 + .07 + .03 = 0.96$.

3.21 Note that $E(N) = E(8\pi R^2) = 8\pi E(R^2)$. So, $E(R^2) = 21^2(.05) + 22^2(.20) + \ldots + 26^2(.05) = 549.1$. Therefore $E(N) = 8\pi(549.1) = 13{,}800.388$.

3.33 **a.** $E(aY+b) = E(aY) + E(b) = aE(Y) + b = a\mu + b.$

b. $V(aY+b) = E[(aY+b-a\mu-b)^2] = E[(aY-a\mu)^2] = a^2E[(Y-\mu)^2] = a^2\sigma^2.$

3.38 Note that Y is binomial with $n = 4$, $p = 1/3 = P$(judge chooses formula B).

a. $p(y) = \binom{4}{y}\left(\frac{1}{3}\right)^y\left(\frac{2}{3}\right)^{4-y}$, $y = 0, 1, 2, 3, 4$.

b. $P(Y \geq 3) = p(3) + p(4) = 8/81 + 1/81 = 9/81 = 1/9$.

c. $E(Y) = 4(1/3) = 4/3$.

d. $V(Y) = 4(1/3)(2/3) = 8/9$

3.56 Using expression for the mean and variance of Y = # of successful explorations, a binomial random variable with $n = 10$ and $p = .1$, $E(Y) = 10(.1) = 1$, and $V(Y) = 10(.1)(.9) = 0.9$.

3.57 If Y = # of successful explorations, then $10 - Y$ is the number of unsuccessful explorations. Hence, the cost C is given by $C = 20{,}000 + 30{,}000Y + 15{,}000(10 - Y)$. Therefore, $E(C) = 20{,}000 + 30{,}000(1) + 15{,}000(10 - 1) = \$185{,}000$.

3.67 $(.7)^4(.3) = 0.07203$.

3.80 Let Y = # of tosses until the first 6 appears, so Y has a geometric distribution. Using the result from Ex. 3.77,

$$P(B \text{ tosses first } 6) = P(Y = 2, 4, 6, \ldots) = 1 - P(Y = 1, 3, 5, \ldots) = 1 - p\frac{1}{1-q^2}.$$

Since $p = 1/6$, $P(B$ tosses first 6$) = 5/11$. Then,

$$\text{P(Y = 4 | B tosses the first 6)} \ \frac{P(Y=4)}{5/11} = \frac{(5/6)^2(1/6)}{5/11} = 275/1296.$$