Improving Recession Probability Forecasts in the U.S. Economy

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Abstract

There are two margins to improve forecasting models of the U.S. recession probability: including additional variables and using different functional form. Using out-of-sample and cross validation methods, I systematically compare the performance of various forecasting models that differ in terms of variables included and functional forms used. I find substantial gains from including additional variables, such as the S&P 500 and non-farm employment growth, together with the term spread. In addition, there is a room to further improve forecasting accuracy by utilizing a non-Normal cumulative distribution function. I also explore this possibility by using the generalized Edgeworth expansion. Resulting predictions outperform ones from a typical probit model in all three measures of forecasting accuracy considered.

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1 Introduction

There are many attempts to forecast recessions in the U.S. economy. Information contained in forward-looking variables can be used for making predictions about the future state of the economy. Stock and Watson (1989) report that the slope of the Treasury yield curve provides useful information in constructing their leading indicator. Estrella and Mishkin (1998) examine the usefulness of financial variables in making predictions about future recession probabilities at various forecasting horizons. They find that the slope of yield curve is the best single predictor of future recession probabilities.[1] Other studies also utilize the predictive content of the term spread in order to forecast recession probabilities.[2] Although developments in the literature achieve some success in making predicting recession, there is a room to improve forecasting accuracy.

To obtain further improvements, there are two margins that we can exploit. One is the question of selecting a set of predictor variables. The other is a choice of functional form. Typical studies on forecasting recession probabilities utilize a simple binary response model, such as a probit or logit model. In this paper, I will explore the importance of the two margins in improving forecasting accuracy.

Within the class of probit/logit models, there is favorable evidence for benefits from including additional information, other than the term spread. For example, Estrella and Mishkin (1998) find some evidence on usefulness of including an additional variable, such as a New York Stock Exchange index or GDP growth. Wright (2006) reports that including the level of the federal funds rate together with the term spread results in better forecasting performance than the term spread alone.[3] King, Levin, and Perli (2007) recently find superior predictive power of the 5-year corporate credit spread on AA-rated firms compared with the 10-year-3-month Treasury term spread over the Great Moderation period (since the mid 1980's). Furthermore, they also report that once the corporate spread is augmented by the term spread, the forecasting performance dramatically improves by reducing "false positive" predictions of recession.

There are also some attempts to extend a simple probit model in the recent literature. For ex-

[1] Their finding is consistent with a simple rule of thumb that the Treasury yield curve inversions or negative term spreads are likely followed by recessions in subsequent periods and the term spread has been cited as a leading economic indicator.

[2] Other studies include Dotsey (1998), Birchenhall, Jessen, Osborn, and Simpson (1999), Estrella, Rodrigues, and Schich (2003), Stock and Watson (2003) and Clements and Galvão (2006).

[3] He also finds some evidence favorable for controlling for a term premium proxy.

ample, Chauvet and Potter (2005) extend a probit model in a way that it allows for business-cycle dependent coefficients (i.e., multiple breaks) and/or autocorrelated errors. Kauppi and Saikkonen (2008) develop dynamic probit models that include lagged explanatory variables and lagged recessionary dummies. Dueker (2005) presents a framework (a Qual VAR) that enables us to treat a qualitative variable as endogenous in a typical VAR framework, so that we can obtain dynamic forecasts of the qualitative variable.

Going beyond the typical univariate probit model with the term spread, such as allowing additional features in a forecasting model and/or including additional predictor variables most likely would improve in-sample forecasting performance. However, at the same time, we also have to worry about a potential over-fitting problem. Hansen (2008) examines the pitfalls of relying on in-sample fit. He shows that too good in-sample fit (over-fitting) tends to be associated poor out-of-sample fit and that model selection based on in-sample fit is not reliable. In addition, he demonstrates that tendencies of yielding spurious results are much higher if we use in-sample evaluation and are more pronounced when we compare performance of a large number of alternative models. In this paper, therefore, I will use out-of-sample and cross validation methods to examine the importance of the two margins that can contribute to improve forecasting performance.

First, I systematically compare the forecasting performance of 6-month-ahead recession probability predictions in the U.S. economy. I look at all possible combinations of 33 variables up to trivariate models with 6 different functional forms. Results highlight the importance of variable selection. Finding the best combination of predictor variables greatly improve forecasting performance. Especially, among variables considered, the combination of the term spread, changes in the S&P 500 stock price index, and growth rate of non-farm employment is found to achieve the best forecasting accuracy. Furthermore, additional gains can be obtained by using non-Normal cumulative distribution functions. However, there is some mixed evidence on what features of functional form are necessary, in order to improve forecasting performance. Then, I proceed to allow for more flexibility in the functional form by utilizing the generalized Edgeworth expansion of Jarrow and Rudd (1982).

The rest of the paper is organized as follows. Section 2 presents basic binary response models and associated results. Section 3 introduces the generalized Edgeworth expansion and applies it to making predictions of the U.S. recession probability. Finally, Section 4 concludes.

2 Basic Models and Results

2.1 Recession Probability Forecasting Model

Let y_t represent an NBER recession binary variable, which equals 1 when the economy is in recession in month t and equals 0 in expansion. Typical models of forecasting h-period-ahead recession probabilities using the information available at time t assume that

$$\text{Prob}(y_{t+h} = 1|\boldsymbol{x}_t) = F(\boldsymbol{\beta}'\boldsymbol{x}_t), \tag{1}$$

where $F(\cdot)$ is a monotonically increasing function, whose range is the unit interval, $\boldsymbol{\beta}$ is a vector of coefficients associated with a vector of predictors $\boldsymbol{x}_t' = [1, x_{1,t}, \cdots, x_{k,t}]$, and k is the number of variables included.

It is commonly assumed that y_{t+h} is a conditionally independent Bernoulli random variable, so that the likelihood function is given by:

$$L = \prod_{t=1}^{T} \left[F(\boldsymbol{\beta}'\boldsymbol{x}_t)\right]^{y_{t+h}} \left[1 - F(\boldsymbol{\beta}'\boldsymbol{x}_t)\right]^{1-y_{t+h}}. \tag{2}$$

In an empirical analysis, I will set $h = 6$ and focus on 6-month-ahead predictions.

In this formulation, predicting recession probability involves two issues that are possibly related to each other. The first one is to choose a set of predictor variables, so that we can obtain useful signals from data. A workhorse predictor in the literature is the Treasury term spread between 10-year and 3-month bonds, which is due to the finding in Estrella and Mishkin (1998).

Given a choice of $F(\cdot)$, finding a better combination of predictor variables obviously helps improve forecasting accuracy if those variables contain different information and jointly provide useful signals. Since there are no *a priori* variable selection procedures available, our approach is to try all possible combinations of variables, in order to find a better combination of variables that are useful in forecasting recession probabilities. Although knowing the best single predictor is helpful, it is unlikely that we can obtain some insights about a better combination of variables by just looking at forecasting performance of single predictors because those variables that show relatively good forecasting performance tend to contain similar information.

The second issue is how to translate signals into a probability measure between 0 and 1, which is related to a shape of $F(\cdot)$. In order to guarantee that $F(\cdot)$ is monotonically increasing and takes values between 0 and 1, we typically use a known cumulative distribution function (CDF). A popular choice is to use either the Standard Normal CDF (a probit model) or the Logistic CDF (a logit model), or some variants of those (e.g., Chauvet and Potter, 2005; Kauppi and Saikkonen, 2008). However, the shape of $F(\cdot)$ is not necessarily restricted to typical ones. In principle, the CDF of any continuous probability random variable will suffice.

Differences in the shape of CDFs can be characterized in terms of "skewness" and "excess kurtosis". It should be noted that it is not appropriate to use terms skewness and excess kurtosis here because we are not talking about characteristics of underlying statistical distributions, but just utilizing functional forms. However, for expositional simplicity, I will use those terms in describing the shape of a CDF. The consequence of allowing skewness and excess kurtosis is as follows. Higher excess kurtosis makes a CDF steeper around the median of $\boldsymbol{\beta}'\boldsymbol{x}$. On the other hand, allowing non-zero skewness makes a CDF asymmetric around $F(\boldsymbol{\beta}'\boldsymbol{x}) = 0.5$. With zero skewness, $F(x) = 1 - F(-x)$ for any $x \in \mathbb{R}$. However, non-zero skewness implies $F(x) \neq 1 - F(-x)$.[4]

In order to understand what features of a CDF are helpful in improving forecasting accuracy, I will consider 6 different CDFs. Table 1 summarizes characteristics of the CDFs considered. In addition to the Standard Normal CDF and the Logistic CDF, I consider Student-t, Laplace, Gumbel, and Type III Generalized Extreme Value (GEV3). All location parameters and scale parameters, if applicable, are set to 0 and 1, respectively. These CDFs are chosen and configured to incorporate non-zero skewness and/or higher excess kurtosis that the Standard Normal CDF does not have. The first four CDFs have zero skewness and differ in terms of degree of excess kurtosis. The Logistic CDF has excess kurtosis of 1.2. The degrees of freedom parameter for Student-t is set to be 6.5, such that its excess kurtosis equals 2.4. The excess kurtosis of Laplace is 3. The last two CDFs also have positive skewness. For the Gumbel CDF, skewness is equal to 1.1395 and excess kurtosis is 2.4. The shape parameter of the Type III GEV, s, is set such that its excess kurtosis equals 3 (i.e., $s = -0.1732$). The associated skewness becomes 0.3492. Thus, for a given set of predictor variables, I will be able to infer the importance of allowing for excess kurtosis

[4] In general, changing location and scale parameters does not affect forecasting results and they are fixed to avoid an identification problem. Changing the location parameter just changes the estimate of the constant term and changing the variance just results in different scaling of coefficients $\boldsymbol{\beta}$.

Table 1: List of the Cumulative Distribution Functions Considered

Type of CDF	$F(x)$	Skewness	Excess Kurtosis
Standard Normal	$\frac{1}{\sqrt{2\pi}} \int_{\infty}^{x} \exp\left(\frac{-u^2}{2}\right) du$	0	0
Logistic	$\frac{\exp(x)}{1+\exp(x)}$	0	1.2
Student-t with $\nu = 6.5$	$\int_{-\infty}^{x} \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1+\frac{u^2}{\nu}\right)^{\frac{-(\nu+1)}{2}} du$	0	2.4
Laplace	$\frac{1}{2}\left[1+\mathrm{sgn}(x)\left\{1-\exp\left(-\lvert x\rvert\right)\right\}\right]$	0	3
Gumbel	$1-\exp\left\{-\exp(-x)\right\}$	1.1395	2.4
Type III GEV with $s = -0.1732$	$\exp\left\{-\left(1+sx\right)^{-1/s}\right\}$	0.3492	3

Note: All location parameters are set to be 0 and all scale parameters are set to be 1. ν represents degrees of freedom parameter for the Student-t distribution. $\Gamma(\cdot)$ is a gamma function. Excess kurtosis of Student-t is given by $6/(\nu-4)$ for $\nu > 4$. Skewness of the Gumbel distribution is $\frac{12\sqrt{6}\zeta(3)}{\pi^3}$, where $\zeta(\cdot)$ is a zeta function. s denotes a shape parameter of the generalized extreme value distribution. For the Type III GEV, skewness is given by $\frac{-\Gamma(1-3s)+3\Gamma(1-s)\Gamma(1-2s)-2(\Gamma(1-s))^3}{\left(\Gamma(1-2s)-(\Gamma(1-s))^2\right)^{3/2}}$ and excess kurtosis is give by $\frac{\Gamma(1-4s)-4\Gamma(1-s)\Gamma(1-3s)+6\Gamma(1-2s)(\Gamma(1-s))^2-3(\Gamma(1-s))^4}{\left(\Gamma(1-2s)-(\Gamma(1-s))^2\right)^2}$.

in $F(\cdot)$ by looking at results based on the first four CDFs. Furthermore, comparing the Gumbel with the Student-t or the Type III GEV with the Laplace enables us to obtain some insight on whether allowing positive skewness is helpful in improving forecasting performance.[5]

2.2 Data

Table 2 lists all 33 monthly variables considered in this paper. The sample period starts from April 1960, which corresponds to the beginning of the 1960-61 recession. It is chosen to maximize data availability and include as many recession episodes as possible. Since it is often the case that a decision of the NBER Business Cycle Dating Committee has a time lag,[6] the sample period is ended at the end of December, 2005. The data set contains the term spreads, the credit spreads, various

[5] It is also possible to introduce negative skewness by using the Type III GEV. However, preliminary estimation indicated that forecasting performance based on the Type III GEV with negative skewness is inferior to others in general. So, I do not report their results.

[6] A substantial time lag of the announcement might be one disadvantage of using the NBER recessions to characterize the state of the economy. This motivates Chauvet and Hamilton (2006) to study algorithms that enable us to construct a real-time business cycle turning point.

interest rates, employment data, stock price indices, monetary aggregates, and other macroeconomic variables. Most of those are investigated in the earlier studies or used in constructing a composite leading indicator. It also includes variables, to which the NBER Business Cycle Dating Committee pays particular attention in deciding business cycle peaks and troughs. Since Kane (2008) documents the usefulness of employment data in predicting occurrence of recessions, the data set includes employment-related variables as well.

Recently, King, Levin, and Perli (2007) report that, in the period of the Great Moderation, the credit spread on AA-rated firms has particularly good forecasting performance. However, the corporate bond yields used in their study (maturity of 5-year and 10-year) do not cover the entire sample. Since there are not many recessionary periods, especially after the 1980's, using shorter sample periods may have considerable effects on evaluating forecasting performance. Thus, I have decided not to include the credit spreads that King, Levin, and Perli (2007) have used. Instead, as a crude proxy, the data set include spreads between Moody's AAA-, AA-, or A- rated corporate bond yield (20 years or longer) and the 10-year Treasury bond yield.[7]

When we focus on financial variables as predictors, we do not need to consider a gap between when an observation is made and when it is available for forecasting. However, as shown in Table 2, some series are not reported immediately and we need to take account of the information lag, in order to accurately assess forecasting models. In this paper, any variable z_t represents the latest data on z available at month t, instead of an observation at month t. For example, industrial production (IP) has 1 month of the information lag. So, $\mathrm{IP}_{2000:01}$ refers to the industrial production data on December 1999.

Because the total number of models increases exponentially as the number of variables included (k) increases, I will restrict my attention to $k \leq 3$. This will result in examining a total of $\sum_{k=1}^{3} \frac{33!}{k!(33-k!)} \times 6 = 36102$ forecasting models.

[7]Since the 20-year Treasury bond yield data has discontinuity between January 1987 and September 1993, I use the 10-year T-bond rate, instead. Thus, precisely speaking, this "credit spread" is a combination of the "true" credit spread and the term spread.

Table 2: List of Variables

Predictor	Description	Info. Lag
Interest Rates		0
FF	Federal Funds rate	0
3M	3-month Treasury Bill rate	0
5Y	5-year Treasury Bond rate	0
10Y	10-year Treasury Bond rate	0
AAA	Moody's corporate bond yield, AAA 20 years or longer	0
AA	Moody's corporate bond yield, AA 20 years or longer	0
A	Moody's corporate bond yield, A 20 years or longer	0
Term Spreads		
TS10YFF	10Y-FF Treasury term spread	0
TS10Y3M	10Y-3M Treasury term spread	0
TS10Y5Y	10Y-5Y Treasury term spread	0
Credit Spreads		
CSAAA	AAA - 10Y spread	0
CSAA	AA - 10Y spread	0
CSA	A - 10Y spread	0
Employment Data		
EMP	Non-agricultural employment (log-differenced)	0
CEMP	Civilian employment (log-differenced)	0
UICLAIM	Initial unemployment insurance claims (log-differenced)	1
UNEMP	Unemployment rate	0
UNEMPD	Changes in unemployment rate	0
HOURS	Average weekly hours in manufacturing (log-differenced)	0
Stock Price Indices		
DJ30	Dow Jones 30 average (% changes over 3 months)	0
SP500	S&P 500 stock price index (% changes over 3 months)	0
Monetary Aggregates		
M0	Monetary base (log-differenced)	1
M1	M1 (log-differenced)	1
M2	M2 (log-differenced)	2
Other Macroeconomic Variables		
CLI11	Composite leading indicators (11 series, 1987=100, log-differenced)	1
CPI	CPI, all urban, all items (log-differenced)	1
EXP	Consumer expectation (1966.1 = 100)	0
EXPD	Changes in consumer expectation	0
HOUSE	New private housing units authorized by building permits (log-differenced)	1
VENDOR	Vendor performance (slower deliveries diffusion index, %)	0
INCOME	Personal income less transfer payments (log-differenced)	2
IP	Industrial production (log-differenced)	1
SALES	Manufacturing & trade sales (log-differenced)	1

Note: Information lag is measured at the end of month. Strictly speaking, those employment data with zero information lag and vendor performance are not available at the end of the month. However, they will be available at the very beginning of the next month and there are virtually no considerable lags. Thus, they are categorized in the zero information lag variable.

2.3 Evaluating Forecasting Performance

In order to evaluate various forecasting models, I will primarily focus on out-of-sample results. Hansen (2008) shows that in-sample and out-of-sample fits are negatively correlated, which implies that good in-sample performance is not a useful indicator of out-of-sample accuracy and that relying on in-sample fit is highly misleading. This over-fitting problem is particularly important, since the likelihood of obtaining spurious results is more pronounced when we search a large number of alternative models. For this reason, I will focus on recursive out-of-sample forecasting evaluation and I will use cross validation as a robustness check.

For recursive (pseudo) out-of-sample forecasting exercises, the out-of-sample prediction starts from January 1989 and ends at the end of the full sample. The out-of-sample period covers the last two recessions in the U.S. economy. By adding observations one by one, I estimate a forecasting model again to produce a forecast for the next month. In reality, when we forecast future recession probability, we are not certain about the true state of the economy in recent months. It is because the decision of the NBER Business Cycle Dating Committee typically involves substantial time lag. To be realistic, I assume that forecasters do not know the true state of the economy for a year and assume that $y_t = \cdots = y_{t-11} = y_{t-12}$, that is, the economy is in the same state as a year ago.

Following Clements and Galvão (2006), I will use three different measures for evaluating accuracy of out-of-sample recession probability predictions. The first measure is the probability-analogue of mean squared error, the quadratic probability score (QPS), which is commonly used in evaluating probability forecasts. The QPS is defined as

$$QPS = \frac{2}{T}\sum_{t=1}^{T}(\hat{p}_t - y_t)^2, \tag{3}$$

where $\hat{p}_t$ is the recession probability forecast month t. The QPS takes values between 0 and 2 and smaller value indicates more accurate forecasts.

The second measure is the log probability score (LPS), which is defined as

$$LPS = -\frac{1}{T}\sum_{t=1}^{T}\big\{y_t \log(\hat{p}_t) + (1 - y_t)\log(1 - \hat{p}_t)\big\}. \tag{4}$$

The LPS ranges from 0 to $+\infty$ and a smaller value corresponds to more accurate predictions and penalizes larger mistakes more heavily than the QPS.

The last measure is the Kuipers Score (KS), which is given by

$$\begin{aligned} KS &= \frac{\sum_{t=1}^{T} y_t 1_{(\hat{p}_t > 0.5)}}{\sum_{t=1}^{T} y_t} - \frac{\sum_{t=1}^{T} (1 - y_t) 1_{(\hat{p}_t > 0.5)}}{\sum_{t=1}^{T} (1 - y_t)}, \\ &= \text{hit rate} - \text{false rate}, \end{aligned} \tag{5}$$

where $1_{(\cdot)}$ is an indicator function that equals 1 if its argument is true and 0 otherwise. The KS calculates the difference between the hit rate and the rate of false signals by using 50% probability of recession as a cutoff. The KS takes values between -1 and 1. A score of 1 corresponds to making perfect predictions. The KS evaluates the recession predictions from a slightly different aspect, compared with other two measures. Even when recession predictions never show "strong" indications (say, higher than 50% probability), it is possible to have seemingly good results based on the QPS and LPS. The KS discounts such "weak" predictions. In this sense, the KS captures the strength and accuracy of predictions by using the 50% probability cutoff.

There is a potential problem of just relying on the recursive out-of-sample exercises described above, especially in the context of recession probability forecasting. Since there are not many recessions in the out-of-sample period, it is possible to select a forecasting model that has particularly good performance for the last two recessions, but not for other recessions or a next recession.

In order to robustify the results, I will also carry out a cross-validation type exercise, called leaving 2-years out. The detailed procedures of the leaving 2-years out exercises are as follows. Let $S = \{(y_t, \boldsymbol{x}_{t-h}) : t = 1, \cdots, T\}$ denote a full sample and $L_\tau = \{(y_t, \boldsymbol{x}_{t-h}) : t = \tau - 12, \cdots, \tau + 12\}$ represent a set of excluding observations. For each $\tau = 13, \cdots, T - 12$,

(i) Take $E_\tau = S \setminus L_\tau$ as a training sample.

(ii) Estimate parameter values $\boldsymbol{\beta}_\tau$ based on E_τ.

(iii) Make a prediction for y_τ by using $\boldsymbol{\beta}_\tau$ and $\boldsymbol{x}_{\tau-h}$ and store it.

(iv) Repeat steps (i) – (iii).

Then calculate QPS, LPS, and KS based on $\{\hat{y}_\tau\}_{\tau=13}^{T-12}$.

2.4 Results

2.4.1 Univariate Probit Models

First, we will start off by looking at forecasting performance within a class of univariate probit models of predicting 6-month-ahead recession probabilities ($h = 6$) as a benchmark. Table 3 summarizes the out-of-sample forecasting performance of univariate probit models.

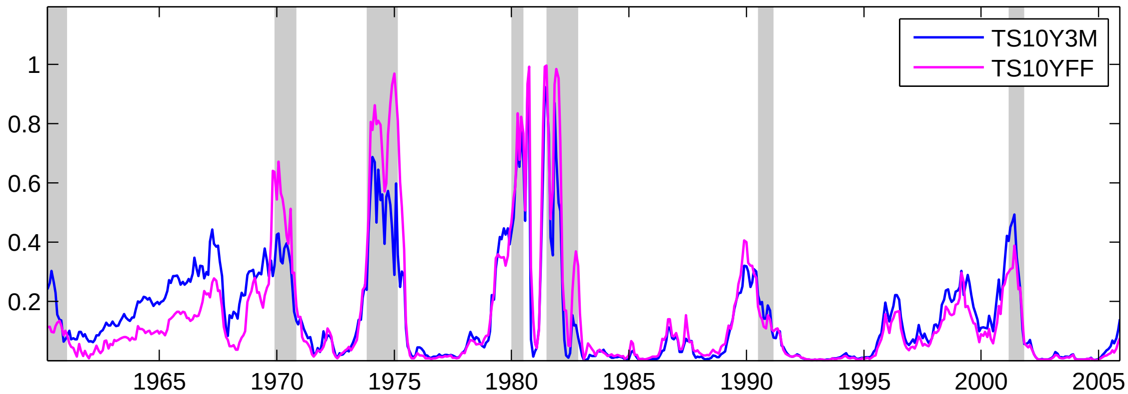
The rankings of out-of-sample forecasting accuracy based on the QPS and the LPS have similar patterns, with a few exceptions. The term spread between 10-year and 3-month Treasury bonds (TS10Y3M), which is commonly used, and the term spread between 10-year bond and the Federal Funds rate (TS10YFF) are the best predictors based on the QPS and LPS, respectively. However, the term spread between 10-year and 5-year bonds (TS10Y5Y) has poorer out-of-sample performance in the both measures. In contrast to the term spread, the credit spreads (CSA, CSAA, and CSAAA) have poor forecasting performance. Among other variables, the levels of interest rates (FF and 3M) and the composite leading indicators (CLI11) might be useful predictors as well. However, employment related variables, those variables that the NBER Business Cycle Dating Committee is paying attention to, and monetary aggregates have considerably poorer results.

It is interesting to point out that, in terms of quadratic loss, the conventional term spread (TS10Y3M) outperforms the one based on the Federal Funds rate. However, if we penalize larger mistakes more heavily, then the out-of-sample forecasting performance improves by using TS10YFF. That is, TS10Y3M minimizes the quadratic loss, but it results some larger mistakes. The same thing is true for the levels of FF and 3M.

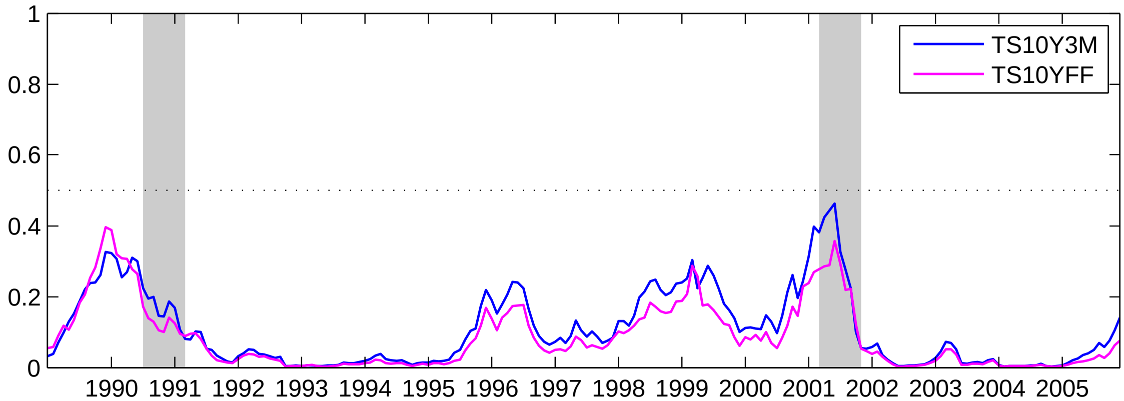
The out-of-sample forecasting performance evaluated by using the KS gives us a completely different picture. Most predictor variables have non-positive values for the KS. In other words, they usually do not give us strong predictions about the occurrence of future recessions or correct predictions are largely offset by false predictions. In the worst case, we will get more false signals, which indicate more than 50% probability of a recession during an expansion, than correct ones. The only variable that has a positive value for the KS is the growth rate of M2. However, other measures of forecasting performance suggest that the performance of M2 is one of the poorest, among predictor variables considered. The opposite can also happen. Although CLI11 has relatively

Table 3: Variable Rankings with Univariate Probit Models

QPS Ranking		LPS Ranking		KS Ranking	
TS10Y3M	0.1399	TS10YFF	0.2396	M2	0.0179
TS10YFF	0.1444	TS10Y3M	0.2401	TS10YFF	0.0000
3M	0.1488	FF	0.2607	TS10Y3M	0.0000
FF	0.1503	3M	0.2637	FF	0.0000
CLI11	0.1541	CLI11	0.2733	3M	0.0000
CPI	0.1579	SP500	0.2877	CPI	0.0000
5Y	0.1610	CPI	0.2964	5Y	0.0000
SP500	0.1627	5Y	0.2993	10Y	0.0000
10Y	0.1650	10Y	0.3081	A	0.0000
A	0.1663	A	0.3094	AAA	0.0000
AAA	0.1665	AAA	0.3098	AA	0.0000
AA	0.1669	AA	0.3108	UICLAIM	0.0000
UICLAIM	0.1695	UICLAIM	0.3131	EXPD	0.0000
CSAAA	0.1712	DJ30	0.3146	IP	0.0000
TS10Y5Y	0.1725	EXPD	0.3216	UNEMPD	0.0000
EXPD	0.1733	IP	0.3227	SALES	0.0000
IP	0.1736	UNEMPD	0.3237	UNEMP	0.0000
CSAA	0.1740	TS10Y5Y	0.3241	EMP	0.0000
UNEMPD	0.1742	SALES	0.3277	HOURS	0.0000
UNEMP	0.1759	UNEMP	0.3284	M0	0.0000
HOURS	0.1765	EMP	0.3284	CEMP	0.0000
M0	0.1767	HOURS	0.3292	HOUSE	0.0000
DJ30	0.1768	M0	0.3302	VENDOR	0.0000
CEMP	0.1769	CEMP	0.3303	M1	0.0000
SALES	0.1775	HOUSE	0.3325	CSA	0.0000
EMP	0.1778	VENDOR	0.3349	CSAA	0.0000
HOUSE	0.1782	EXP	0.3362	CSAAA	0.0000
VENDOR	0.1791	INCOME	0.3480	CLI11	-0.0054
CSA	0.1808	M1	0.3494	EXP	-0.0054
EXP	0.1854	CSA	0.3511	INCOME	-0.0108
INCOME	0.1923	CSAA	0.3694	DJ30	-0.0161
M1	0.1927	M2	0.4214	TS10Y5Y	-0.0215
M2	0.2498	CSAAA	0.4261	SP500	-0.0323



(a) In-Sample Fit



(b) Out-of-Sample Prediction

Figure 1: Predictions with Univariate Probit Models

Note: The shaded areas represent the NBER recessions. Horizontal axes measures recession probabilities.

good performance based on other measures, it has a negative value for the KS.

Figure 1 presents in-sample (i.e., the entire sample) and out-of-sample 6-month-ahead predictions from univariate probit models with TS10YFF and TS10Y3M. The shaded areas indicate the recession periods. The top panel shows in-sample fit. As can be seen, the term spreads well capture recessions and expansions in early periods, especially those in the 1970's and 1980's. However, it seems that they do not provide strong predictions for the last two recessions.[8] Particularly, for the 1990-91 recession, the predictions made by the term spreads miss the timing of the recession and

[8]In conjunction with the Great Moderation, some researchers find instability of the predictive power of the term spread (for example, Chauvet and Potter, 2005). However, Estrella, Rodrigues, and Schich (2003) do not find evidence of a structural break in the binary response model.

Table 4: Top 20 Univariate Models

QPS Ranking			LPS Ranking			KS Ranking		
CDF	x_1	QPS	CDF	x_1	LPS	CDF	x_1	KS
Laplace	TS10Y3M	0.1360	Gumbel	TS10YFF	0.2362	Laplace	TS10Y3M	0.1111
Student-t	TS10Y3M	0.1388	GEV3	TS10YFF	0.2370	Laplace	CLI11	0.0448
Logistic	TS10Y3M	0.1388	Laplace	TS10Y3M	0.2392	Laplace	SP500	0.0233
Normal	TS10Y3M	0.1399	Normal	TS10YFF	0.2396	GEV3	SP500	0.0233
GEV3	TS10Y3M	0.1410	Normal	TS10Y3M	0.2401	Gumbel	SP500	0.0233
Gumbel	TS10Y3M	0.1421	GEV3	TS10Y3M	0.2401	Normal	M2	0.0179
Normal	TS10YFF	0.1444	Logistic	TS10Y3M	0.2406	Gumbel	TS10YFF	0.0000
GEV3	TS10YFF	0.1446	Gumbel	TS10Y3M	0.2408	GEV3	TS10YFF	0.0000
Logistic	TS10YFF	0.1453	Student-t	TS10Y3M	0.2410	Normal	TS10YFF	0.0000
Student-t	TS10YFF	0.1455	Logistic	TS10YFF	0.2430	Normal	TS10Y3M	0.0000
Gumbel	TS10YFF	0.1459	Student-t	TS10YFF	0.2436	GEV3	TS10Y3M	0.0000
Laplace	TS10YFF	0.1459	Laplace	TS10YFF	0.2463	Logistic	TS10Y3M	0.0000
Gumbel	3M	0.1482	Gumbel	FF	0.2564	Gumbel	TS10Y3M	0.0000
GEV3	3M	0.1484	GEV3	FF	0.2582	Student-t	TS10Y3M	0.0000
Normal	3M	0.1488	Normal	FF	0.2607	Logistic	TS10YFF	0.0000
Gumbel	FF	0.1491	Gumbel	3M	0.2615	Student-t	TS10YFF	0.0000
Logistic	3M	0.1494	GEV3	3M	0.2623	Laplace	TS10YFF	0.0000
Student-t	3M	0.1495	Normal	3M	0.2637	Gumbel	FF	0.0000
GEV3	FF	0.1496	Logistic	FF	0.2640	GEV3	FF	0.0000
Normal	FF	0.1503	Student-t	FF	0.2644	Normal	FF	0.0000

they do not have strong signals. This difficulty is also noted in other studies.[9]

The same is true for the out-of-sample forecasts, which are shown at the bottom panel of Figure 1. Although both the QPS and LPS select TS10YFF and TS10Y3M as the first two univariate probit models, actual predictions are relatively poor and never exceed 50%. Furthermore, as in the in-sample case, out-of-sample predictions also miss the timing of the 1990-91 recession. This illustrates importance of paying attention to the KS, according to which both models score zero.

2.4.2 Univariate Models

Now we will look at the performance of univariate models based on alternative CDFs, which allow positive skewness and/or excess kurtosis. Table 4 summarizes the ranking of univariate models based on out-of-sample fit.

In terms of the QPS, as in the probit model, models with TS10Y3M provide better out-of-sample forecasting accuracy than those with TS10YFF. For TS10Y3M, there is a clear indication

[9]See Stock and Watson (2003) for a more detailed discussion.

of a better prediction for CDFs with excess kurtosis and the Laplace CDF achieves the best QPS. On the other hand, allowing for skewness deteriorates forecasting accuracy, such as GEV3 and Gumbel. However, such a pattern does not hold for TS10YFF and, in fact, the Normal CDF outperforms other CDFs among univariate models with TS10YFF.

If we penalize larger mistakes more heavily than the QPS, the variable ranking based on the LPS suggests some roles played by allowing skewness and excess kurtosis. For TS10YFF, allowing positive skewness and excess kurtosis contributes to reducing large mistakes, compared with the Normal. However, just incorporating excess kurtosis, but not skewness, appears to perform worse than the Normal CDF and also than univariate models with TS10Y3M. Although, within the probit models, TS10Y3M is inferior to TS10YFF, using the Laplace CDF improves the LPS and achieves a better result than TS10YFF with the Normal. It is possible to obtain better forecasting performance by employing non-Normal CDFs. However, it is not clear what additional features of the functional form are helpful and it seems to be variable specific.

The rankings based on the QPS and LPS suggest that information contained in 3M helps reduce quadratic loss, whereas information contained in FF tends to reduce larger mistakes.

In general, most of predictors do not make correct and strong predictions, in terms of the 50% probability cutoff. Only 6 models have positive values for the KS. Especially, the Laplace with TS10Y3M has superior out-of-sample forecasting performance, compared with other univariate models. Although based on other measures, including skewness and/or excess kurtosis improves out-of-sample fit slightly, using the Laplace CDF for TS10Y3M makes significant difference in the KS (increasing from 0 to 0.1111). It is also important to point out that other predictor variables that do not appear in the other rankings are ranked relatively higher in this KS ranking, namely CLI11, SP500 and M2.

2.4.3 Bivariate Models

Now we look at forecasting results based on bivariate models, in order to see the importance of additional information. Tables 5 – 7 show the top 20 bivariate forecasting models based on the QPS, LPS, and KS, respectively.

The combination of TS10Y3M and SP500 is the best predictor variables based on the QPS

Table 5: Top 20 Bivariate Models by the QPS

CDF	x_1	x_2	QPS
Laplace	TS10Y3M	SP500	0.1144
Student-t	TS10Y3M	SP500	0.1144
Logistic	TS10Y3M	SP500	0.1144
Normal	TS10Y3M	SP500	0.1151
GEV3	TS10Y3M	SP500	0.1160
Gumbel	TS10Y3M	SP500	0.1173
GEV3	TS10YFF	SP500	0.1226
Student-t	TS10YFF	SP500	0.1228
Logistic	TS10YFF	SP500	0.1228
Gumbel	TS10YFF	SP500	0.1230
Normal	TS10YFF	SP500	0.1230
Laplace	TS10YFF	SP500	0.1236
Laplace	TS10Y3M	EMP	0.1254
Logistic	TS10Y3M	EMP	0.1268
Student-t	TS10Y3M	EMP	0.1268
Normal	TS10Y3M	EMP	0.1277
GEV3	TS10Y3M	EMP	0.1283
Gumbel	3M	SP500	0.1290
Gumbel	TS10Y3M	EMP	0.1295
Gumbel	FF	SP500	0.1298

Table 6: Top 20 Bivariate Models by the LPS

CDF	x_1	x_2	LPS
GEV3	TS10Y3M	SP500	0.1984
Normal	TS10Y3M	SP500	0.1985
Gumbel	TS10Y3M	SP500	0.1993
Logistic	TS10Y3M	SP500	0.1994
Student-t	TS10Y3M	SP500	0.1997
Laplace	TS10Y3M	SP500	0.2020
Gumbel	TS10YFF	SP500	0.2027
GEV3	TS10YFF	SP500	0.2037
Normal	TS10YFF	SP500	0.2064
Logistic	TS10YFF	SP500	0.2084
Student-t	TS10YFF	SP500	0.2087
Laplace	TS10YFF	SP500	0.2112
Gumbel	FF	SP500	0.2113
Gumbel	3M	SP500	0.2129
GEV3	FF	SP500	0.2146
GEV3	3M	SP500	0.2149
GEV3	TS10Y3M	EMP	0.2152
Gumbel	TS10Y3M	EMP	0.2154
Normal	TS10Y3M	EMP	0.2166
Logistic	TS10Y3M	EMP	0.2172

and LPS. All three measures of out-of-sample accuracy suggest that it is important to include a measure of term spread (either TS10Y3M or TS10YFF). Interestingly, there are some variables that have relatively poor performance in univariate models and produce good out-of-sample forecasting accuracy, *together with* the term spread measure. According to the rankings based on the univariate models, SP500 and EMP are not a useful single predictor. Especially, EMP has relatively poor out-of-sample forecasting performance. However, they become an important companion variable to the term spread. In fact, those variables with the term spread are better than a combination of two term spreads (TS10Y3M and TS10YFF), which are in the best two variables among univariate models. This suggests that those variables contain some useful information that the term spread does not have and also indicates that the univariate ranking is not a helpful guide for choosing multiple predictors.

The importance of SP500 together with TS10Y3M is consistent with the finding of Estrella and Mishkin (1998). In our bivariate models, even without being combined with the term spread, some bivariate models that contain SP500 perform relatively well. However, it should be mentioned that King, Levin, and Perli (2007) do not find superiority of SP500 in conjunction with the term spread. Rather, they report that a combination of variables, which have better performance in univariate

Table 7: Top 20 Bivariate Models by the KS

CDF	x_1	x_2	KS
Laplace	TS10Y3M	EMP	0.2724
GEV3	TS10Y3M	SP500	0.2115
Normal	TS10Y3M	SP500	0.2115
Gumbel	TS10Y3M	SP500	0.2115
Logistic	TS10Y3M	SP500	0.2115
Student-t	TS10Y3M	SP500	0.2115
Laplace	TS10Y3M	SP500	0.2115
Laplace	UICLAIM	SP500	0.1900
Laplace	TS10Y3M	EXPD	0.1667
Laplace	TS10Y3M	VENDOR	0.1667
Laplace	TS10Y3M	CSA	0.1613
Laplace	TS10Y3M	M1	0.1559
Gumbel	UICLAIM	SP500	0.1344
GEV3	UICLAIM	SP500	0.1344
Normal	UICLAIM	SP500	0.1344
Student-t	UICLAIM	SP500	0.1344
Logistic	UICLAIM	SP500	0.1344
Laplace	TS10Y3M	M2	0.1344
Gumbel	EMP	SP500	0.1290
Laplace	IP	SP500	0.1290

models, also perform better in bivariate models.[10]

Unlike the univariate models, combining SP500 with TS10Y3M always achieves the better performance both in terms of the QPS and LPS than those models with TS10YFF. For the combination of TS10Y3M and SP500, allowing only excess kurtosis helps improve the QPS, while including both skewness and excess kurtosis contributes to improving the LPS and KS. Allowing both skewness and excess kurtosis, in general, improves the LPS by reducing large false predictions. However, there is some mixed evidence for the QPS in terms of the role of skewness and excess kurtosis. At least, for the models with TS10Y3M, it seems better to allow only excess kurtosis, but not positive skewness.

Although forecasting performance generally improves by adding one more predictor, probably the biggest gain appears in the KS. In univariate models, only 6 models out of 198 have positive scores. In bivariate models, 348 models out of 3168 have at least positive values for the KS. Furthermore, the best model (TS10Y3M and EMP with the Laplace) scores more than twice as

[10]This could be because of a couple of reasons. First, it could be attributed to the difference in periods used for the out-of-sample forecasting exercises and forecasting horizon. Second, it might be because of the fact that my data set does not include the credit spread measures that they use and perform quite well in their univariate models. Finally, it could be due to the difference in evaluating out-of-sample forecasting performance. They look at average out-of-sample predictions over two test periods, the 2001 recession and the post-2001 expansion.

Table 8: Top 20 Models by the QPS

CDF	x_1	x_2	x_3	QPS
Laplace	TS10Y3M	SP500	EMP	0.1051
Logistic	TS10Y3M	SP500	EMP	0.1074
Student-t	TS10Y3M	SP500	EMP	0.1074
Student-t	TS10Y3M	SP500	A	0.1079
Laplace	TS10Y3M	SP500	A	0.1080
Logistic	TS10Y3M	SP500	A	0.1081
Student-t	TS10Y3M	SP500	AAA	0.1084
Logistic	TS10Y3M	SP500	AAA	0.1086
Student-t	TS10Y3M	SP500	AA	0.1087
Logistic	TS10Y3M	SP500	AA	0.1088
Laplace	TS10Y3M	SP500	AAA	0.1088
Normal	TS10Y3M	SP500	EMP	0.1089
Laplace	TS10Y3M	SP500	AA	0.1090
GEV3	TS10Y3M	SP500	A	0.1092
Normal	TS10Y3M	SP500	A	0.1093
Laplace	TS10Y3M	SP500	IP	0.1093
GEV3	TS10Y3M	SP500	AAA	0.1098
Normal	TS10Y3M	SP500	AAA	0.1098
GEV3	TS10Y3M	SP500	AA	0.1100
GEV3	TS10Y3M	SP500	EMP	0.1100

Table 9: Top 20 Models by the LPS

CDF	x_1	x_2	x_3	LPS
GEV3	TS10Y3M	SP500	A	0.1832
Gumbel	TS10Y3M	SP500	A	0.1834
GEV3	TS10Y3M	SP500	AAA	0.1843
GEV3	TS10Y3M	SP500	AA	0.1845
Gumbel	TS10Y3M	SP500	AAA	0.1847
Gumbel	TS10Y3M	SP500	AA	0.1848
Gumbel	5Y	SP500	3M	0.1848
Student-t	TS10Y3M	SP500	A	0.1852
Logistic	TS10Y3M	SP500	A	0.1852
Normal	TS10Y3M	SP500	A	0.1853
GEV3	TS10Y3M	SP500	EMP	0.1857
GEV3	5Y	SP500	3M	0.1857
Gumbel	TS10Y3M	SP500	10Y	0.1858
Gumbel	TS10Y3M	SP500	3M	0.1858
Gumbel	10Y	SP500	3M	0.1858
Gumbel	TS10Y3M	SP500	EMP	0.1859
Student-t	TS10Y3M	SP500	AAA	0.1861
Logistic	TS10Y3M	SP500	AAA	0.1861
Normal	TS10Y3M	SP500	AAA	0.1861
GEV3	TS10Y3M	SP500	10Y	0.1862

high as the best univariate model.

2.4.4 Trivariate Models and Overall Rankings

Now we turn our attention to the overall forecasting performance, including all of univariate, bivariate, and trivariate models with 6 different CDFs. Tables 8 – 10 list the top 20 forecasting models out of 36102 forecasting models, based on different criteria.

The Laplace with TS10Y3M, SP500, and EMP is the best forecasting model based on the QPS and KS. Regardless of the functional form used, this variable combination records relatively good out-of-sample performance. This is consistent with the bivariate results that SP500 and EMP perform relatively better in conjunction with the term spread measure.

However, the combination of TS10Y3M, SP500, and EMP tend to make larger prediction errors, compared to the combination that includes corporate bond yields on A-rated firms (A), instead of EMP. The combination of TS10Y3M, SP500, and A is the best in terms of the LPS. It is also the second best in the QPS ranking. However, this variable combination does not result in good

Table 10: Top 20 Models by the KS

CDF	x_1	x_2	x_3	KS
Laplace	TS10Y3M	SP500	EMP	0.3835
Normal	TS10Y3M	SP500	EMP	0.3333
Logistic	TS10Y3M	SP500	EMP	0.3280
Student-t	TS10Y3M	SP500	EMP	0.3280
Logistic	TS10Y3M	SP500	IP	0.3280
Student-t	TS10Y3M	SP500	IP	0.3280
Laplace	TS10Y3M	SP500	IP	0.3280
GEV3	TS10Y3M	SP500	EMP	0.3172
Laplace	UNEMP	SP500	A	0.3172
Laplace	UNEMP	SP500	AA	0.3172
Gumbel	TS10Y3M	SP500	EMP	0.3118
GEV3	TS10Y3M	M2	CSA	0.2957
Gumbel	TS10Y3M	M2	CSA	0.2957
Laplace	TS10Y3M	EMP		0.2724
GEV3	TS10Y3M	SP500	UNEMPD	0.2724
Normal	TS10Y3M	SP500	UNEMPD	0.2724
Logistic	TS10Y3M	SP500	UNEMPD	0.2724
Gumbel	TS10Y3M	SP500	UNEMPD	0.2724
Student-t	TS10Y3M	SP500	UNEMPD	0.2724
Laplace	TS10Y3M	SP500	UNEMPD	0.2724

performance in terms of the KS.

It is worthwhile to point out the importance of TS10Y3M and SP500, regardless of the forecasting accuracy measures. Thus, improving out-of-sample forecasting accuracy could be a task of choosing a good companion variable to this combination. In this sense, other corporate bond yields (AA and AAA) are also good companion variables. Although in the bivariate models, the term spread measure TS10YFF was one of the best predictors together with SP500, augmenting the combination of TS10Y3M and SP500 with other variables outperforms the TS10YFF counterparts.

Except for the combination of TS10Y3M, SP500, and EMP, the variable combinations that are ranked relatively high in the KS ranking do not have good out-of-sample forecasting performance in terms of the QPS or LPS. In other words, making strong predictions during recessions involves risks of making positive false signals during expansions, which worsens the QPS and LPS. In out-of-sample forecasting exercises, there is some favorable evidence for allowing excess kurtosis to improve the QPS and KS, whereas adding positive skewness could improve forecasting accuracy in terms of the LPS.

Table 11: Importance of the Two Margins

Models	QPS			LPS			KS		
	Normal	Laplace	GEV3	Normal	Laplace	GEV3	Normal	Laplace	GEV3
Univariate	0.1399	0.1360	0.1410	0.2401	0.2392	0.2401	0.0000	0.1111	0.0000
Bivariate	0.1151	0.1144	0.1160	0.1985	0.2020	0.1984	0.2115	0.2115	0.2115
Trivariate	0.1089	0.1051	0.1100	0.1868	0.1870	0.1857	0.3333	0.3835	0.3172

Note: The univariate model includes TS10Y3M. The bivariate model refers to models with TS10Y3M and SP500 and the trivariate models add EMP to the bivariate models.

2.4.5 Discussion

Forecasting performance based on all of models indicates that the term spread measure (TS10Y3M) is one of the most important variables, as widely documented in other studies. However, there are two margins to improve forecasting performance. One is to include additional predictors. By combining different information, we can increase accuracy of recession probability predictions. As demonstrated, a particular combination of variables greatly outperforms a typical univariate probit model with TS10Y3M. Based on the forecasting performance of bivariate models, the importance of the stock price index (SP500) is emerged as a companion variable to the term spread measure. In addition, among the all combinations of variables considered in this paper, augmenting the term spread measure by SP500 and EMP results in the best out-of-sample forecasting performance. Another margin is to change a functional form of $F(\cdot)$, in order to improve how to translate signals to a recession probability measure.

It is important to understand how these two margins contribute to better forecasting performance. Table 11 summarizes how forecasting performance improves by changing the two margins and illustrates different effects of the margins. We compare probit models with the Laplace and GEV3 counterparts. In general, including additional predictors has greater impact on improving all three out-of-sample forecasting measures. However, it is possible to obtain non-negligible gains from changing the functional form.

For example, starting from the univariate probit model with TS10Y3M, provided that we know SP500 as the best companion variable to the term spread, adding SP500 drastically reduces the QPS and LPS by 17.7% and 17.3%, respectively. Furthermore, the KS increases from 0 to 0.2115. Using the Laplace in the univariate model moderately improves the QPS and results in a large gain in the KS (from 0 to 0.1111), whereas it deteriorates the LPS.

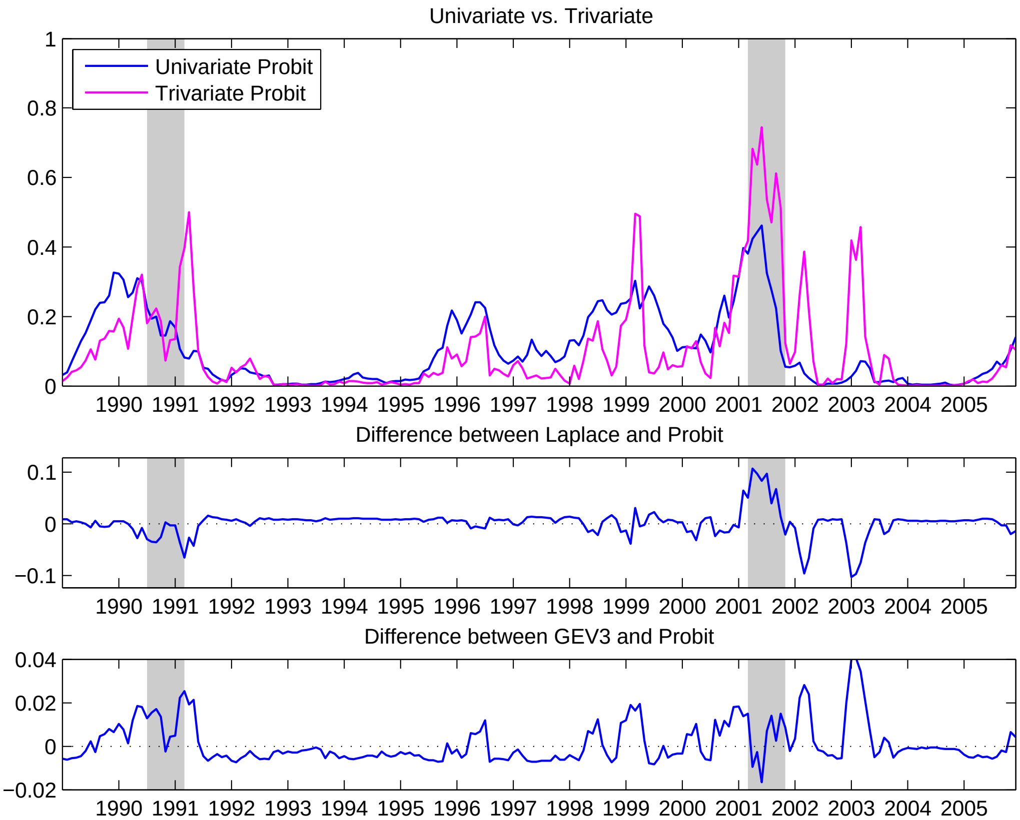


Figure 2: Comparison of Out-of-Sample Predictions

Note: Shaded areas indicate the NBER recession episodes. The top panel compares predictions from the univariate probit model with TS10Y3M with those from the trivariate models with TS10Y3M, SP500, and EMP. The middle and bottom panels show differences in predictions by changing the functional form. The middle panel plots the difference between the trivariate Laplace and the trivariate probit and the bottom panel depicts the difference between the trivariate GEV3 and the trivariate probit.

Although marginal gains become smaller, adding the third variable (EMP) further improves the out-of-sample forecasting performance of the probit model. Using the Laplace improves the QPS and KS of the trivariate model. While only allowing for the excess kurtosis (Laplace) cannot improve the LPS, allowing both skewness and excess kurtosis (GEV3) can improve the LPS, compared with the probit model, however. In sum, the QPS and KS can be improved by using the Laplace, whereas using the GEV3 can improve the LPS.

The top panel of Figure 2 compares out-of-sample predictions from the univariate probit model with the term spread with the trivariate probit model (TS10Y3M, SP500, and EMP). As can be seen, including additional variables reduces forecasting errors during expansions. Especially, predictions from the univariate model suggest moderate possibilities of recessions during the last half of the 1990's, when the economy was in fact in the longest expansion. Those positive predictions are dampened by including additional variables. At the same time, it contributes to making stronger predictions during recessions, especially for the last recession. However, in exchange for such improvements, the trivariate model also contains a couple of relatively strong false positive signals during expansions, which are not present in the univariate model.

The middle panel of Figure 2 plots differences in predictions between the trivariate Laplace model and the trivariate probit model. This suggests some benefits from changing functional form. Using the Laplace, instead of the Normal, will amplify positive signals during the 2001 recession and also will dampen strong false positive signals that happen right after the 1990-91 recession and before and after the 2001 recession. In general, using the Laplace will contribute to amplifying strong signals and to dampening weak signals, so that on average there will be less forecasting errors. However, such a feature works in an unfavorable way during the 1990-91 recession, since the underlying predictions are somewhat weak and miss the correct timing.

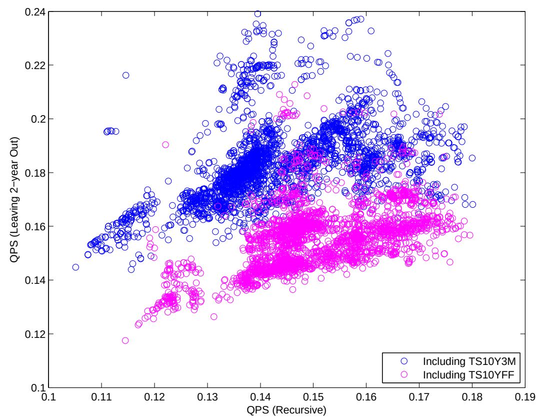
The bottom panel of Figure 2 plots differences in predictions between the trivariate GEV3 model and the trivariate probit model. Overall, differences between the GEV3 model and the probit model are somewhat small. Unlike the Laplace model, using the GEV3 improves predictions for the 1990-91 recession. However, early periods of the 2001 recession have weaker predictions than the probit model. Although using the GEV3 will reduce false positive predictions during expansions, strong false positive signals are, in fact, amplified.

Although there are some trade-offs in using different functional forms, as indicated in Table 11, additional features in a forecasting model can improve out-of-sample forecasts.

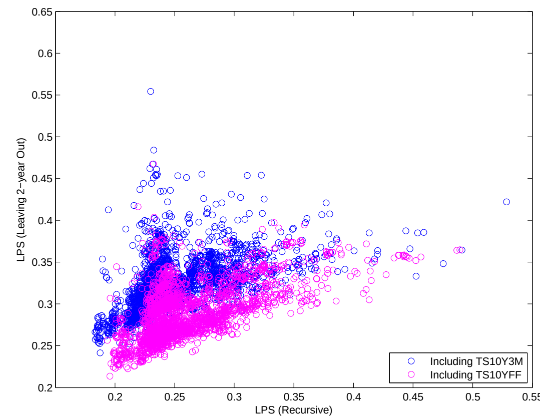
2.4.6 Robustness Check

As a robustness check, I also evaluate forecasting performance of all models using the leaving 2-years out exercises. Differences in the results that the leaving 2-years out typically selects TS10YFF as a measure of a term spread, instead of TS10Y3M. Figure 3 compares forecasting performance of models including TS10Y3M and TS10YFF in the both exercises.

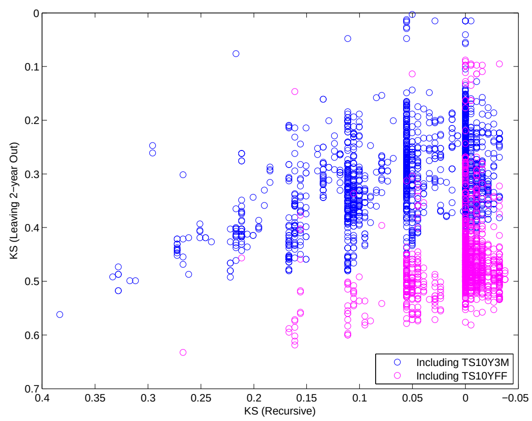
In general, those models that include TS10Y3M work relatively better in the out-of-sample



(a) QPS



(b) LPS

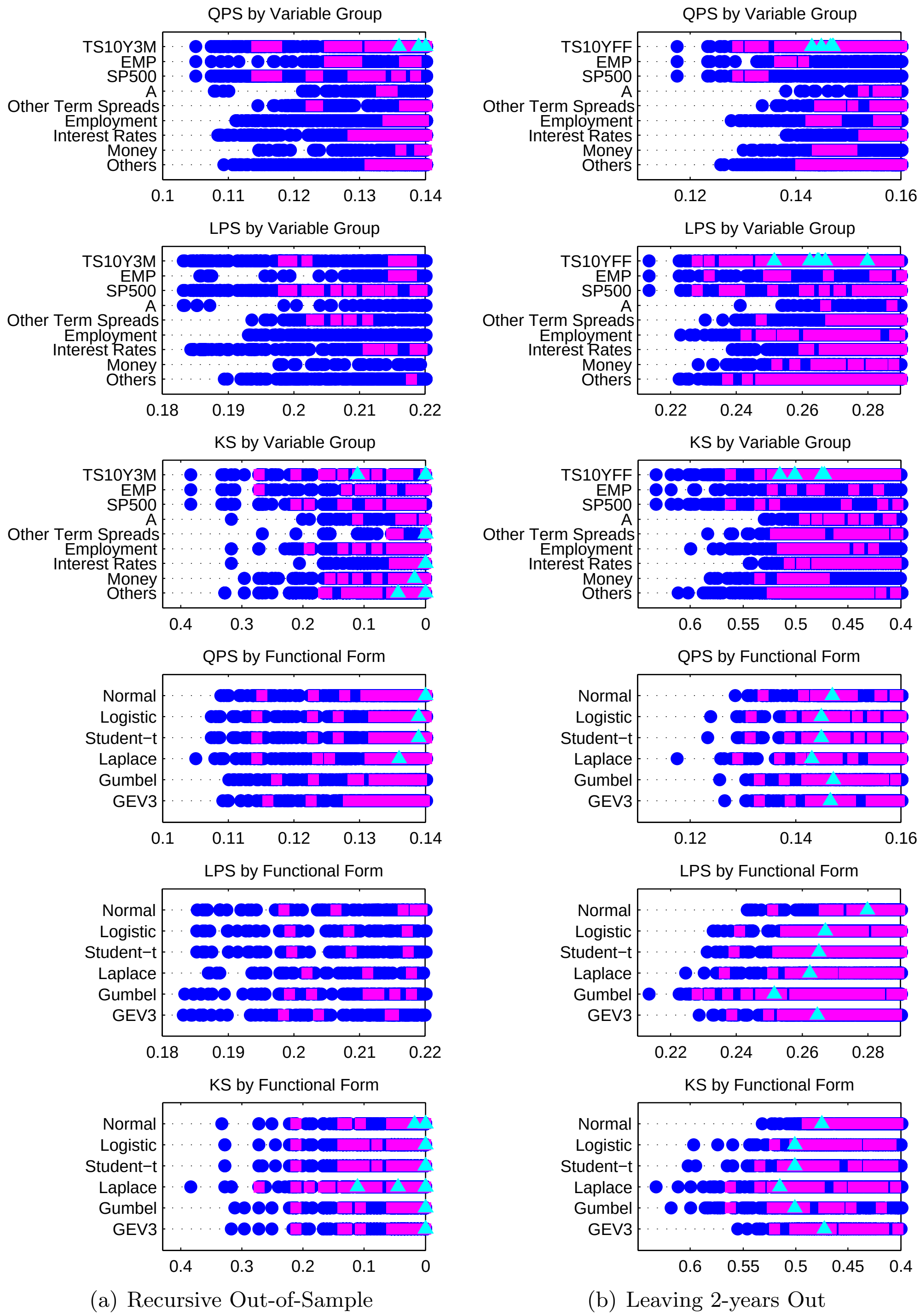


(c) KS

Figure 3: Recursive Out-of-Sample vs. Leaving 2-years Out

Note: Horizontal axes take forecasting accuracy measures based on the recursive out-of-sample forecasting. Vertical axes represent those based on the leaving 2-years out exercises. Other models are not plotted here since their performance is inferior to those include the term spread measures.

forecasting exercises, whereas those with TS10YFF have better performance in the leaving 2-years out exercises. A primary reason is that models with TS10YFF make smaller prediction errors in the earlier periods, which are included in the leaving 2-years out exercises and are not covered in the recursive out-of-sample forecasting exercises. As shown in the top panel of Figure 1, after the 1980's, both term spreads show similar predictions. However, in earlier periods, predictions based on TS10Y3M are associated with larger forecasting errors. Although it is clear that including a measure of a term spread is important, relative performance could depend on which term spread is used.



(a) Recursive Out-of-Sample (b) Leaving 2-years Out

Figure 4: Visual Comparison of Forecasting Accuracy: Recursive vs. Leaving 2-years Out

Note: The left panels show forecasting accuracy based on the recursive out-of-sample predictions. The right panels display the based on the leaving 2-years out exercises. Within each exercise, the top three panels and bottom three panels summarize results by variable groups and by functional forms, respectively. Circles represent trivariate models. Squares indicate bivariate models and triangles are univariate models. Horizontal axes for the KS is in the reverse order, such that better models appear on the left.

Regardless of the choice of the term spread measure, the importance of SP500 and EMP is quite robust in both exercises. Figure 4 graphically compares forecasting accuracy based on the recursive out-of-sample predictions with the one based on the leaving 2-years out. The left panels of Figure 4 summarize forecasting performance based on the recursive out-of-sample predictions and the right panels plot the one based on the leaving 2-years out. Within each exercise, the top three panels categorize three measures of forecasting accuracy by variable groups and the bottom three panels are based on functional forms. Each panel of Figure 4 covers roughly the top 10% forecasting models. For the top three panels, markers associated with a particular value of the out-of-sample accuracy measure (horizontal axis) indicate that the corresponding forecasting model contains variables listed on the vertical axis. For the bottom three panels, the vertical axis indicates which functional form is used.

With the leaving 2-years out exercises, the variable combination of TS10YFF, SP500, and EMP achieves the best results in all three measures of forecasting accuracy. Although the out-of-sample forecasting results suggest that it is helpful to use the corporate bond yield on A-rated firms (A) to improve the LPS, the results based on the leaving 2-years out do not support the importance of A. Despite its good performance in the out-of-sample predictions that covers the last two recessions, the leaving 2-years out results suggest the predictability of A together with the term spread and SP500 might be somewhat fragile.

In both exercises, there are not many trivariate models that outperform the best bivariate model. However, there are non-trivial gains from including the third predictor variable. In terms of the functional form, it appears that the Laplace CDF contributes to improving the QPS and the KS in both out-of-sample and leaving 2-years out exercises, regardless of the number of variables included. On the other hand, there is favorable evidence for introducing both skewness and excess, in order to improve the LPS.

3 Allowing More Flexible Skewness and Excess Kurtosis

3.1 Generalized Edgeworth Expansion

Results in the previous section suggest some benefits of allowing for positive skewness or excess kurtosis, in order to improve recession probability forecasts. However, there is some mixed evidence on what features in the functional form are important, compared with a typical probit model. It is possibly because of the fact that I just examine a relatively restrictive set of combinations of skewness and excess kurtosis. In this section, I will examine the effect of allowing for more flexibility in the functional form.

In particular, I will allow for more flexibility in skewness and excess kurtosis. In order to do so, I will utilize the generalized Edgeworth series expansion, which is introduced by Jarrow and Rudd (1982).[11] The generalized Edgeworth expansion (GEE) will approximate the "true" distribution $F(s)$ by using a series expansion involving higher moments of the approximating distribution $A(s)$. I assume that both $F(s)$ and $A(s)$ are continuous, that is,

$$\frac{dF(s)}{ds} = f(s) \quad \text{and} \quad \frac{dA(s)}{ds} = a(s) \tag{6}$$

exist. Furthermore, I assume that the moments of $F(s)$, $\alpha_j(F)$, exist for $j \leq n$. Given $\alpha_n(F)$ exists, the first $n-1$ cumulants $\kappa_j(F)$ from $j = 1, \cdots, n-1$ also exist. Jarrow and Rudd (1982) show that

$$f(s) \approx a(s) + \sum_{j=1}^{N} E_j \frac{(-1)^j}{j!} \frac{d^j a(s)}{ds^j}, \tag{7}$$

where E_j is a function of the cumulants of $F(s)$ and $A(s)$ up to the jth cumulant, which will be given below. By taking integral of (7), the "true" $F(x)$ can be approximated by:

$$F(x) \approx \int_{\infty}^{x} a(s)ds + \int_{\infty}^{x} \left[\sum_{j=1}^{N} E_j \frac{(-1)^j}{j!} \frac{d^j a(s)}{ds^j} \right] ds. \tag{8}$$

Since I am interested in having more flexibility in skewness and excess kurtosis, it is sufficient to set $N = 4$. The third and fourth cumulants are related to skewness and excess kurtosis, respectively, as

$$\kappa_3 = \gamma_1 (\kappa_2)^{3/2} \qquad \text{and} \qquad \kappa_4 = \gamma_2 (\kappa_2)^2, \tag{9}$$

where γ_1 and γ_2 denote skewness and excess kurtosis, respectively, and κ_2 denotes the 2nd cumulant

[11] They use the generalized Edgeworth expansion to study the problem of option valuation, where the underlying security distribution is unknown.

which corresponds to the variance. Then, E_j for $j = 1, \cdots, 4$ are given by:

$$E_1 = \kappa_1(F) - \kappa_1(A), \tag{10}$$

$$E_2 = \kappa_2(F) - \kappa_2(A) + E_1^2, \tag{11}$$

$$E_3 = \kappa_3(F) - \kappa_3(A) + 3E_1\left(\kappa_2(F) - \kappa_2(A)\right) + E_1^3, \tag{12}$$

$$\begin{aligned} E_4 = {} & \kappa_4(F) - \kappa_4(A) + 4\left(\kappa_3(F) - \kappa_3(A)\right)E_1 \\ & + 3\left(\kappa_2(F) - \kappa_2(A)\right)^2 + 6E_1^2\left(\kappa_2(F) - \kappa_2(A)\right) + E_1^4. \end{aligned} \tag{13}$$

Given a choice of the approximating distribution $A(x)$ (i.e., $\kappa_j(A)$ for $j = 1, \cdots, 4$), E_j is a function of the cumulants of $F(x)$ from $\kappa_1(F)$ to $\kappa_j(F)$. For my purpose of using this approximation in the binary response model, location and scale parameters are needed to be fixed for normalization. Otherwise, the coefficient vector $\boldsymbol{\beta}$ cannot be uniquely estimated. Without loss of generality, I assume that $F(x)$ have zero mean and unit variance and also assume that $A(x)$ has zero mean, so that $\kappa_1(F) = \kappa_1(A)$ and $E_1 = 0$. Then the approximation equation becomes:

$$\widetilde{F}(x) \equiv A(x) + \frac{E_2}{2!}\int_{\infty}^{x} \frac{d^2 a(s)}{ds^2} ds - \frac{E_3}{3!}\int_{\infty}^{x} \frac{d^3 a(s)}{ds^3} ds + \frac{E_4}{4!}\int_{\infty}^{x} \frac{d^4 a(s)}{ds^4} ds. \tag{14}$$

Although it is easily shown that $\lim_{x\to\infty} \widetilde{F}(x) = 1$ and $\lim_{x\to-\infty} \widetilde{F}(x) = 0$, there is no guarantee that $\widetilde{F}(x)$ is monotonically increasing. In order for (14) being monotonically increasing, I need to impose a condition

$$a(x) + \sum_{j=2}^{4} E_j \frac{(-1)^j}{j!}\frac{d^j a(s)}{ds^j} \geq 0 \qquad \text{for all } x \in \mathbb{R}. \tag{15}$$

This condition (15) determines admissible combinations of skewness and excess kurtosis of $\widetilde{F}(x)$. This admissible set depends on a choice of $A(x)$. In order to cover all CDFs considered in the previous section, I will use the Logistic distribution whose mean equals zero and variance is 1.2 as the approximating distribution $A(x)$. Figure 5 illustrates the admissible set of (γ_1, γ_2) with the Logistic distribution, together with skewness and excess kurtosis of other CDFs.[12] Since I cannot estimate the variance of $A(x)$ due to an identification problem, I will restrict my search of skewness

[12]Basically, the bottom corner of the admissible set depicted in Figure 5 moves up and the angle of the bottom corner becomes smaller as the variance increases. For example, with unit variance the area is slightly below the skewness and excess kurtosis of the Gumbel and with larger variance the admissible set passes by the Gumbel's combination.

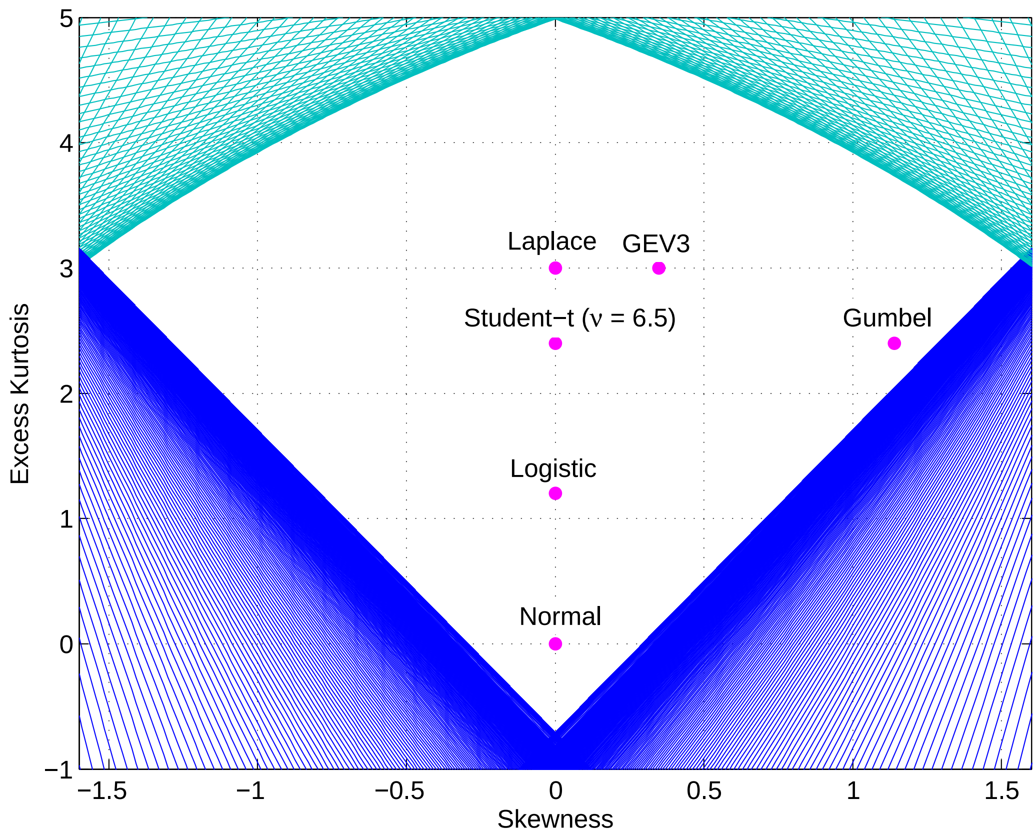


Figure 5: Admissible Set of Skewness and Excess Kurtosis with the Logistic ($\mu = 0, \sigma^2 = 1.2$)

Note: The white area bounded by lines approximates the admissible combinations of skewness and excess kurtosis that satisfy (15) when the Logistic with mean =0 and $\sigma^2 = 1.2$ is used as the approximating distribution. This is drawn by discretizing x.

and excess kurtosis to combinations within this admissible set in Figure 5.

3.2 The GEE Results

I will use (14) for $F(\cdot)$ in (1). Since E_3 and E_4 are functions of γ_1 and γ_2, respectively, there are 2 additional parameters to be estimated. The exact expression of (14) is presented in the Appendix. This is a parsimonious way to cover a relatively large space of skewness and excess kurtosis.[13]

By using the GEE, I will estimate two trivariate models that are found to be the best forecasting

[13]Instead of using the GEE, we could use more complicated CDFs that can cover a much wider set of skewness and excess kurtosis, such as the one based on the skewed generalized t distribution of Theodossiou (1998). See Hansen, McDonald, and Theodossiou (2007) for other flexible distributions. However, there are basically two problems. First, those functions have more parameters and moments are complicated functions of those parameters, so that it is relatively difficult to control the identification issue in the binary response model. Second, it is often the case that there are no closed form CDFs available, which could increase computational burden.

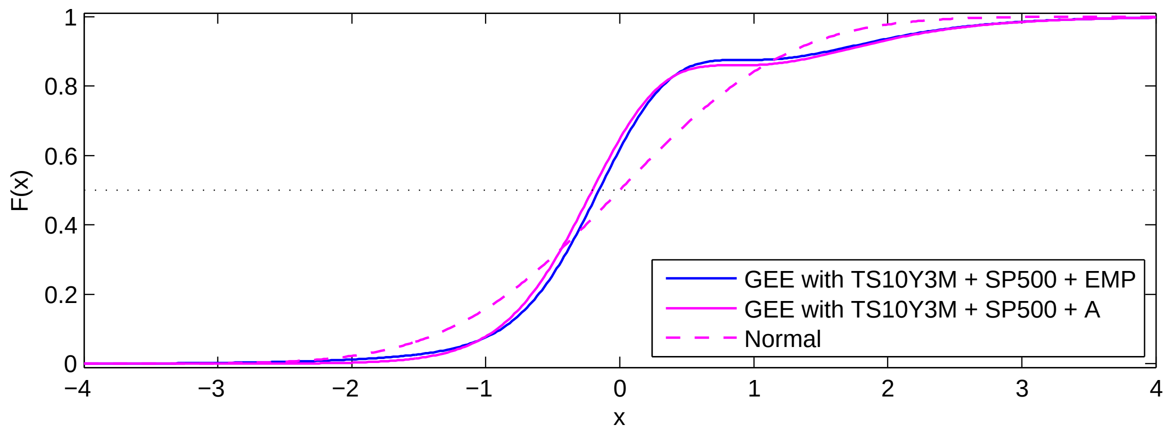


Figure 6: Comparison of Functional Forms

Table 12: Out-of-Sample Forecasting Performance with the Generalized Edgeworth Expansion

Variables	CDF	QPS	LPS	KS
TS10Y3M + SP500 + EMP	GEE	0.1067 [0.1401]	0.1852 [0.2358]	0.4229 [0.5898]
	Normal	0.1089 [0.1486]	0.1868 [0.2560]	0.3333 [0.4925]
	Laplace	0.1051 [0.1420]	0.1870 [0.2488]	0.3835 [0.5629]
	GEV3	0.1100 [0.1486]	0.1857 [0.2510]	0.3172 [0.5000]
TS10Y3M + SP500 + A	GEE	0.1059 [0.1412]	0.1818 [0.2357]	0.1667 [0.5075]
	Normal	0.1093 [0.1516]	0.1853 [0.2772]	0.1667 [0.4599]
	Laplace	0.1080 [0.1498]	0.1872 [0.2656]	0.1667 [0.4815]
	GEV3	0.1092 [0.1487]	0.1832 [0.2603]	0.1667 [0.4555]

Note: Results from the Normal, Laplace, and GEV3 counterparts from the previous section are reproduced for comparison. Values reported in brackets are the corresponding values obtained from the leaving 2-years out exercises.

models in the out-of-sample forecasting exercises. These two trivariate models include TS10Y3M and SP500. As the third variable, one includes EMP, which resulted in the best QPS and KS, and the other includes A, which resulted in the best LPS.

Figure 6 graphically compares functional forms estimated by the GEE with the Normal CDF. In-sample estimates of γ_1 and γ_2 based on the variable combination of TS10Y3M, SP500, and EMP are 1.2486 and 3.5795, respectively. Those estimates using TS10Y3M, SP500, and A, are 1.5588 and 3.0864, respectively. The GEE estimates from the both trivariate models suggest to include both positive skewness and excess kurtosis. The GEE makes smaller predictions around $x = -1$. While the GEE predictions will be more aggressive than the probit ones at somewhere between $x = -0.5$ and 0.5, its prediction becomes modest around $x > 1$, given the same information.

Table 12 reports performance of predictions made by using the GEE. Regarding the out-of-sample forecasting performance, the GEE models always outperform the probit counterparts.[14] Furthermore, the GEE models almost always have better out-of-sample forecasting performance than other models, except for the QPS with the model that includes EMP. In the previous section, there are no single non-Normal models that outperform the Normal counterpart for the variable combination of TS10Y3M, SP500, and EMP. For example, with TS10Y3M, SP500, and EMP, the Laplace model is better than the probit model in the QPS and KS, but not in the LPS. The GEV3 model is better in the LPS, but not in other two. However, the GEE model in fact beats the probit model in all three measures of out-of-sample forecasting performance.

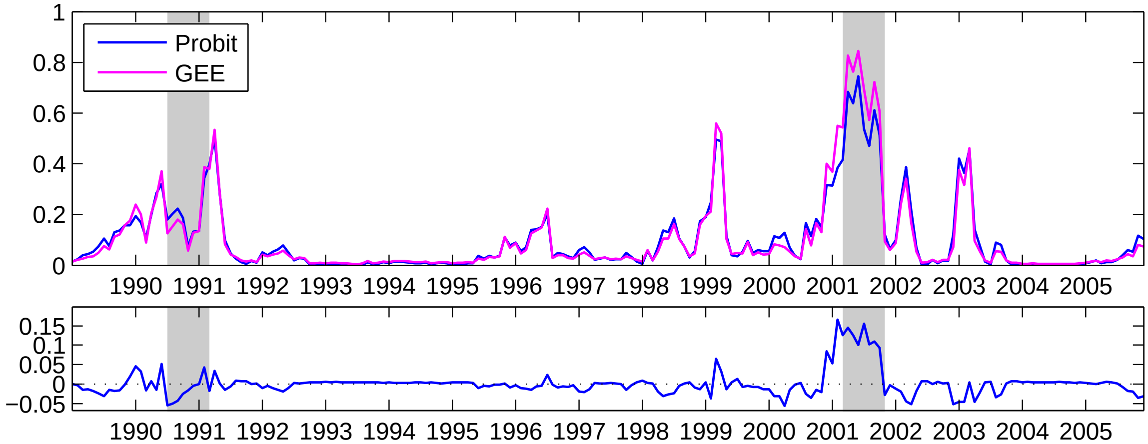
In addition to the recursive out-of-sample forecasting exercises, I also perform the leaving 2-years out exercises as a robustness check. Table 12 also reports results from the leaving 2-years out exercises. As in the recursive out-of-sample exercises, results based on the leaving 2-years out exercises also confirm the superiority of the GEE model over other models.

Figure 7 compares predictions made by the GEE models with those from the probit models. Predictions from the GEE models amplify the positive signals for the 2001 recession. Furthermore, false positive signals that arise from the trivariate model in 1999 and 2003 are effectively dampened by the GEE. However, for the 1990-91 recession, as in other models, the GEE models do not indicate occurrence of the recession correctly. This suggests that allowing flexibility in the functional form will effectively improve recession probability forecasts. Although there is a possibility to make false predictions worse, there are net gains in forecasting accuracy from using the GEE. Using more flexible functional form can be combined with other extensions to probit models, in order to achieve further improvements in forecasting accuracy.

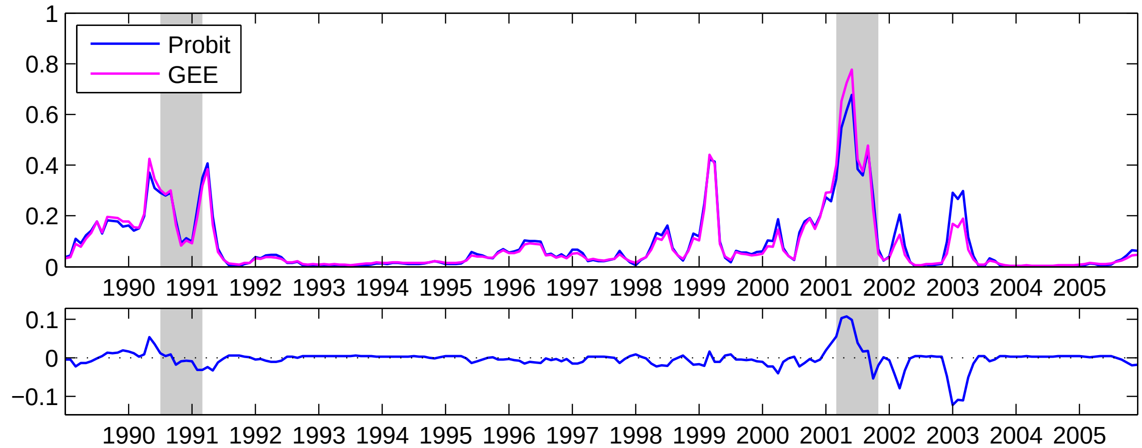
3.3 Current Predictions by the GEE

Results based on the known CDFs suggest to use the forecasting model that includes TS10Y3M, SP500, and EMP, together with the Laplace CDF, which is robust both in out-of-sample and leaving 2-years out forecasting exercises. As we have seen in this section, we can obtain further gains in forecasting accuracy, by using the GEE. It may be of interest to use the GEE model to infer the

[14] Although the value of the KS for the model with A, improvements in both the QPS and LPS conditional on the same value of the KS imply that the GEE model is superior than the probit counterpart.



(a) Trivariate Model with TS10Y3M, SP500, and EMP



(b) Trivariate Model with TS10Y3M, SP500, and A

Figure 7: Out-of-Sample Predictions by the GEE

recent state of the U.S. economy.

Figure 8 shows current predictions of the U.S. recession probabilities that are beyond the sample period used in this paper.[15] Predictions made by the GEE suggest moderate slowdown in the U.S. economy starting from the late 2006. However, they do not indicate strong signals of recession. Although we are still uncertain about the state of the economy in this time period, it is likely that there was no recession during 2006 and 2007 if we apply a conventional rule of thumb that more than two consecutive quarters of negative GDP growth are regarded as a recession. Thus, the predictions from the GEE models are perhaps consistent with the state of the economy in the last two years.

[15]Parameter estimates based on the data up to 2008:03 are reported in the Appendix.

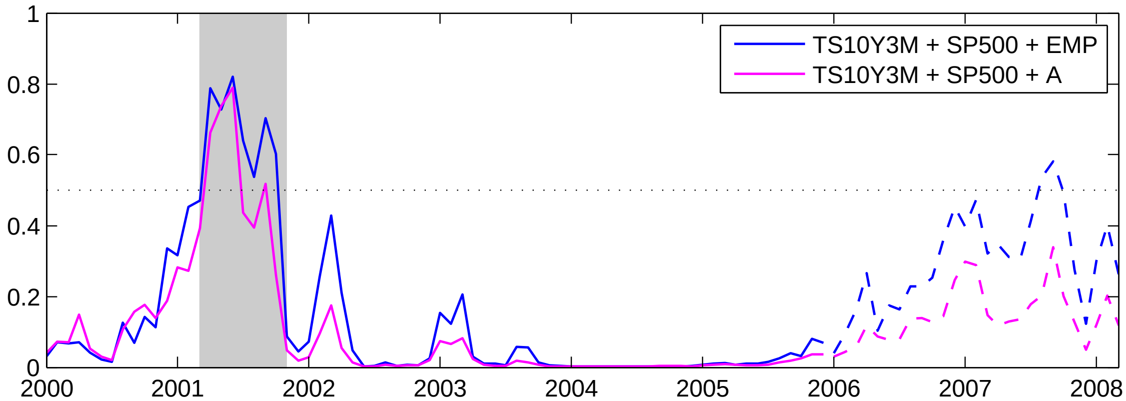


Figure 8: Current Predictions by the GEE

Note: Solid lines represent in-sample probabilities and dashed lines indicate recursively calculated current predictions.

4 Conclusion

Systematically comparing forecasting models of recession probability in the U.S. economy, I have found substantial gains in out-of-sample forecasting accuracy from including additional variables in a typical univariate probit model with the term spread. Furthermore, there is a room to further improve forecasting performance by utilizing non-Normal CDFs, which allow for positive skewness and/or excess kurtosis. Among 36102 forecasting models considered, the combination of the term spread between 10-year and 3-month Treasury bonds, S&P 500, and non-farm employment growth with the Laplace CDF records the best out-of-sample result, in terms of the QPS and KS. On the other hand, the combination of the term spread, S&P 500, and the corporate bond yield on A-rated firms achieves the best out-of-sample LPS. Especially, its superior forecasting performance of S&P 500 and employment growth, together with a term spread measure is robust in the out-of-sample and leaving 2-years out exercises. Although it is always better to use non-Normal CDF, there is some mixed evidence on a better functional form. These models do not always beat the probit counterparts in all three measures of out-of-sample accuracy.

In order to allow for more flexibility in the functional form, I have applied the generalized Edgeworth expansion and I have found that predictions from the GEE are always superior to the probit counterparts. In principle, we can amplify correct predictions and dampen false signals by allowing for more flexible functional form.

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Appendix

A Detailed Expression for (14)

Let ς denote a scale parameter of the Logistic distribution, such that its variance equals $\frac{\pi^2}{3}\varsigma^2$, and also let $z = x/\varsigma$. Then, the GEE model (14) can be expressed as:

$$\begin{aligned}\widetilde{F}(x) = \frac{\exp(z)}{1+\exp(z)} &+ \frac{0.1\exp(z)\left\{-1+\exp(z)\right\}}{\varsigma^2\left\{1+\exp(z)\right\}^3} + \frac{\gamma_1}{6}\frac{\exp(z)\left\{-1+4\exp(z)-\exp(2z)\right\}}{\varsigma^3\left\{1+\exp(z)\right\}^4} \\ &- \left(\frac{\gamma_2}{24} - 0.067\right)\frac{\exp(z)\left\{-1+11\left(\exp(z)-\exp(2z)\right)+\exp(3z)\right\}}{\varsigma^4\left\{1+\exp(z)\right\}^5} \end{aligned} \tag{16}$$

B Parameter Estimates based on the Recent Data

Parameter estimates based on the data from 1961:04 to 2008:03 are presented in Table 13. The first two columns are those from the GEE models. For reference, the last two columns also report estimates based on the Laplace and GEV3, respectively.

Table 13: Parameter Estimates based on the Recent Data

	GEE		Laplace	GEV3
Constant	−0.1821	−1.2764	−0.1443	−2.2979
TS10Y3M	−0.5257	−0.4704	−1.2593	−0.7643
SP500	−0.0426	−0.0445	−0.0997	−0.0774
EMP	−1.3923		−3.2104	
A		0.1061		0.2467
γ_1	1.5470	1.5670	n.a.	n.a.
γ_2	3.1069	3.0722	n.a.	n.a.

Note: Estimates are based on the data from 1961:04 to 2008:03. γ_1 and γ_2 denote skewness and excess kurtosis in the GEE model, respectively.