# A Competitive Theory of Equilibrium Mismatch

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**Abstract**

This paper studies why unemployment and vacancies coexist in an Arrow-Debreu equilibrium model in which jobs and workers are assigned to heterogeneous markets. Markets are segmented and jobs and workers are assigned before knowing if trade will take place in a given market. Job assignments to markets where trade is uncertain represent vacancies. Unemployment is of two kinds. Some workers search across markets whereas others waited in markets were trade was expected but not realized. The Welfare Theorems are established and it is shown that, in response to productivity shocks, the calibrated model traces out a Beveridge curve –a negative co-movement between unemployment and vacancies.

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# 1 Introduction

A central concern in business cycles is the behavior of the labor market. Not only is the labor market one of the largest sectors of the economy but most of the output change in modern business cycles is accounted for by changes in the labor input. While the importance of the labor market is widely acknowledged, most of the literature that studies labor market mismatch –why worker unemployment and job vacancies coexist– considers the labor market in isolation and assumes the existence of an "aggregate matching function." The purpose of this paper is to provide a simple, yet general, theoretical framework for examining the business cycle fluctuations of unemployment and vacancies in a setting in which the aggregate matching function is *microfounded* and prices and quantities are simultaneously determined in an competitive *Arrow-Debreu equilibrium*.

The model presented here proposes one possible microeconomic foundation for mismatch based on *trading frictions* inspired by the work of Prescott [48], Butters [10], Gale [23], and Eden [18].[1] In the model, jobs and workers are assigned to heterogeneous and segmented markets but trading in some of these markets is uncertain.[2] This gives rise to a market equilibrium in which unemployed workers and vacant jobs are associated with optimal assignments to markets where profitable trade is expected but not realized.

[1] In Prescott [48], sellers of motel rooms set prices before they know how many buyers will appear. (In Butters [10], sellers send price offers to potential costumers.) When the realization of demand is low, capacity is low and rooms will be vacant. In the paper, we follow the competitive version of the uncertain and sequential trade model by Eden [18]. (See also Eden [20].) The model is also related to Gale [23]'s analysis of trading uncertainty in competitive economies.

[2] The role of job assignments has a long tradition in economics but analyses in the labor market have focused mainly on understanding the determinants of the structure of wages and the distribution of earnings. (Sattinger [51] provides a survey of assignment models in the labor market. See also Akerlof [1] and Lucas [38] for models of the labor market.) Additional examples of assignment models include Becker [7], Koopmans and Beckmann [33], Shapley and Shubik [53], and Shimer [54]. A general description of nonatomic assignment models can be found in Grestky et al. [25].

The following example describes the workings of the model. Consider two markets. Trading in the first market is certain while trading in the second one is uncertain. If the second market opens, trading will occur separately from trade in the first market. Workers can move between locations but assignments take place before uncertainty is fully resolved (although not necessarily before the beginning of the trading process as typically considered in Arrow-Debreu markets). Not all jobs will be assigned to the first market under the possibility that the second market will open. Workers might also speculate on higher-paying jobs in the second market. This speculation is essential for mismatch because some jobs and some workers will be *waiting* for the second market to open. If this market opens, the market will clear and there will be no unused factors. If the market does not open, these jobs will remain vacant and these workers will be unemployed.[3]

Typical examples of segmented markets include spatial and temporal segmentation although skills or occupations are perhaps more useful in the context of business cycles and labor market fluctuations. In the economic development literature, Harris and Todaro [30] considered a *spatial* market segmentation in which migrant workers from rural areas speculated on finding jobs in cities. In their model, however, wages and job assignments are exogenous. Eden [18] considered a monetary model and temporal segmentation; i.e., some markets traded *before* others. Skill heterogeneity has also been recognized in business cycle research, see, e.g., Cooley ([14], chapter 5). It is known, for example, that unskilled workers are relatively more likely to become unemployed during and economic downturn that do skill workers. In the model, the uncertainty associated with trade in a given

[3]Some workers speculated on receiving a job in any given location but at the end of the trading round were not able to work. Following the empirical literature, see, e.g., Jones and Riddell [31], this notion of unemployment can be seen as *waiting unemployment*. Later on, we will also consider *search* and the notion of *rest unemployment* as described in Jovanovic [32], Gouge and King [24], and Alvarez and Shimer [4].

micro-market (i.e., geographic location, skill, firm, occupation, or industry) is a primitive.

It is perhaps important to highlight that the trading frictions central to this paper can be understood as affecting *markets* but not any specific bilateral trade opportunity within a market, which is the main consideration in search and matching models, e.g., Mortensen and Pissarides [46]. Because agents here interact with the market, wages are set competitively. Moreover, despite the fact that the assignment of jobs is irreversible and made before uncertainty is resolved, the plans are time consistent and socially efficient. Plans can also be interpreted as contingencies in Arrow-Debreu markets so one may think of job assignments as posted vacancies and of market prices as advertised market wages.

We enrich the basic assignment model in two main ways. First, we allow for worker search and, second, we consider capital accumulation. We examine an assignment version of Lucas and Prescott's [39] *search unemployment* in which there are aggregate productivity shocks, job vacancies, and worker mobility between locations (whereas in Lucas and Prescott [39] only worker mobility is allowed). Because the aggregate distribution of workers becomes a state variable, we describe the dynamic economy using sequences of aggregate distributions of the labor force and prove existence as in anonymous sequential games with aggregate uncertainty (see, e.g., Bergin and Bernhardt [8]; [9]; Miao [40]). As recognized by the literature, see, e.g., Duffie et al. [17]; Kubler and Schmedders [34], a recursive representation of frictional models with individual stochastic heterogeneity and aggregate shocks may be problematic because decisions and equilibrium prices generally depend on expectations about the distribution of people across locations.

The focus on how productivity shocks affect the labor market is interesting because the response is intuitive in this paper. The main effect of a positive productivity shock is to increase wages and to make worker search less attractive. Productivity shocks also increase job assignments to all markets including those were trade may not take place. Both changes suggest a *Beveridge curve*. Quantitatively, we show that the model produces such a Beveridge curve. The model also produces highly volatile unemployment and vacancies. As a shortcoming of the model, unemployment is not persistent whereas in the data unemployment is highly persistent. The model also delivers implications for consumption, investment, and output typical of the real business cycle literature, see, e.g., Kydland and Prescott [48] and Cooley [14].

An important class of alternative models are those based on Mortensen and Pissarides [45] and [46]. In this class of models, informational frictions also prevent workers from meeting suitable employers. Frictions, however, are introduced through a *reduced-form* aggregate matching function. (Pissarides [47] presents a general treatment of this literature.) This approach has been criticized by Lagos [36] because a reduced-form matching function fails to provide an explicit account of how informational frictions affect the labor market and hence it may not be policy-invariant. A reduced form, as we show later on, also implies important assumptions about the treatment of labor market *heterogeneity*.[4]

While this paper is explicit about the informational problems in the labor market, its focus is not just on the microfoundations for mismatch. Attention is confined here to the relationship between the labor market and the rest of the economy. To study these

[4] Shimer [55] argues that a calibrated version of the Mortensen and Pissarides [45] model can account for only a small fraction of the business cycle volatility of unemployment and vacancies. Several attempts have been proposed to reconcile this model with business cycle fluctuations based on endogenous labor supply decisions, alternative bargaining arrangements for wages or wage rigidity, alternative calibrations, or the introduction of additional productivity shocks. (Hall [28] presents a detailed survey not attempted here. See also Mortensen and Nagypál [44] and Hagedorn and Manovskii [27].)

interactions, we rely on a classical *competitive analysis*. General competitive analysis has been very useful in examining business cycles, monetary nonneutralities, unemployment, and asset pricing (among some other macroeconomic issues), but it has not to been applied to study labor market mismatch. To the best of our knowledge, there is currently no Beveridge curve consistent with an Arrow-Debreu equilibrium. The class of models based on Mortensen and Pissarides [45] does not use the Arrow-Debreu type of equilibrium. Instead, it uses a notion of Nash equilibrium. Models of competitive search and wage-posting, see, e.g., Moen [42], also rely on a Nash equilibrium concept and arrive to Walrasian outcomes only as a limiting result. (Some additional related literature is discussed later on.)

The paper is also related to the recent models of Lagos [36], Shimer [56], and Mortensen [43], but the treatment proposed here differs in many ways from previous studies. Mismatch in Lagos [36] and Shimer [56] assume that workers and jobs are related in *fixed proportions* or that the short-side of the market is always served. In both previous settings, prices are exogenous and mismatch takes place because there are locations with excess supply of workers and locations with excess supply of jobs. In this class of models, mismatch exists if and only if the meeting technology in micro-markets is Leontief.

The rest of the paper proceeds as follows. Section 2 first studies a static version the assignment model and then generalizes the model to study equilibrium search and business cycles. Section 3 presents a quantitative exploration. Section 4 reviews previous theories of mismatch. Section 5 concludes this paper. The Appendix discusses the recursive representation of the model and the solution method of the quantitative section.

## 2 The model

The *space of locations* is $(\mathbf{X},\mathcal{A})$ which can be thought as the unit interval equipped with the Lebesgue measure; we write $x$ for a typical individual location. Time is discrete, indexed by $t \in \{0, 1, ..., +\infty\}$. Workers are represented by the nonnegative and measurable function $l_t(x)$ which gives the *labor force* in location $x$ and period $t$. The mass of workers in $A \in \mathcal{A}$ is $\mu_t(A) = \int_A l_t(x)dx$. The initial distribution of the labor force $\mu_0$ is nonstochastic and given. By convention, the total mass of workers is normalized to one, i.e., $\mu_0(\mathbf{X}) = 1$. *Employment* in location $x$ is $e_t(x)$ but employment may differ from the labor force in location $x$ in ways that will be specified later on.

At the beginning of period $t$, each location $x$ experiences an *idiosyncratic* and an *aggregate* productivity shock. Idiosyncratic shocks are defined on $(\mathbf{W}, \mathcal{W})$ and given as a measurable function $\phi : \mathbf{X} \times \mathbf{W} \to \mathbb{R}_+$.[5] The transition function is $\Phi(A, x) = \Pr(\{\omega : \phi_{t+1}(x, \omega) \in A\})$ for all $A \in \mathcal{A}$. A location's productivity and its type are closely related. The interpretation of idiosyncratic shocks is that if $\omega$ is drawn from $\mathbf{W}$, location $x$'s new 'type' is $\phi_{t+1}(x, \omega)$. The initial value $\phi_0(x)$ is a given and nonstochastic.

Aggregate productivity shocks $\mathbf{z}_t$ are defined on $(\mathbf{Z}, \mathcal{Z})$. Let $\mathbf{z}^t = (\mathbf{z}_t, \mathbf{z}^{t-1})$ be its history in $\mathbf{Z}^t = \mathbf{Z} \times \mathbf{Z}^{t-1}$ and assume $\mathbf{Z}$ is countable and that $\mathbf{Z}$ and $\mathbf{W}$ are compact metric spaces. The value of $\mathbf{z}_0$ is also given. Aggregate shocks are a Markov process with transition function, $Q : \mathbf{Z} \times \mathcal{Z} \to [0, 1]$. The transition functions have the Feller property as defined by Stokey, Lucas, and Prescott ([58], chap. 12).

[5] That is, for any $x \in \mathbf{X}$, the function $\phi(x, \cdot)$ is $(\mathcal{W}-)$ measurable and for any $\omega \in \mathbf{W}$, the function $\phi(\cdot, \omega)$ is $(\mathcal{A}-)$ measurable. For convenience, the use of phrases such as "almost always" or "almost everywhere" is omitted with the convention that measure concepts are referred back to $(\mathbf{W}, \mathcal{W})$ or other additional probability spaces described later on.

Jobs are assignments of capital to each location, $k_t(x)$. If the employment rate in location $x$ is $e_t(x)$, and if trade takes place (in a sense that we will make precise in the next paragraphs), the output of location $x$ will be $\mathbf{z}_t\phi_t(x)f(k_t(x), e_t(x))$. The production technology has diminishing returns to scale in $k$ and $e$ but it is standard otherwise; $f(k, e)$ is continuously differentiable, strictly increasing, and strictly concave, satisfies $f_{ke}(k, e) > 0$, $\lim_{k\to+\infty} f_k(k, e) = \lim_{e\to1} f_e(k, e) = 0$ and $\lim_{e\to0} f_e(k, e) = \lim_{k\to0} f_k(k, e) = +\infty$. The mass of jobs assigned to a subset $B \in \mathcal{A}$ of locations is $\displaystyle\int_B k_t(x)dx$. Feasibility for the job assignment then requires $\displaystyle\int_{\mathbf{X}} k_t(x)dx \leq K_t$ for a given aggregate number of available jobs $K_t$ whose law of motion will be specified later on.

The economy sketched above is somewhat general but, thus far, the assumptions listed will not generate unused factors in a Walrasian equilibrium. To that end, suppose *trade* is uncertain and *order* locations in terms of their probability of trade. Suppose also that worker and job assignments are decided before knowing if trade will take place in a given location $x$ but after knowing the value of productivity shocks. Let $(\mathbf{\Omega}, \mathcal{F}, \Pi)$ be an arbitrary measurable space representing states of nature. A point $\omega$ in $\mathbf{\Omega}$ is a description of all the random environmental factors that might influence the trading process.

The trading mechanism to be discussed is based on a simple *queue*. At each period, the nonstochastic location $\underline{x} \in \mathbf{X}$ is chosen to be the head of the queue and the queue forms to the right. Locations $x \leq \underline{x}$ trade with certainty. Trade for locations $x \geq \underline{x}$ is uncertain; at a certain point or in certain location $N_t(\omega)$ trade will stop and all assigned jobs and workers in locations 'after' $N_t(\omega)$ in the queue will remain vacant and unemployed. Figure 1 has a description of the timing of trade.

Let $\tilde{\mathbf{X}} = \{x \in \mathbf{X} : x \geq \underline{x}\}$ and consider $\{\mathcal{F}_x : x \in \tilde{\mathbf{X}}\}$ as the sigma-field of events that depend only on the 'past' up to location $x$. Assume $\mathcal{F}_{\tilde{\mathbf{X}}} = \mathcal{F}$ and define the positive random variable $N_t(\omega)$ taking on values in $\tilde{\mathbf{X}}$ as a *nonanticipating random time* (occasionally referred to as a *stopping time*, see, e.g., Bhattacharya and Waymire ([6], section 2.13)). That is, assume the condition $\{\omega : N_t(\omega) \leq x\} \in \mathcal{F}_x$ is satisfied for all locations $x \in \tilde{\mathbf{X}}$. In other words, assume it is possible to determine whether or not a location $x \geq \underline{x}$ has traded on the basis of the knowledge of $\mathcal{F}_x$. Trade opportunities for location $x$ are represented by a *stopping process* $\tau_t : \mathbf{X} \times \mathbf{\Omega} \to \{0, 1\}$ derived from $N_t(\omega)$ by $\tau_t(x, \omega) = 1$ if $N_t(\omega) \geq x$ and $\tau_t(x, \omega) = 0$ if $N_t(\omega) < x$.

The assignment of jobs and workers will clearly depend on the probability that trade will take place in location $x$, i.e., $\Pr(\{\omega : \tau_t(x, \omega) = 1\})$. This probability is assumed stationary and given by $q_t(x) = 1$ if $x \leq \underline{x}$ and:

$$q_t(x) = \Pr(\{\omega : \tau_t(x, \omega) = 1\}) = \Pr(\{\omega : N_t(\omega) \geq x\}) = \int_{x \in \tilde{\mathbf{X}}}^{1} \Pi(y) dy,$$

if $x \geq \underline{x}$ with $\int_{\tilde{\mathbf{X}}}^{1} \Pi(y) dy = 1$ as a needed normalization. To represent locations subject to trading uncertainty, i.e., locations that need to post vacancies, we use the *indicator function* $\chi_q(x) = 1$ if $x \geq \underline{x}$ and $\chi_q(x) = 0$ otherwise.

The following are some remarks on the trading uncertainty in the paper. Consider two locations $x' > x \geq \underline{x}$. Trade is uncertain in both locations but the probability of trade in $x'$ is smaller than in $x$. In a spatial interpretation of the model along the lines of Harris and Todaro [30], worker mobility would represent rural-urban migration and the absence of trade would represent urban unemployment. In models of temporal segmentation, because

$q_t(x)$ is weakly decreasing in $x$, trading is commonly seen as 'sequential' with location $x$ trading before $x'$. (For example, some costumers arrive *before* others in Prescott [48].) This is a notion of sequential trade not based on real time, see, e.g., Eden [20].[6]

The previous trading frictions have been defined in terms of *market* trade as in Gale [23] but an alternative interpretation closer to Prescott [48], Butters [10], and Eden [18] can be given in terms of any particular side of the market. Assume workers are admitted into the labor market in order and that there is uncertainty about the total number of workers who will be admitted. Let $\mu_t([\underline{x}, x]) = \int_{\underline{x}}^{x} l_t(y)dy$ be the mass of workers in locations between $\underline{x}$ and $x$. The probability that workers in location $x \geq \underline{x}$ are admitted is:

$$q_t(x, \mu_t) = \Pr(\{\omega : N_t(\omega) \geq \mu_t([\underline{x}, x])\}) = \int_{\mu_t([\underline{x}, x])}^{1} \Pi(y)dy.$$

If uncertainty is about the number of jobs available in location $x$, i.e., uncertainty about job openings that may appear in some locations, worker-job matches will be possible for locations in which $N_t(\omega) \geq \int_{x \in \tilde{\mathbf{X}}}^{1} k_t(y)dy$. The case of uncertainty on both sides of the market is a simple generalization of previous cases. Also, we have considered a situation in which the head of the queue $\underline{x}$ is known. If there is uncertainty about the beginning and end of the queue, the probability of trade will be $q_t(x) = \Pr(N_t^-(\omega) \geq x) + (1 - \Pr(N_t^-(\omega) \geq x)) \Pr(N_t^+(\omega) \geq x)$, in which $N_t^-(\omega)$ and $N_t^+(\omega) \geq N_t^-(\omega)$ are random variables that define the head and end of the queue. As evident from the previous discussion, $q_t(x)$

[6] A sequential resolution of uncertainty is also related to first-come first-serve sequential services typical of models of bank runs. In models of banking, customers form a queue outside of the bank and are admitted sequentially to make their withdrawals. By the sequential service constraint, customers attempt to move near the head of the queue to maximize the probability of a withdrawal. In our model, similar forms of *congestion* will exist if there is uncertainty about the number of jobs or workers that will be able to trade. Finally, one can consider coordination frictions and strategic considerations to make the trading probabilities endogenous objects. Those ideas are not pursued here.

serves as a general representation of the trading frictions in this economy. Each case, however, may carry different welfare and quantitative implications. In all these settings, however, markets will be *incomplete* as long as trade cannot be made contingent on $N_t(\omega)$.

Finally, the relationship between the probability of trade and economic decisions is different from that typical of matching models. In matching models (e.g., Mortensen and Pissarides [46]), the probability of trade within a market is determined by a matching function that depends on a given market tightness. Loosely speaking, under uncertain trade, the probability of trade is given and market tightness responds to this probability.

## 2.1 The basic assignment

Next we describe a static job assignment in which we assume no worker mobility and a constant number of jobs $K_t$ to be assigned. The next sub-section allows for worker mobility and accumulation although they are not essential to observe mismatch. We consider a Planner problem and then describe the equivalence with a competitive market. The basic properties of the competitive wages that decentralize the assignment are also discussed later on.

Recall that employment in location $x$ is $e_t(x)$ which here equals the available and given labor force, $l_t(x)$. Since firms are risk neutral and job assignments are decided before $N_t(\omega)$ is known, the assignment problem is:

$$Y_t(K_t, \mathbf{z}_t, \mu_t) = \max \left\{ \int_{\mathbf{X}} \mathbf{z}_t \phi_t(x) q_t(x) f(k_t(x), l_t(x)) dx : \int_{\mathbf{X}} k_t(x) dx \leq K_t \right\}, \quad (1)$$

with $Y_t(K_t, \mathbf{z}_t, \mu_t)$ as the value function that represents total output in the economy.

(This value function will in general depend on $\phi_t(x)$ and $q_t(x)$ but here we have listed only aggregate state variables.)

In the next Theorem we establish the basic existence result:

**Theorem 1 (Existence and uniqueness)** *For a given $(K_t, \mathbf{z}_t, \mu_t)$, $\phi_t(x)$, and a trading friction $q_t(x)$, there is a unique job assignment $k_t : \mathbf{X} \to \mathbb{R}_+$ which is optimal for $Y_t(K_t, \mathbf{z}_t, \mu_t)$.*

Because of strict concavity in $f(k, l)$, there is a unique solution to the job assignment problem.[7] Moreover, notice that $\partial Y_t(K_t, \mathbf{z}_t, \mu_t)/\partial K_t = r_t$ and that $Y_t(K_t, \mathbf{z}_t, \mu_t)$ is strictly concave in $K_t$. Finally, since the feasibility constraint is an isoperimetric side condition, the assignment can be characterized by the Kuhn-Tucker Theorem. For example, the first order condition for job assignments to $x$ is: $\mathbf{z}_t \phi_t(x) f_k(k_t(x), l_t(x)) = r_t/q_t(x)$, with $r_t$ as the opportunity cost of a job or the Lagrange multiplier on the feasibility constraint.[8] As the previous expression shows, the cost of a job and the probability of trade are *traded-off*. If trading is unlikely for location $x$, a few jobs will be assigned to that location to minimize expected unused capacity. Notice, however, that there are incentives to advertise more jobs than the number of jobs one is absolutely certain of filling. This generates unemployment and vacancies:

[7] There are no special difficulties with this problem. The setting is a simplification of nonatomic assignment models, e.g., Grestky et al. [25]. In Grestky et al. [25], the solution is characterized in a space of nonnegative measures. Grestky et al. [25] show that $Y_t(K_t, \mathbf{z}_t, \mu_t)$ is continuous, increasing, bounded and differentiable in $K_t$. $Y_t(K_t, \mathbf{z}_t, \mu_t)$ is linearly homogeneous in $\mu_t$ (i.e., $Y_t(K_t, \mathbf{z}_t, \lambda\mu_t) = \lambda Y_t(K_t/\lambda, \mathbf{z}_t, \mu_t)$ for $\lambda > 0$) so the problem can be seen as a linear optimization on a convex domain. Overall, the analysis of Grestky et al. [25] encompasses a larger class of objects than the ones considered in this paper because jobs are homogeneous here. They also present a game theoretic version via the core and a Walrasian version of the assignment and establish the effective equivalence of the solution concept of the three versions.

[8] To highlight the difference of the model and standard treatments of uncertainty, assume a nonordered random fraction of locations $h_t(\omega)$ will be able to trade. Jobs must be assigned before $h_t(\omega)$ is known. The optimal assignment of jobs satisfies: $\int_0^1 \mathbf{z}_t \phi_t(x) h_t(\omega) f_k\left(k_t(x), l_t(x)\right) \Pi(\omega) d\omega = r_t$. Notice that the model can be generalized by allowing both sources of uncertainty play a role.

**Proposition 1 (Mismatch)** *For a given* $(K_t, \mathbf{z}_t, \mu_t)$, $\phi_t(x)$, *and a trading friction* $q_t(x)$, *the solution to the static job assignment in equation (1) features mismatch. Unemployment and vacancies are:*

$$U_t(K_t, \mathbf{z}_t, \mu_t) = 1 - \int_{\mathbf{X}} q_t(x) l_t(x) dx, \tag{2}$$

$$V_t(K_t, \mathbf{z}_t, \mu_t) = \int_{\mathbf{X}} \chi_q(x) k_t(x) dx, \tag{3}$$

*with the opportunity cost of a job uniquely determined from the feasibility conditions.*

**Proof.** First notice that there is a unique opportunity cost that ensures that all jobs are assigned ex-ante. Employment is the mass of workers with jobs in locations with trade. Unemployed workers in equation (2) are the mass of workers in locations that were not able to trade. This notion of unemployment we have considered as *waiting unemployment* as these workers were waiting to trade. (Unemployment is measured at the end of the trading period, as typically measured in the data.)

Vacancies are described next. Locations with $\chi_q(x) = 1$ are subject to trading uncertainty and need to post a vacancy if production is to take place. The expected number of worker-job matches is the number of advertised jobs to locations that open:

$$M_t(K_t, \mathbf{z}_t, \mu_t) = \int_{\mathbf{X}} \chi_q(x) k_t(x) q_t(x) dx, \tag{4}$$

while unfilled vacancies are the expected value of job assignments to locations that failed to open, $\int_{\mathbf{X}} \chi_q(x) k_t(x)[1 - q_t(x)] dx$. This expression together with $M_t(K_t, \mathbf{z}_t, \mu_t)$ is equation (3). Notice also that the jobs for the set of locations that do not need to post vacancies is: $\int_{\mathbf{X}} [1 - \chi_q(x)] k_t(x) dx$, and that:

$$V_t(K_t, \mathbf{z}_t, \mu_t) + \int_{\mathbf{X}} [1 - \chi_q(x)] k_t(x) dx = K_t(\mathbf{z}_t, \mu_t). \tag{5}$$

(Vacancies, as in the data, are measured at the beginning of the trading period.) ■

The previous proposition describes an assignment in which some jobs need to be advertised while other are filled with certainty. Out of those advertised jobs, some are filled but others remain unused at the end of the trading period. Thus, some workers *match* with their advertised jobs but others remain unemployed. Both unused factors are a consequence of trading frictions and hence they will not arise if $q_t(x) = 1$ or if $\chi_q(x) = 0$ for all $x$. One can also *derive* an aggregate matching function $M_t(K_t, \mathbf{z}_t, \mu_t)$ based on the aggregation over locations. Among the *microeconomic* determinants of the aggregate matching function are market tightness $K_t$, the distribution of the labor force $\mu_t$, and the extent of trading frictions and productivity shocks, $q_t(x)$, $\mathbf{z}_t$, and $\phi_t(x)$. Similar aspects are important in alternative models that examine the microfoundations of mismatch, see, e.g., Lagos [36], Shimer [56], and Mortensen [43], and also in the aggregate production function $Y_t(K_t, \mathbf{z}_t, \mu_t)$.

The assignment can also be interpreted as a competitive *market economy*:

**Definition 1 (Competitive market equilibrium)** *For a given* $(K_t, \mathbf{z}_t, \mu_t)$*,* $\phi_t(x)$*, and a trading friction* $q_t(x)$*, a competitive equilibrium for the job assignment (1) is a set of state-dependent market wages, a market price for jobs, and an ex-ante assignment of jobs such that, taking prices and the probability of trade as given, job assignments maximize* *profits and markets that open clear.*

In terms of the *welfare implications*, the two welfare Theorems hold:

**Theorem 2 (Welfare Theorems)** *The competitive assignment of jobs in equation (1) is constrained Pareto optimal and the Pareto assignment can be implemented in a competitive market.*

**Proof.** The proof is standard as the relevant Pareto optimum is the one that maximizes the value of the job assignment. Finding the prices that decentralize the assignment is also a standard exercise. In a competitive economy, $r_t$, the opportunity cost of a job (or the shadow price of capital), coincides with the market price. Wages are given by $w_t(x) = \mathbf{z}_t \phi_t(x) q_t(x) f_e(k_t(x), l_t(x))$. ■

The structure of market wages are discussed next. Wages are heterogeneous if trading opportunities are different for equally productive workers.[9] Also, advertised wages are determined in competitive markets and hence flexible but they will not change in response to trading in the labor market or to the partial resolution of uncertainty. Assume trading has taken place in location $x \geq \underline{x}$. Consider markets in locations $x' > x$. By Bayes' rule, the job assignments for $x'$, conditional on location $x$ opening, is:

$$\mathbf{z}_t \phi_t(x') q_t(x'|x) f_k(k_t(x'), l_t(x')) = \frac{\mathbf{z}_t \phi_t(x') q_t(x') f_k(k_t(x'), l_t(x'))}{q_t(x)}. \tag{6}$$

The expected cost for a job assigned to workers in locations $x'$ conditional on trade

[9] An equilibrium distribution of prices is an efficient response to trading uncertainty because it reduces the expected costs of excess capacity (Prescott [48] and Butters [10]). Additional aspects of the distribution of prices have been considered by Carlton [12], Eden and Griliches [21], and Eden [18]. See also Weitzman [60] for a model of wage rigidity. Rotemberg and Summers [50] also employ Prescott's [48] model to study labor hoarding and labor productivity in a model in which prices are set before demand is known.

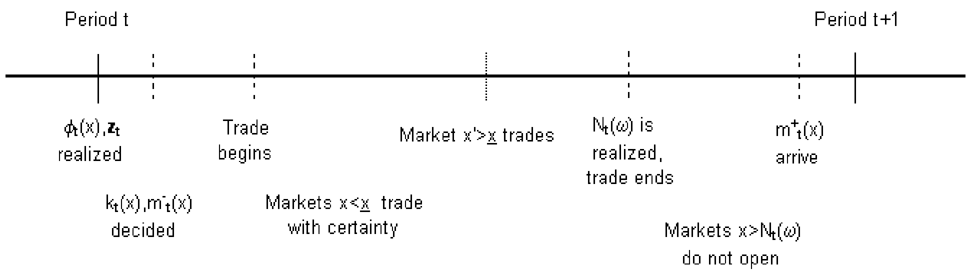


Figure 1: Trade process under uncertain trade and segmented markets.

in $x$, is given by $r_t/q_t(x)$. As it follows from equation (6), observing trade in $x$ does not provide an incentive to reassign jobs to $x'$. As job assignments are unchanged, advertised wages will be unresponsive to observed trade in the labor market. Wages in location $x'$, conditional on trade in $x$ are $w_t(x'|x) = w_t(x')/q_t(x)$. Moreover, since only wages actually *paid* in locations with trade are observed, average wages for actual transactions would also seem to be 'too high,' see Landsburg and Eden [37] and Eden [18].[10]

## 2.2 The general assignment

This sub-section of the paper generalizes the earlier analysis in directions that make the model applicable to quantitatively study the business cycle properties of the labor market. The main contribution of this sub-section is to allow for capital accumulation and worker search.

Let $\mathcal{P}(\mathbf{X}, \mathcal{A})$ be the set of probability measures on $\mathbf{X}$ and define $(\mathbf{S}, \mathcal{S})$ by $(\mathbf{K}\times\mathbf{Z}, \mathcal{K}\times\mathcal{Z})$. The *state* in this economy is the aggregate number of jobs $K_t$, the aggregate productivity shock $\mathbf{z}_t$, and the distribution of the labor force across locations $\mu_t$. We write the state

[10] As Eden [18] notes, it is important to distinguish between advertised and observed wages. Advertised wages in a market equilibrium are given by $w_t(x)$ while observed wages only reflect a partial list of contracts (those for locations with actual trade). If only observed wages are considered to describe the labor market, the commodity space will be misspecified.

as a pair $(\mathbf{s}_t, \mu_t)$ in $\mathbf{S}^t \times \mathcal{P}(\mathbf{X}, \mathcal{A})$. The assignment of jobs to $x$, for instance, is a function $k_t(x, \mathbf{s}^t, \mu_t)$ with $k_t : \mathbf{X} \times \mathbf{S}^t \times \mathcal{P}(\mathbf{X}, \mathcal{A}) \to \mathbb{R}_+$ whereas employment is $e_t(x, \mathbf{s}^t, \mu_t)$.

To properly treat the distribution of the labor force as a state variable, we assume that idiosyncratic shocks do not induce aggregate uncertainty. Because of idiosyncratic shocks, the distribution of the labor force in the future $\tilde{\mu}_{t+1}(\{\phi_{t+1}(x, \omega) \in A\})$ is a *random measure*, i.e.,

$$\tilde{\mu}_{t+1}(A) = \int_{\mathbf{\Omega}} \tilde{\mu}_{t+1}(\{\phi_{t+1}(x, \omega) \in A\}) \Phi(x, d\omega).$$

Under the *no aggregate uncertainty* assumption put forward by Bergin and Bernhardt [8] and [9], the distribution of the labor force becomes deterministic $\tilde{\mu}_{t+1}(A) = \mu_{t+1}(A) = \int_{\mathbf{X}} \Phi(A, x) \mu_{t+1}(dx)$, for all realizations $\omega \in \mathbf{W}$ and all $A \in \mathcal{A}$. For this assumption to hold, the distribution of individual shocks across locations $\mathbf{X}$, for a given $\omega$, should be the same as the distribution over states $\mathbf{W}$ for any given location. This interpretation is in the spirit of the Law of Large Numbers for the continuum, see, e.g., Bergin and Bernhardt [8] and Feldman and Gilles [22].[11]

Decisions about job creation are analogous to decisions about capital accumulation. There is an initial aggregate number of jobs $K_0$. Given a history of aggregate variables $\mathbf{s}^t$ and a distribution of the labor force $\mu_t(\mathbf{s}^{t-1}, \mu^{t-1})$, the investment rate is $i_t(\mathbf{s}^t, \mu_t)$. Capital depreciates at a rate $\delta \in (0, 1)$ regardless the number of used or unused jobs, i.e., $K_{t+1}(\mathbf{z}^t, \mu_t) = i_t(\mathbf{s}^t, \mu_t) + (1 - \delta) K_t(\mathbf{z}^{t-1}, \mu_{t-1})$, for all possible histories.

[11] Abusing notation, one can write $x_{t+1} = \phi_{t+1}(x_t, \omega)$. Then, $\tilde{\mu}_{t+1}(\{\phi_{t+1}(x_t, \omega) \in A\})$ can be seen as $\tilde{\mu}_{t+1}(\{x_{t+1} \in A\})$ which is a random measure. The no aggregate uncertainty condition assumes a stochastic process whose transition $\Phi(A, x_t)$ can be chosen independent of $x_t$. That is, it assumes that $\tilde{\mu}_{t+1}(\{x_{t+1} : \phi_{t+1}(x_t, \omega) \in D\}) = v(D)$ for all $\omega \in \mathbf{W}$ and that $\Pr(\{\omega : \phi_{t+1}(x_t, \omega) \in D\}) = v(D)$ for all $x_t \in \mathbf{X}$. The no aggregate uncertainty condition we consider, as in Bergin and Bernhardt [8], is conditional on a given history of aggregate shocks.

The following discussion about worker *search* is inspired by Lucas and Prescott [39] although our representation is not recursive and we are not interested in a stationary equilibrium. Workers are allowed to move between locations before trading begins and after observing $\mathbf{z}_t$ and $\phi_t(x)$. (Figure 1 illustrates the sequential trade process and timing in the model.) Movements are of two types. Workers who move *out* of location $x$ are represented by $m_t^-(x, \mathbf{s}^t, \mu_t)$ while workers who move *in* are $m_t^+(x, \mathbf{s}^t, \mu_t)$. No location will have workers leaving and arriving in the same period. Search is frictional because workers who move in forego their employment opportunities for one period.

Employment is:

$$e_t(x, \mathbf{s}^t, \mu_t) = l_t(x, \mathbf{s}^{t-1}, \mu_{t-1}) - m_t^-(x, \mathbf{s}^t, \mu_t), \tag{7}$$

while workers who arrive to any given location due so just before the beginning of the next round of trade, i.e.,

$$l_{t+1}(x, \mathbf{s}^t, \mu_t) = l_t(x, \mathbf{s}^{t-1}, \mu_{t-1}) + m_t^+(x, \mathbf{s}^t, \mu_t) - m_t^-(x, \mathbf{s}^t, \mu_t). \tag{8}$$

In order to close the model, assume a representative consumer whose choices are given by $c_t(\mathbf{s}^t, \mu_t)$ and based on $\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t(\mathbf{s}^t, \mu_t))$, with $u : \mathbb{R}_+ \to \mathbb{R}$ as an increasing, strictly concave, continuously differentiable and bounded utility function. The discount factor satisfies $\beta \in (0, 1)$.[12]

[12] Notice that the state of the economy differs from the one in Lucas and Prescott [39] since the opportunity cost of a job depends on the *distribution* of the labor force. (Lucas and Prescott [39] studied a *representative* micro-market.) Gouge and King [24] introduced aggregate productivity shocks but no capital accumulation so the distribution of the labor force is not a state variable in their setting either.

The *general* assignment problem is defined next:

**Definition 2 (General assignment)** *A worker-job assignment is a sequence of opportunity costs of search and opportunity costs of a job, $\{\theta_t(\mathbf{s}^t, \mu_t), r_t(\mathbf{s}^t, \mu_t)\}$, job accumulation decisions $\{K_{t+1}(\mathbf{z}^t, \mu_t)\}$, ex-ante worker search decisions $\{m_t^+(x, \mathbf{s}^t, \mu_t), m_t^-(x, \mathbf{s}^t, \mu_t)\}$, and ex-ante job assignments $\{k_t(x, \mathbf{s}^t, \mu_t)\}$, that maximizes:*

$$\mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t(\mathbf{s}^t, \mu_t)) \right\}, \tag{9}$$

*subject to feasibility in job assignments,* $\int_{\mathbf{X}} k_t(x, \mathbf{s}^t, \mu_t)dx = K_t(\mathbf{z}^{t-1}, \mu_{t-1})$, *and:*

$$K_{t+1}(\mathbf{z}^t, \mu_t) = Y_t(\mathbf{s}^t, \mu_t) - c_t(\mathbf{s}^t, \mu_t) + (1-\delta)K_t(\mathbf{z}^{t-1}, \mu_{t-1}), \tag{10}$$

*and feasibility in worker assignments, equations (7), (8), and*

$$\int_{\mathbf{X}} m_t^+(x, \mathbf{s}^t, \mu_t)dx = \int_{\mathbf{X}} m_t^-(x, \mathbf{s}^t, \mu_t)dx, \tag{11}$$

*for all* $t \geq 0$ *and for all* $\mathbf{s}^t$ *in* $\mathbf{S}^t$.

The aim of the general problem is to assign jobs and workers. Feasibility in job assignments is as before. The only new decision is for $K_{t+1}(\mathbf{z}^t, \mu_t)$ which is typical for accumulation programs. Search decisions for workers are also similar to those in Lucas and Prescott [39].[13] Here, however, decisions in period $t$ will in general depend on the

[13] We study a sequential problem and defer the recursive characterization to the Appendix. In the recursive case, it is natural to include the aggregate distribution $\mu_t$ as a state variable. But the current distribution of the labor force may not be enough to describe the history of the state of the economy, see, e.g., Duffie et al. [17]. As discussed in Miao [40] and Kubler and Schmedders [34], if the assignment is

expectation of the distribution of the labor force in $t+1$ (and of course of expectations about other variables).

The rest of this sub-section examines the solution to the general assignment. The proof is constructive and it describes the solution and its properties in two steps. To characterize the assignment, we first condition on a *sequence* of aggregate distributions and then we study the consistency requirements of the assignment. (This approach is essentially the one in Bergin and Bernhardt [8] and Miao [40].) Let $\{\mu\}$ represent $\{\mu_t(\mathbf{s}^t, \mu_{t-1}; \mathbf{s}_0, \mu_0)\}$. Recall that $\mu_t$ is a function from $\mathbf{S}^t \times \mathcal{P}(\mathbf{X}, \mathcal{A})$ to $\mathcal{P}(\mathbf{X}, \mathcal{A})$. Let $\mathcal{P}(\mathbf{X}, \mathcal{A})^{\mathbf{S}^t}$ denote the set of such functions. Let $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A}) = \times_{t=1}^{\infty} \mathcal{P}(\mathbf{X}, \mathcal{A})^{\mathbf{S}^t}$. The sequence $\{\mu\}$ lies in $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$.

The next proposition describes optimal job creation decisions given $\{\mu\}$:

**Proposition 2 (Job accumulation)** *For a given $\{\mu\}$ and a history $\mathbf{s}^t \in \mathbf{S}^t$, job creation decisions satisfy the following Euler equation:*

$$u_c(c_t(\mathbf{s}^t, \mu_t)) = \beta \int_{\mathbf{S}^{t+1}} u_c(c_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1})) \left(1 + r_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1}) - \delta\right) P_\mu(\mathbf{s}^t, d\mathbf{s}_{t+1}), \quad (12)$$

*with $P_\mu : (\mathbf{S}, \mathcal{S}) \rightarrow [0,1]$ as the transition function for $\mathbf{s}_t = (K_t, \mathbf{z}_t)$. Moreover, job creation decisions are continuous and the transition function $P_\mu(\mathbf{s}^t, d\mathbf{s}_{t+1})$ has the Feller property.*

**Proof.** Given the fact that we have conditioned the solution on $\{\mu\}$, the construction of the transition function for $\mathbf{s}_t$ is standard, see Stokey, Lucas, and Prescott ([58], Theorem 9.13). The Feller property follows from Stokey, Lucas, and Prescott ([58], Theorem 9.14). ■

globally unique for all possible initial values, the current distribution of the labor force would be a sufficient statistic for the state of economy. Uniqueness, however, cannot be checked from primitives.

The Euler equations for worker search and feasibility fully characterize the worker assignment in this economy. To economize on notation, define a function $w_t(x, \mathbf{s}^t, \mu_t) = \mathbf{z}_t\phi_t(x)q_t(x)f_e(k_t(x, \mathbf{s}^t, \mu_t), e_t(x, \mathbf{s}^t, \mu_t))u_c(c_t(\mathbf{s}^t, \mu_t))$. It is perhaps obvious that this function is related to the value of wages in a competitive market but here it can be seen just as a convenient representation. Let also $R_t(\mathbf{s}^t, \mu_t) = 1 + r_t(\mathbf{s}^t, \mu_t) - \delta$.

**Proposition 3 (Search unemployment)** *For a given $\{\mu\}$ and a history $\mathbf{s}^t \in \mathbf{S}^t$, there exists a continuous worker assignment $\{m_t^-(x, \mathbf{s}^t, \mu_t), m_t^+(x, \mathbf{s}^t, \mu_t)\}$ that solves the general assignment problem. The assignment of workers satisfies the following Euler equations:*

$$\theta_t(\mathbf{s}^t, \mu_t) \geq w_t(x, \mathbf{s}^t, \mu_t) + \beta\mathbb{E}_t\left[R_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1})w_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^{t+1}, \mu_{t+1})\right], \tag{13}$$

*with equality if $m_t^-(x, \mathbf{s}^t, \mu_t) > 0$, and*

$$\beta\mathbb{E}_t\left[R_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1})w_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^{t+1}, \mu_{t+1})\right] \geq \theta_t(\mathbf{s}^t, \mu_t), \tag{14}$$

*with equality if $m_t^+(x, \mathbf{s}^t, \mu_t) > 0$, for all $x \in \mathbf{X}$. Moreover, expectations are given by: $\mathbb{E}_t[w_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^{t+1}, \mu_{t+1})] = \int_{\mathbf{W}}\int_{\mathbf{S}^{t+1}} w_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^{t+1}, \mu_{t+1})P_\mu(\mathbf{s}^t, d\mathbf{s}_{t+1})\Phi(x, d\omega)$ and defined over idiosyncratic and aggregate shocks.*

**Proof.** Since feasibility in worker search involves an isoperimetric constraint, the assignment can also be characterized by the Kuhn-Tucker Theorem. The opportunity cost of search $\theta_t(\mathbf{s}^t, \mu_t)$ is the Lagrange multiplier on equation (11). Substitute the definition of employment (7) and equation (8) into the objective function. Then, the previous job assignment becomes a standard dynamic optimization problem. To establish continuity in

$m_t^-(x, \mathbf{s}^t, \mu_t)$ and $m_t^+(x, \mathbf{s}^t, \mu_t)$, first use the fact that the transition functions satisfy the Feller property and that continuity for $\mathbf{s}^t$ is preserved by integration, see, e.g., Bergin and Bernhardt ([9], Lemmas 1 and 2). Second, continuity with respect to $\mu_t$ follows from the continuity in $f(k, e)$ since $\mu_t$ affects $w_t$ only through its influence in the opportunity cost of a job $r_t$. (Normally, in a constrained allocation, the Planner would also need to form expectations about $\mu_{t+1}$ but we have conditioned on $\{\mu\}$.) ■

Proposition 3 describes the search decisions in the economy. A market representation of the assignment is not problematic because wages (also conditional on $\{\mu\}$) are easily computed. In a market, search decisions are perhaps easier to describe. For instance, Proposition 3 can be understood as in Lucas and Prescott [39] because there are 3 cases to consider:

*Case A:* Some or all current workers in $x$ leave, $m_t^-(x, \mathbf{s}^t, \mu_t) > 0$. For these workers, the expected benefit of staying, current and future advertised wages, is smaller than the opportunity cost of moving, $\theta_t(\mathbf{s}^t, \mu_t)$, see equation (13).

*Case B*: No additional worker arrives and no worker leaves location $x$. If current and future wages exceed the return to search in equation (13), $m_t^-(x, \mathbf{s}^t, \mu_t) = 0$. In this case, workers are not willing to lose their expected wage in order to search and $l_{t+1}(x, \mathbf{s}^t, \mu_t) = l_t(x, \mathbf{s}^{t-1}, \mu_{t-1})$.

*Case C:* Workers move into $x$ if the future expected value of the location, is larger than the opportunity cost of searching, $\theta_t(\mathbf{s}^t, \mu_t)$, see equation (14).

Finally notice that for each history $\mathbf{s}^t$, the assignment of jobs and workers needs to be consistent with the evolution of the labor force so $\theta_t(\mathbf{s}^t, \mu_t)$ must ensure that the workers who search for jobs are ‘absorbed’ by the future labor market.

Next we examine the evolution of the aggregate distribution of the labor force to establish the existence of a solution to the assignment problem. At the end of period $t$, when searchers arrive, the mass of agents in $A \in \mathcal{A}$ is $\int_A l_{t+1}(x, \mathbf{s}^t, \mu_t)dx$. Recall Figure 1. The distribution of the labor force at the beginning of period $t+1$, when shocks are realized, is then:

$$\begin{aligned} \mu_{t+1}(A, \mathbf{s}^t, \mu_t) &= \int_{\mathbf{W}} \int_A l_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^t, \mu_t)\Phi(x, d\omega)dx, \\ &= \int_{\mathbf{X}} l_{t+1}(x, \mathbf{s}^t, \mu_t)\Phi(A, x)dx, \end{aligned} \tag{15}$$

where the second line follows by the no aggregate uncertainty condition.

All the relevant information needed to describe a solution to the assignment problem in this economy is contained in $\mu_{t+1}(A, \mathbf{s}^t, \mu_t)$. For example, a solution requires consistency between the previous economic decisions about job creation, job assignments, and worker search with the sequence of distributions $\{\mu\}$ we have considered thus far. That is, any distribution $\mu_{t+1}$ on $\mathcal{P}(\mathbf{X}, \mathcal{A})$ whose marginal distribution agrees with the distribution $\mu_t$ is consistent with $\mu_t$ at time $t+1$. Since equation (15) defines a mapping $T : \mathcal{P}^\infty(\mathbf{X}, \mathcal{A}) \to \mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$, a topological fixed point $\mu^* = T(\mu^*)$ is a solution to the assignment problem:

**Theorem 3 (Existence and Welfare Theorems)** *For a given $(K_0, \mathbf{z}_0, \mu_0)$, $\phi_0(x)$, and a trading friction $q_t(x)$, there is a solution to the general assignment problem. Moreover, any solution is constrained Pareto optimal and it can be implemented in a competitive market.*

**Proof.** See Appendix. ■

Characterizing the general assignment is difficult because the distribution of the labor force is a state variable. Because workers need to form expectations about the future distribution of the labor force, any future equilibrium must be consistent with these expectations. If there are multiple equilibria, workers need additional information to determine which equilibrium was expected last period. The fact that more than one distribution $\mu_t$ may be consistent with current conditions is an unresolved problem in dynamic economies with frictions and heterogeneous agents, see, e.g., Duffie et al. [17], Kubler and Schmedders [34], and Miao [40].

## 2.3 The Beveridge curve

This sub-section examines the predicted response of unemployment and vacancies to productivity shocks. Throughout this section we consider assignments along a solution $\{\mu_t^*(\mathbf{s}^t, \mu_{t-1}^*; \mathbf{s}_0, \mu_0)\}$. The employment rate is $E_t(\mathbf{s}^t, \mu_t^*) = \int_{\mathbf{X}} q_t(x) e_t(x, \mathbf{s}^t, \mu_t^*) dx$. The unemployment rate is:

$$U_t(\mathbf{s}^t, \mu_t^*) = 1 - \int_{\mathbf{X}} q_t(x) e_t(x, \mathbf{s}^t, \mu_t^*) dx,$$

which is the result of *search unemployment*, $U_t^{\text{search}}(\mathbf{s}^t, \mu_t^*) = \int_{\mathbf{X}} m_t(x, \mathbf{s}^t, \mu_t^*) dx$, and *waiting unemployment*, $U_t^{\text{waiting}}(\mathbf{s}^t, \mu_t^*) = \int_{\mathbf{X}} [1 - q_t(x)] e_t(x, \mathbf{s}^t, \mu_t^*) dx$.

As expected, $E_t(\mathbf{s}^t, \mu_t^*) + U_t(\mathbf{s}^t, \mu_t^*) = 1$. Notice that in contrast to search, where workers are not unemployed in any particular locale, waiting unemployment takes place *in* the labor market while workers wait for trade to take place. Furthermore, as in equation

(3), vacancies are $V_t(\mathbf{s}^t, \mu_t^*) = \int_{\mathbf{X}} \chi_q(x) k_t(x, \mathbf{s}^t, \mu_t^*) dx$.

Consider the response to a positive productivity shock for any single location $x$. An increase in $\mathbf{z}_t$ or $\phi_t(x)$ makes speculation in $k_t^*(x)$ less costly and this increases assignments to all locations including those with $\chi_q(x) = 1$. This makes vacancies increase. A positive productivity shock also makes work more attractive in comparison to search for two reasons. First, current advertised wages increase when productivity increases (due to the increase in capital but also by the direct effect of productivity). Second, if shocks are persistent, a positive productivity shock also increases expected future productivity and induces households to substitute consumption intertemporally. This increases the number of jobs in the future and future expected wages. Both effects cause $U_t^{\text{search}}(\mathbf{s}^t, \mu_t^*)$ to decline, see, e.g., equation (13).

If no additional effect was present, this economy will trace out a *Beveridge curve* or a negative comovement between vacancies and unemployment. However, because employment $e_t^*(x)$ increases, waiting unemployment increases and this offsets the effects of search. (In our calibrated example, this offsetting effect would actually produce a positively sloped Beveridge curve when there is low substitution in the production function.) Moreover, positive productivity shocks also make potential wages across locations more unequal and this increases the benefit of searching. Thus, whereas vacancies are increasing in productivity, the overall response in unemployment rates is ambiguous.

## 2.4 Rest unemployment and some generalizations

The general equilibrium model presented here can be generalized in several directions including, but not limited to, firm entry and exit, capacity utilization, labor market participation, fiscal policies, partially observed shocks, adjustment costs, costly vacancy posting, and stock-flow matching.[14] In here, we suggest a simple extension to examine the role of *rest unemployment* as studied by Jovanovic [32], Gouge and King [24], and Alvarez and Shimer [4]. The analysis is related to labor market participation because we do not distinguish the return to rest from the return to nonmarket production.

Assume once workers enter the market they receive the following options: wait for trade to take place (as before), search for an alternative location (as before), or *rest* in their current location. If rest carries no benefit, workers will not rest because positive expected wages make rest unattractive. Assume workers receive a return $\mathbf{b}$ if they decide to rest. Workers will then opt to enter the queue and wait if $w_t(x, \mathbf{z}^t, \mu_t) \geq \mathbf{b}$. Search decisions will also change because the benefit of staying is now $\max\{w_t(x, \mathbf{z}^t, \mu_t), \mathbf{b}\} + \beta \mathbb{E}_t \left[ R_{t+1}(\mathbf{s}^{t+1}, \mu_{t+1}) \max\{w_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^{t+1}, \mu_{t+1}), \mathbf{b}\} \right]$. Rest works like a *truncation* on the distribution of the labor force. For example, the distribution of the labor force may be written as $\hat{\mu}_t(A|\mathbf{b}) = \mu_t(\{x : x \in A, w_t(x, \mathbf{z}^t, \mu_t) \geq \mathbf{b}\})$, see, e.g., Gouge and King [24], Jovanovic [32], and Alvarez and Shimer [4]. In this case, the solution to the assignment will be a fixed point for $\{\hat{\mu}\}$.

[14] We have assumed that all jobs depreciate at the same rate whether they are filled or not and that labor supply decisions are inelastic. Similarly, trading uncertainty exists for all potential trades but one can consider the case in which only new jobs face such uncertainty or the case in which only searchers face trading uncertainty. Both cases can be seen as rough generalizations of the *stock-flow matching* proposed by Coles and Smith [13]. In stock-flow matching models, workers match only with the inflow of new jobs whereas firms may also have to wait for a suitable new worker to enter the labor market in order to fill a position.

Search, rest, and waiting are interesting because they offer a formal description of the various degrees of separation seen in actual labor markets, see, e.g., Jones and Riddell [31]. *Searchers* are clearly not in the market because, as in Lucas and Prescott [39], they are in transition. Workers who *rest* are marginally attached to the labor market because they are not participating in the current round of trade whereas workers who are *waiting* are in the market and in the current round of trade so they are clearly more likely to find employment than workers who rest and searchers in the current period.

# 3 A quantitative exploration

This section provides a parametrization designed to explore the quantitative properties of the model. While the paper is mainly concerned with methodological aspects, it is useful to examine the quantitative predictions in terms of the Beveridge curve and the volatility of unemployment and vacancies. Both aspects have been the subject of a recent controversy in macroeconomics, see, e.g., Shimer [56], Mortensen and Nagypál [44], and Hagedorn and Manovskii [27].

The model is parametrized next. Assume $\Pi$ is exponential. Then, $q_t(x) = \exp\{-\eta(x - \bar{x})\}$ for $x \geq \bar{x}$. Assume a Cobb-Douglas production function with a constant unit of land in each location, $f(k,e) = k^{\alpha}e^{1-\alpha-b}$ with $\alpha \in (0,1)$ and $b \in (0, 1-\alpha)$. Assume also an isoelastic utility function, $u(c) = (1-\varphi)^{-1}(c^{1-\varphi} - 1)$. Wages under the previous parametrization are:

$$\hat{w}_t(x, \mathbf{s}^t, \mu_t) = (1-\alpha-b)\left[\mathbf{z}_t\phi_t(x)q_t(x)\left(\frac{r_t(\mathbf{s}^t,\mu_t)}{\alpha}\right)^{-\alpha} e_t(x,\mathbf{s}^t,\mu_t)^{-b}\right]^{1/(1-\alpha)}, \tag{16}$$

with $r_t(\mathbf{s}^t, \mu_t)$ determined from the static job assignment. In our previous notation, i.e., (13) and (14), wages are $w_t(x, \mathbf{s}^t, \mu_t) = \hat{w}_t(x, \mathbf{s}^t, \mu_t) c_t(\mathbf{s}^t, \mu_t)^{-\varphi}$. Productivity shocks are given by $\ln \mathbf{z}_t = \rho \ln \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t$ with $\sigma_{\boldsymbol{\varepsilon}}^2$ as the variance of $\boldsymbol{\varepsilon}_t$. Idiosyncratic shocks are also AR(1) with the same correlation but variance $\sigma_\phi^2$.

Table 1. Calibrated parameter values in the baseline model.

| Parameter | Baseline value | Description | | Source |
|---|---|---|---|---|
| $\beta$ | 0.989 | Time discount factor | | Cooley [14] |
| $\varphi$ | 1 | Intertemporal elasticity of substitution | | Cooley [14] |
| $\delta$ | 0.012 | Depreciation rate | | |
| $\alpha$ | 0.30 | Capital share | Labor share in production, 0.60 | Cooley [14] |
| $b$ | 0.10 | 'Land' share | | |
| $\rho$ | 0.895 | Persistence of productivity shocks | | BLS data |
| $\sigma_{\boldsymbol{\varepsilon}}, \sigma_\phi$ | 0.0083 | Volatility of shocks, $\mathbf{z}_t$ and $\phi_t(x)$ | | |
| $\underline{x}$ | 0.7 | Fraction of locations not subject to trade frictions | | JOLTS |
| $\eta$ | 0.05 | Probability of trade | | Calibrated |

Note: The parameters are discussed in the text. We assume parameters as in typical real business cycle models, see, e.g., Cooley ([14], 22). The estimates of the persistence and volatility of shocks are based on the quarterly real average output per person in the non-farm business sector from the BLS. The solution algorithm is described in the Appendix.

The following parameters need to be determined. The time discount factor $\beta$, the depreciation rate $\delta$, the persistence of productivity shocks $\rho$, the variance of innovations $\sigma_{\boldsymbol{\varepsilon}}^2$ and $\sigma_\phi^2$, the shares $\alpha$ and $b$, the intertemporal elasticity of substitution $\varphi$, and the trading friction parameters $\eta$ and $\underline{x}$. Additionally, we need to specify a model period. We consider each period to be a quarter and assume a depreciation rate of $\delta = 0.012$. This measure is the conventional value for quarterly capital depreciation in real business cycle models. The discount factor is assumed to be $\beta = 1/1.045^{1/4}$ and $\varphi = 1$ or a log-utility. For the shares in production we assume $\alpha = 0.30$ and $b = 0.10$ with $b$ representing the role of 'land.' The labor share, 0.60, is also the value typical of real business cycle models, see Cooley ([14], 22).

Table 2. Summary statistics for the labor market.

| | $U$ | $V$ | $V/U$ | Job finding | Job separation | $\mathbf{z}$ |
|---|---|---|---|---|---|---|
| | Quarterly US data, 1951 to 2003 | | | | | |
| Standard deviation | 0.190 | 0.202 | 0.382 | 0.118 | 0.075 | 0.020 |
| Quarterly autocorrelation | 0.936 | 0.940 | 0.941 | 0.908 | 0.733 | 0.878 |
| | Correlation matrix | | | | | |
| $U$ | 1 | -0.894 | -0.971 | -0.949 | 0.709 | -0.408 |
| $V$ | | 1 | 0.975 | 0.897 | -0.684 | 0.364 |
| $V/U$ | | | 1 | 0.948 | -0.715 | 0.396 |
| Job finding rate | | | | 1 | -0.574 | 0.396 |
| Job separation rate | | | | | 1 | -0.524 |
| $\mathbf{z}$ | | | | | | 1 |

Source: Shimer [56]. The data has been filtered by the Hodrick-Prescott filter.

The parameters that govern $\ln \mathbf{z}_t$ can be estimated directly from quarterly data on labor productivity. As suggested by Shimer [55], we employ the seasonally adjusted quarterly real average output per person in the non-farm business sector constructed by the BLS. We also employ the Hodrick-Prescott filter with a smoothing parameter of $100,000$. The estimates obtained are $\rho = 0.895$ and $\sigma_{\varepsilon} = 0.0083$.[15] The parameters $\sigma_\phi$, $\eta$, and $\underline{x}$ are key determinants of unemployment and vacancies. For the benchmark model, we assume

[15] The difference between total factor productivity and labor productivity is not important as both are proportional to each other. In typical real business cycle models (Kydland and Prescott [35]), the previous parameters are not very different. The estimates of $\rho$ and $\sigma_{\varepsilon}$ are also similar to those in Hagedorn and Manovskii [27].

that each location's volatility of productivity is the same as the volatility of aggregate productivity measured from the data.

To determine $\underline{x}$, we assume that 70 percent of the locations do not need to post a vacancy. In JOLTS from 2000 to 2005, the percent of worker hires with zero vacancies was 42.4 percent and the percent of establishments with zero vacancies was 87.6, see, e.g., Davis et al. ([16], Table 2). We take 70 percent as a rough average of both numbers. In the model, in locations $\underline{x} \leq x$, firms produce without hires or hire workers without posting a vacancy. Finally, we calibrate $\eta$ to obtain an average measure of the vacancy rate of 3 percent. The average value of the vacancy rate in the US, between 2000 and 2006, is 2.3 percent, see, e.g., Shimer ([55], 1083). We, however, measure vacancies relative to the capital stock. Because the employment rate is close to one in the simulations, the vacancy rate taken with respect to employment levels is close to 3 percent as well. Also, we treat waiting and search unemployment as a single measure in part because in the data, marginally attached workers' behavior is closer to the behavior of unemployed workers. Empirically, waiting is as productive as searching in terms of finding employment, see, e.g., Jones and Riddell [31].

Because $\mu_t$ is a state variable, we need to reduce the dimensionality of this distribution. Several alternatives are possible, but here we discretize $\mathbf{X}$ and treat $\mu_t$ as a vector of dimension $1 \times L$. Then, the state of the economy lies in $\mathbf{S} \times [0,1]^{L \times 1}$. We assume $L = 10$ or that there are 10 locations. Alternatively, one can use a limited number of moments of the distribution, or parameterize the distribution itself. Since the current stage is exploratory, and since there is no agreed method on how to quantitatively study frictional economies with heterogeneous agents, we leave alternative strategies for future work. In terms of the assignment of searchers, we assume, as in Lucas and Prescott [39], that searchers are randomly assigned (i.e., search is not coordinated or directed to locations with rents).

Table 3. Summary statistics and simulations for the labor market.

| | $U$ | $V$ | $V/U$ | Job finding | Job separation | $Y$ |
|---|---|---|---|---|---|---|
| | Simulations from the model | | | | | |
| Standard deviation | 0.3215 | 0.118 | 0.3696 | 0.5347 | 0.0277 | 0.0206 |
| | (0.012) | (0.008) | (0.016) | (0.027) | (0.000) | (0.000) |
| Quarterly autocorrelation | -0.1250 | 0.6360 | -0.1317 | -0.4800 | -0.5251 | 0.4472 |
| | (0.0273) | (0.0312) | (0.0248) | (0.0171) | (0.0159) | (0.0453) |
| | Correlation matrix | | | | | |
| $U$ | 1 | -0.2552 | -0.9511 | -0.5744 | 0.7239 | -0.6114 |
| | | (0.0318) | (0.0054) | (0.0123) | (0.0099) | (0.0000) |
| $V$ | | 1 | 0.5410 | 0.3383 | -0.4770 | 0.2098 |
| | | | (0.0192) | (0.0210) | (0.0185) | (0.0000) |
| $V/U$ | | | 1 | 0.6074 | -0.7818 | 0.5987 |
| | | | | (0.0102) | (0.0078) | (0.0000) |
| Job finding rate | | | | 1 | -0.8957 | 0.3288 |
| | | | | | (0.063) | (0.0000) |
| Job separation rate | | | | | 1 | -0.4582 |
| | | | | | | (0.0000) |

Note: Each sample is based on 2500 periods. Cross-sample moments from 250 samples that assign workers and jobs across 10 locations. The data has been filtered by the Hodrick-Prescott filter. The solution method is described in the Appendix.

The solution method used for the simulations is described in detail in the Appendix. Shocks are drawn first. The sequence of shocks is taken as given. We parameterize the expected wages in different locations using a flexible nonlinear function that depends on the state of the economy. We take those expectations as given and solve for the worker and job assignment. The solution is used to update the expectations and then we solve the dynamic problem again until convergence is achieved. We create 250 samples each of 2500 periods (the first/last 25 percent of the sample is removed) and report model moments and cross-sample standard deviations of those moments. The average measure of unemployment associated with the previous parameters is 4.47 percent and the average vacancy rate 2.94 percent. The standard errors in both cases are 0.0006. In the post-World War II period, the average unemployment rate in the US has been 5 percent.

Table 2 summarizes labor-market data and Table 3 summarizes the model-generated data. The last column in Table 3 shows aggregate output. Measures of productivity are highly correlated with output as Table 4 shows later on. As the last column of Table 3 shows, unemployment is highly countercyclical whereas vacancies are procyclical as suggested also by Table 2. The first two columns show unemployment and vacancies. Both series are more volatile than output and unemployment is almost twice as volatile as in the data. The reason for this high volatility can be understood by looking at equation (16). Because productivity shocks create wage differentials across locations, worker mobility is needed to offset these rents. However, when $b$ is small, the amount of worker mobility needed to equalize the value of a location is relatively large and hence the economy produces a highly volatile unemployment rate. Since vacancies and employment are closely related (in fact, if mobility was frictionless, the capital-labor ratio will be constant across locations), part of the volatility of employment is transmitted to vacancies. (In order to verify the role of $b$ in generating high volatilities in the labor market, we considered a case with $b = 0.30$ in the Appendix. In this case, not only is volatility reduced but the negative correlation between unemployment and vacancies is also closer to zero. The response to changes in other parameters is also studied in the Appendix.)

The model is also consistent with a Beveridge curve. Table 3 shows a negative correlation between unemployment and vacancies although the correlation is weaker than in the data. Because waiting unemployment and vacancies are positively correlated, the observed relationship between total unemployment and vacancies is weaker. For instance, the correlation between vacancies and search unemployment is -0.5259 (s.e.=0.0201). The correlation between waiting unemployment and vacancies is 0.91 while the correlation between search and waiting unemployment is -0.55. The vacancy/unemployment ratio is also highly volatile in the model. Measures of job finding and separation rates have been estimated as the probability an unemployed worker finds employment in the current period and the probability an employed worker becomes unemployed. The difference between the volatility in both measures is also consistent with that in the data.

Table 4. Output and real variables in the simulations.

| | $Y$ | $\mathbf{z}$ | $c$ | $i$ | $\mathbb{E}_x[\hat{w}(x)]$ | $r$ |
|---|---|---|---|---|---|---|
| Correlation with $Y$ | 1 | 0.6524 | 0.0829 | 0.9980 | 0.2540 | 0.9600 |
| | | (0.0220) | (0.1583) | (0.0001) | (0.0417) | (0.0090) |
| Standard deviation | 0.0206 | 0.0148 | 0.0005 | 0.0222 | 0.0201 | 0.0194 |
| | (0.0006) | (0.0007) | (0.0001) | (0.0007) | (0.0008) | (0.006) |

Note: Each sample is based on 2500 periods. Cross-sample moments from 250 samples that assign workers and jobs across 10 locations. The data has been filtered by the Hodrick-Prescott filter. The solution method is described in the Appendix.

The contemporary correlation between unemployment and vacancies with respect to $V/U$ and the finding and separation rates are also consistent with those in the data.

The same is also true for finding and separation rates. The main shortcoming of the model is the absence of persistence in unemployment. Whereas unemployment is highly persistent in the data, the model predicts a negative autocorrelation in unemployment. This is so because worker mobility is driven by idiosyncratic reasons. Because vacancies are associated with capital assignments, vacancies are persistent in the model and have a serial autocorrelation that is closer to the one on productivity shocks. The quarterly autocorrelation of $V/U$ and finding and separation are also the opposite of the ones in the data.

Finally, Table 4 reports moments for output, consumption, investment, the cross-sectional average of advertised wages, and the rate of return to capital. Output is highly correlated with aggregate productivity shocks although it is more volatile than aggregate shocks due to idiosyncratic shocks and worker search. The model also predicts a stable consumption but the correlation with output is weaker than in the data. In part, this is so because the Euler equation for consumption is not precisely estimated in the model. In terms of wages, the model predicts a small correlation with output. The volatility of wages and the return to capital is similar to that of output and considerably smaller than that of unemployment and vacancies. This fact is a prominent feature of the business cycle fluctuations in the labor market.

# 4 Some remarks about previous theories of mismatch

This last section provides an exposition of the conditions under which mismatch exists in models where information is perfect, see, e.g., Lagos [36], Shimer [56], and Mortensen

[43]. These models can be seen as are special cases of the assignment (1) with a Leontief meeting technology in micro-markets.[16] We also provide some remarks on the treatment of labor market heterogeneity in the class of models that follow Mortensen and Pissarides [46].

First we consider an adaptation of Lagos [36] in a familiar assignment setting. Assume throughout that trading frictions are absent, i.e., $q_t(x) = 1$ for all $x$. Assume also that $\mathbf{z}_t = 1$. Since it is very relevant for mismatch, we begin by reviewing the existence of unemployment in models with perfect information:

**Proposition 4 (Unemployment)** *In the static assignment (1), assume* $f_k(0, l) = \boldsymbol{\rho} < +\infty$ *for all* $l$*. Then, it is optimal to assign no jobs to locations with* $\phi_t(x)\boldsymbol{\rho} < r_t$*.*

Optimality requires $[\phi_t(x) f_k(k_t(x), l_t(x)) - r_t]\, k_t(x) = 0$. Thus, if $f_k(0, l_t(x)) = \boldsymbol{\rho} < \infty$, it is optimal to assign $k_t(x) = 0$ to locations for which $\phi_t(x)\boldsymbol{\rho} < r_t$. These locations would misuse scarce jobs, see, e.g., Akerlof [1] and [2]. Define unemployment as the mass of jobless workers, i.e., $U_t(K_t, \mu_t) = 1 - \int_{\phi_t(x)\boldsymbol{\rho} \geq r_t} l_t(x)dx$. Unemployment, however, is consistent with a Walrasian equilibrium since prices ensure that the *value* of excess supply is zero. In a competitive market, wages are: $w_t(x) = \phi_t(x) f_e(k_t(x), l_t(x))$. If $f_e(0, l) = 0$, the labor market *clears* with some markets having excess supply of labor but zero wages.

Because $\boldsymbol{\rho} = 1$, the Leontief technology is known to generate unemployment (see, e.g., Akerlof [1]). Under some special conditions, a Leontief technology also generates

[16] Lagos [36] studied a model of the matching of taxis and passengers in a setting in which one taxi is used by exactly one passenger. In Lagos [36], taxis are free to locate at any point in a linear city but the assignment features vacant taxis that coexist with unserved passengers. Mismatch in Lagos [36], arises when there is a small number of taxis, some locations are more attractive than others, and the technology of the assignment is exactly Leontief (i.e., the short side of the market is always served). Shimer [56] and Mortensen [43] considered a random assignment closely related to Lagos [36].

mismatch. Let $Y_t(r_t, K_t, \mu_t) = \max\left\{\int_{\mathbf{X}} \phi_t(x) \min\left[k_t(x), l_t(x)\right] dx : \int_{\mathbf{X}} k_t(x)dx \leq K_t\right\}$ represent a *Planner* version of the assignment. Lagos [36] considered a similar problem but instead of looking at the Planner problem, Lagos [36] considered a *market* that equalizes average productivity across locations:

$$\left(\phi_t(x) \min\left[1, \frac{l_t(x)}{k_t(x)}\right] - r_t\right) k_t(x) = 0. \tag{17}$$

Notice that workers in locations with $\phi_t(x) < r$ are unemployed (as in Proposition 4). For locations with jobs, assignments are $\phi_t(x)l_t(x)/k_t(x) = r_t$. The opportunity cost $r_t$, however, is *exogenous* and there is no market clearing:

**Proposition 5 (Mismatch)** *The market assignment in equation (17) exhibits excess supply of jobs in locations with $\phi_t(x) > r_t$ simultaneously with excess supply of workers in locations with $\phi_t(x) < r_t$. If $\phi_t(0) \geq r_t$, workers are in excess supply if $r_t K_t < \int_{\mathbf{X}} \phi_t(x) l_t(x) dx$.*

**Proof.** The excess supply of workers is as in Proposition 4. Since $k_t(x) = \phi_t(x)l_t(x)/r_t$ and $\phi_t(x)/r_t > 1$ for $\phi_t(x) > r_t$, there is also an excess supply of jobs regardless of $K_t$, i.e., $k_t(x) > l_t(x)$ in these markets. The total amount of jobs assigned is: $\int_{\phi_t(x) \geq r_t} k_t(x)dx = \frac{1}{r_t}\int_{\phi_t(x)\geq r_t} \phi_t(x)l_t(x)dx$. Notice that when $\phi_t(0) \geq r_t$, all workers are employable but the assignment would generate excess supply of workers if $K_t$ is small.[17] ■

Proposition 5 describes a market with *excess supply of jobs* and *excess supply of work-*

[17] Notice that $M_t(K_t, r_t, \mu_t) = \min\{K_t, r_t^{-1} \int_{\phi_t(x)\geq r_t} \phi_t(x)l_t(x)dx\}$ is an aggregate matching function. If $\phi_t(0) = r_t$, the matching function is $\min\{\boldsymbol{\phi}_t K_t, \mu_t(\mathbf{X})\}$, with $\boldsymbol{\phi}_t^{-1} = \phi_t(0)^{-1} \int_{\mathbf{X}} \phi_t(x)l_t(x)dx$, which is the exact form presented in Lagos ([36], Section V). Also, market clearing only takes place at the marginally employed location; see equation (17).

*ers*. The proposition is a generalization of Lagos ([36], Proposition 2) but here workers and jobs are in excess supply because the cost of a job is too high *or* because there are few jobs to be assigned (which is the case Lagos [36] discussed originally). One can argue that $K_t$ and $r_t$ should be related but since there is no market equilibrium, variations in $K_t$ and in $r_t$ are independent. The allocation, however, is clearly *inefficient* and holds only for a Leontief technology in micro-markets:

**Corollary 1 (Efficiency)** *The market assignment in equation (17) is socially inefficient.*

**Corollary 2 (Knife-edge)** *Let* $f(k_t, l_t) = [\alpha k_t^{(\boldsymbol{\sigma}-1)/\boldsymbol{\sigma}} + (1-\alpha) l_t^{(\boldsymbol{\sigma}-1)/\boldsymbol{\sigma}}]^{\boldsymbol{\sigma}/(\boldsymbol{\sigma}-1)}$ *denote the production function of the job assignment. If* $\boldsymbol{\sigma} \to 0$*, workers and jobs are in excess supply in a market assignment as in Proposition 5. If* $0 < \boldsymbol{\sigma} < 1$*, workers are in excess supply as in Proposition 4 but there is no excess supply of jobs, while if* $\boldsymbol{\sigma} \geq 1$ *there is no excess supply of jobs or workers.*

In a Planner assignment, locations with $\phi_t(x) < r_t$ will receive no jobs while locations with $\phi_t(x) \geq r_t$ will have $k_t(x) = l_t(x)$ and no excess supply of jobs. This intensive margin differs in a Planner assignment because a competitive case equalizes *average* productivity.[18] Moreover, as Corollary 2 shows, if some substitution is allowed in micro-markets, mismatch disappears. This will also be true for Shimer [56] and Mortensen [43] where job and worker assignments are random but the production function is still $\min[k_t(x), l_t(x)]$.[19]

[18] The inefficiency in Lagos [36] is typical of directed search or common resource problems (e.g., in the 'tragedy of the commons'). In the directed search literature, an additional job offer lowers the probability of having a successful match in a given micro-market and this generates an externality, see Moen [42].

[19] Additional models that emphasize the spatial nature of mismatch include Coulson et al. [15] and Brueckner and Martin [11]. Sattinger [52] and Stevens [57] also discuss applications of queueing theory relevant for mismatch. In this class of models, an aggregate matching function can be derived from random search at the individual level.

The aggregate matching function derived in this paper also contrasts with matching models that follow the Mortensen and Pissarides [46] tradition. Matching models assume that if there are $U_t$ unemployed workers and $V_t$ vacancies, the number of matches is $M_t(U_t, V_t)$.[20] This reduced-form matching function has been criticized by the absence of microfoundations, see, e.g., Lagos [36]. A reduced-form matching also makes important assumptions about labor market *heterogeneity.* Suppose that we try to study the effects of heterogeneity using an exogenous matching technology. To the extent that unemployment and vacancies are unchanged, an exogenous matching function predicts no change in the economy.

In contrast, the *aggregate* matching function derived in this paper, as well as the *aggregate* production function $Y_t(K_t, \mathbf{z}_t, \mu_t)$ associated with the assignment, depend on the whole distribution of the labor force.[21] Thus, a *pure* increase in heterogeneity generates changes in all the markets in the economy. Let $\tilde{\mu}_t(A) = \int_A \tilde{l}_t(x)dx$ with $\int_{\mathbf{X}} q_t(x)[\tilde{l}_t(x) - l_t(x)]dx = 0$ and $\int_0^x q_t(y)[\tilde{l}_t(y) - l_t(y)]dy \geq 0$. Let also $\int_{\mathbf{X}} \chi_q(x)[\tilde{k}_t(x) - k_t(x)]dx = 0$, and $\int_0^x \chi_q(x)[\tilde{k}_t(y) - k_t(y)]dy \geq 0$. It is obvious that the economies $(\mu_t, k_t(x))$ and $(\tilde{\mu}_t, \tilde{k}_t(x))$

[20] There is a large literature that studies business cycles using a non-Walrasian labor market coupled with Walrasian goods and capital markets, see, e.g., Andolfatto [5] and Mertz [41]. Veracierto [59] considered a model in which firms are subject to a matching function and workers search for jobs as in Lucas and Prescott [39]. Rocheteau and Wright ([49], 188-192) studied search in a monetary economy similar to Lucas and Prescott [39] and introduced a notion of a vacancy under the assumption that not all agents enter into a competitive market. While market entrance is probabilistic, the notion of a vacancy in this paper, a notion based on uncertain trade, is very different from that in Rocheteau and Wright [49].

[21] Consider once again the assignment (1) with uncertain trade in all locations, i.e., $\chi_q(x) = 1$ for all $x$. Under the Cobb-Douglas production function above, if $b = 0$ and $\phi_t(x) = 1$, the assignment of jobs satisfies $k_t(x) = (r_t/\alpha)^{1/(\alpha-1)} q_t(x)^{1/(1-\alpha)} l_t(x)$. Market clearing implies $K_t = (r_t/\alpha)^{1/(\alpha-1)} \int_{\mathbf{X}} q_t(x)^{1/(1-\alpha)} l_t(x)dx$. Using equation (5), the aggregate matching function (4) can be approximated around $\alpha = 0$ by:

$$M_t(U_t, V_t) \approx \frac{V_t \Delta_1}{1 - U_t + \Delta_2},$$

with $\Delta_1 = \int_{\mathbf{X}} q_t^2(x) l_t(x)[1 + \Gamma(q_t(x))]dx$ and $\Delta_2 = \int_{\mathbf{X}} q_t(x) l_t(x) \Gamma(q_t(x))dx$ for $\Gamma(q_t(x)) = \sum_{i=1} [\ln q_t(x)]^i$. The previous is an increasing function of vacancies and unemployment. This matching function depends on $U_t$ and $V_t$ although it is also a function of the distribution of the labor force and the extent of trading frictions collected in $\Delta_1$ and $\Delta_2$.

advertise the same measure of vacancies and have the same measure of unemployment but there is no reason for the two economies to have the same matches or output in the assignment (1).

## 5 Conclusion

This paper examined an economy in which unemployment and vacancies coexist in a competitive Arrow-Debreu equilibrium. A crucial element of the model are informational frictions given by trading uncertainty as in Prescott [48], Butters [10], Gale [23], and Eden [18]. In the model, trade is uncertain and markets are incomplete and segmented. There are some markets that open and clear in the typical sense, while there are also markets where trade was expected but not realized. Because trading frictions are given at the market level (and not within each separate market, as commonly considered in the search and matching literature), the physical environment is consistent with classical Arrow-Debreu economies.

Worker and job assignments are made before knowing if trade will take place so unused factors are expected in a competitive equilibrium as long as firms and workers are willing to speculate on the possibility of trade in these markets. Vacancies represent job assignments to markets were trade is uncertain. Some workers are also assigned to these markets while others are searching for alternative markets (as in Lucas and Prescott [39]). At the end of a trading period, some assigned jobs will remain vacant and some of the workers who speculated on trade will remain jobless. These forms of unemployment and vacancies are perfectly voluntary and socially efficient. Moreover, the presence of trading frictions in micro-markets generates an aggregate matching function given by the mass of locations in which the advertised jobs met the assigned workers.

To study business cycles and labor market fluctuations, the model considered aggregate and idiosyncratic shocks. Because the distribution of the labor force across locations becomes a state variable, the economy cannot be studied using a representative location. Decisions follow the evolution of the entire distribution of the population. To surmount the difficulties that arise in this case, we proved the existence of a solution as in anonymous sequential games with aggregate uncertainty (see, e.g., Bergin and Bernhardt [8]; [9]; Miao [40]). The model was calibrated and it was shown that the business cycle association between unemployment and vacancies was negative; there is a Beveridge curve. While the quantitative analysis is exploratory, in addition to the Beveridge curve, the model produced high volatilities in unemployment and vacancies.

One way to understand the mechanics behind the Beveridge curve and the high volatility in the labor market is the following. In an economy with segmented markets, a positive aggregate productivity shock makes capital and labor more productive across all markets. In markets where trade is uncertain, the response is to advertise more jobs hoping that trade will indeed take place. A positive productivity shock thus induces more vacancies. The response of unemployment is not as simple as in the case of vacancies because a productivity shock also makes speculation across markets more likely and this increases waiting unemployment (i.e., the measure of workers who speculated in locations that failed to open).

Allowing for equilibrium search helps to make the unemployment rate countercyclical.

This response arises because a positive productivity shock increases current and future wages and this makes search unattractive. The quantitative effects depend on the importance of diminishing returns and the volatility of productivity shocks among other parameters. While the marginal product of capital will be equalized across locations, wages will not be equal. When there are decreasing returns to scale and search is costly, the worker's value of a good location will be reduced only after labor movements take place. If large labor movements are needed to reduce these rents, the labor market becomes highly volatile.

# 6 Appendix

This Appendix presents the proof of Theorem 3, additional results regarding the recursive representation of the general assignment, and the parameterized expectation method employed in the quantitative section.

**Proof of Theorem 3.** The proof follows the arguments in the proof of Miao ([40], Theorem 1) and applies Brouwer-Schauder-Tychonoff Fixed-Point Theorem to $T$. One needs to show that the domain of $T$ is compact, that $T$ maps $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$ into itself, and that $T$ is continuous. Because $Y_t(K_t(\mathbf{z}^{t-1}, \mu_{t-1}), \mathbf{z}^t, \mu_t)$ acts as a typical aggregate production technology, the aggregate capital is uniformly bounded or there is a *maximum sustainable* stock of capital. Consider a policy in which consumptions are always zero. The condition that $\lim_{k\to\infty} f_k(k, e) + 1 - \delta < 1$ implies that there is a $\bar{K} = \sup\{K(\mathbf{z}, \mu) : K(\mathbf{z}, \mu) = Y(K(\mathbf{z}, \mu), \mathbf{z}, \mu) + (1 - \delta)K(\mathbf{z}, \mu)\} < +\infty$. For every assignment sequence, $\bar{K} \geq \int_{\mathbf{X}} k_0(x, \mathbf{s}_0, \mu_0)dx$, and $\bar{K} \geq \int_{\mathbf{X}} k_t(x, \mathbf{s}^t, \mu_t)dx$ so job assignments are uniformly bounded.

The space of aggregate shocks $\mathbf{Z}$ is also bounded and compact by assumption. This makes $\mathbf{S}$ bounded and compact. Because $\mathbf{X}$ is assumed compact, $\mathcal{P}(\mathbf{X}, \mathcal{A})$ is also weak* compact, see, e.g., Aliprantis and Border ([3], Th. 15.11). Finally, recall that $\{\mu\}$ is a function from $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$. Endowed with the product (or pointwise convergence) topology, $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$ and the set $\mathbf{S}^\infty \times \mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$ are compact, see, e.g., Aliprantis and Border ([3], Th. 2.69).

By equations (15) and (11), $T(\mu)$ lies in $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$. That is, the cost of search ensures that the mapping $T$ returns a distribution function in $\mathcal{P}^\infty(\mathbf{X}, \mathcal{A})$. Finally, $T$ is continuous because the search decisions and job assignments are continuous, see Propositions 2 and 3. This establishes existence. Pareto optimality follows by definition whereas implementability is as in Theorem 1 and already discussed.

In a competitive equilibrium, the representative consumer maximizes equation (9), subject to: $K_{t+1}(\mathbf{z}^t, \mu_t) = R_t(\mathbf{s}^t, \mu_t)K_t(\mathbf{z}^{t-1}, \mu_{t-1}) + \int_{\mathbf{X}} \hat{w}_t(x, \mathbf{s}^t, \mu_t)[l_t(x, \mathbf{s}^t, \mu_t) - m_t^-(x, \mathbf{s}^t, \mu_t)]dx + \pi_t(\mathbf{s}^t, \mu_t) - c_t(\mathbf{s}^t, \mu_t)$, and equations (8) and (11). Let $\pi_t(\mathbf{s}^t, \mu_t) = \max \int_{\mathbf{X}} \{\mathbf{z}_t \phi_t(x) q_t(x) f(k_t(x, \cdot), e_t(x, \cdot)) - r_t(\mathbf{s}^t, \mu_t)k_t(x, \cdot) - \hat{w}_t(x, \mathbf{s}^t, \mu_t)e_t(x, \cdot)\}dx$ be firms' profits.

A *competitive market equilibrium* is a generalization of the static case. Equilibrium consists of state-dependent market wages $\{\hat{w}_t(x, \mathbf{s}^t, \mu_t) \text{ for all } x\}$, a market price for capital $R_t(\mathbf{s}^t, \mu_t)$, and an opportunity cost of search $\theta_t(\mathbf{s}^t, \mu_t)$; as well as utility maximizing household decisions for capital accumulation, consumption and search $\{K_t(\mathbf{z}^t, \mu_t), c_t(\mathbf{s}^t, \mu_t); m_t^-(x, \mathbf{s}^t, \mu_t), m_t^+(x, \mathbf{s}^t, \mu_t)$ for all $x\}$, and profit maximizing decisions for capital use and employment $\{k_t(x, \mathbf{s}^t, \mu_t), e_t(x, \mathbf{s}^t, \mu_t)$ for all $x\}$ such that aggregate feasibility conditions are satisfied, i.e., $\int_{\mathbf{X}} k_t(x, \mathbf{s}^t, \mu_t)dx = K_t(\mathbf{z}^{t-1}, \mu_{t-1})$ and equations (10), (11), and markets that open clear.

The equivalence of optimality conditions between a market equilibrium and the Planner problem completes the proof. ■

**Recursive equilibrium.** The discussions of the general assignment have thus far only considered a sequential representation of the problem. As show by Miao [40], and Bergin and Bernhardt [9], one can establish an equivalence with a recursive equilibrium if the state-space is rich enough. In particular, if one conditions on $\{\mu\}$:

**Proposition 6 (Recursive assignment)** *Given $\{\mu\}$, for each $t \in \{0, 1., ..., +\infty\}$, there exists a function $\mathsf{W}_t(\mathbf{s}^t, \{\mu\})$ that gives the value of the assignment and satisfies:*

$$\mathsf{W}_t(\mathbf{s}^t, \{\mu\}) = \max\left\{u(c_t(\mathbf{s}^t, \mu_t)) + \beta \mathbb{E}_t \mathsf{W}_{t+1}(\mathbf{s}^{t+1}, \{\mu\})\right\},$$

*subject to equation (10). Moreover, the value of employment in location $x$ satisfies:*

$$\mathsf{v}_t(x, \mathbf{s}^t, \{\mu\}) = \max\left\{\theta_t(\mathbf{s}^t, \mu_t), w_t(x, \mathbf{s}^t, \mu_t) + \min\left[\theta_t(\mathbf{s}^t, \mu_t), \beta \mathbb{E}_t \mathsf{v}_{t+1}(\phi_{t+1}(x, \omega), \mathbf{s}^{t+1}, \{\mu\})\right]\right\},$$

*with wages $w_t(x, \mathbf{s}^t, \mu_t)$ defined as in Proposition 3, with feasibility in labor mobility given by (11), and with jobs based on a static assignment that solves $Y_t(\mathbf{s}^t, \mu_t)$.*

**Proof.** Because we have conditioned on $\{\mu\}$, the job creation representation is standard for dynamic programs, see, e.g., Stokey, Lucas, and Prescott ([58], Chapter 4). The worker assignment can also be described using a recursive representation similar to that of Lucas and Prescott [39].

Let $\hat{f}(e_t(x, \mathbf{s}^t, \mu_t)) = \max_k \{\mathbf{z}_t \phi_t(x) q_t(x) f(k, e_t(x, \mathbf{s}^t, \mu_t)) - r_t(\mathbf{s}^t, \mu_t)k)\} u_c(c_t(\mathbf{s}^t, \mu_t))$, represent the value of a locations once jobs are assigned. Consider an $n$-period truncation. In period $n$, the value of any given location from the worker's perspective is $\mathsf{v}_n(x, \mathbf{s}^n, \{\mu\}) = w_n(x, \mathbf{s}^n, \mu_n)$ because $m^+(x, \mathbf{s}^n, \mu_n) = m^-(x, \mathbf{s}^n, \mu_n) = 0$. For period $n-1$, the worker assignment solves:

$$\max_{m^{-,+}(x, \mathbf{s}^{n-1}, \mu_{n-1})} \left\{\hat{f}(e_{n-1}(x, \mathbf{s}^{n-1}, \mu_{n-1})) + \beta \mathbb{E}_{n-1}[\hat{f}(e_n(x, \mathbf{s}^n, \mu_n)]\right\},$$

subject to the feasibility conditions of the general assignment.

Using the $n$-period truncation and Proposition 3, if some (or all) workers leave location $x$, then $\mathsf{v}_{n-1}(x, \mathbf{s}^{n-1}, \{\mu\}) = \theta_{n-1}(\mathbf{s}^{n-1}, \mu_{n-1})$. Also, $\mathsf{v}_{n-1}(x, \mathbf{s}^{n-1}, \{\mu\}) = w_{n-1}(x, \mathbf{s}^{n-1}, \mu_{n-1}) + \beta \mathbb{E}_{n-1}[\mathsf{v}_n(\phi_n(x, \omega), \mathbf{s}^n, \{\mu\})]$ if there are no movements. If there are workers who arrive to location $x$, $\beta \mathbb{E}_{n-1}[\mathsf{v}_n(\phi_n(x, \omega), \mathbf{s}^n, \mu_n)] = \theta_{n-1}(\mathbf{s}^{n-1}, \mu_{n-1})$. Combining all previous cases yields the functional equation $\mathsf{v}_{n-1}(x, \mathbf{s}^{n-1}, \{\mu\})$ above. Induction will work for periods $0 \leq t \leq n$. If $\lim_{n \to +\infty} \beta \mathbb{E}_{n-1}\left[\mathsf{v}_n(x, \mathbf{s}^j, \{\mu\})\right]$ is defined, the sequential problem can be represented by the value function, see, e.g., Stokey, Lucas, and Prescott ([58], Theorem 4.2). ■

Because equilibrium prices generally depend on the distribution of people across locations, it is natural to include the aggregate distribution $\mu_t$ as a state variable. But the current distribution of the labor force $\mu_t$ may not be enough to describe the state of the economy and for that reason we conditioned on the whole sequence $\{\mu\}$, see Miao [40] and Kubler and Schmedders [34] for additional remarks.

**Solution method.** To solve the model we use a parameterized expectations algorithm, see, e.g., den Haan and Marcet [26] and Cooley ([14], 89-90). First, we generate a sequence of aggregate and idiosyncratic productivity shocks $\{(\mathbf{z}_t, \phi_t(x)) : t = 0, 1, ..., T \text{ and } x = 1, ..., L\}$. These realizations respect the stochastic processes assumed in the model and are held fixed in all subsequent steps. All decisions can be seen as functions of these sequences of realizations.

The general assignment satisfies a series of Euler equations for consumption and worker mobility; equations (12), (13), and (14) with two mobility equations for each location. The number of Euler equations is then $2L + 1$. These equations are summarized by

$$G(\mathbb{E}_t \left[g(\mathbf{s}_{t+1}, \mu_{t+1})\right], \mathbf{s}_t, \mu_t) = 0,$$

with $\mathbb{E}_t \left[g(\mathbf{s}_{t+1}, \mu_{t+1})\right]$ as the conditional expectations of the system of Euler equations. The algorithm approximates the conditional expectations in the Euler equations by a first-order exponentiated polynomial,

$$\boldsymbol{\Psi}(\mathbf{s}_t, \mu_t; \{\overrightarrow{\gamma}, \overrightarrow{\boldsymbol{\eta}}\}) = \exp\left\{\sum\nolimits_{x=1}^{L} \overrightarrow{\gamma} \times \mu_t(x) + \overrightarrow{\boldsymbol{\eta}} \times \ln \mathbf{s}_t\right\},$$

with $\{\overrightarrow{\gamma}, \overrightarrow{\boldsymbol{\eta}}\}$ as a vector of coefficients to be found. (The choice of function is polynomial since such a function can approximate any general function well.) We initialize $\{\overrightarrow{\gamma}, \overrightarrow{\boldsymbol{\eta}}\}$ by drawing a random sequence of worker assignments and using a nonlinear least squares fit of the Euler equations:

$$\{\overrightarrow{\gamma}, \overrightarrow{\boldsymbol{\eta}}\} \in \arg\min T^{-1} \sum\nolimits_{t=1}^{T} \left\|\mathbb{E}_t \left[g(\mathbf{s}_{t+1}, \mu_{t+1})\right] - \boldsymbol{\Psi}(\mathbf{s}_t, \mu_t; \{\overrightarrow{\gamma}, \overrightarrow{\boldsymbol{\eta}}\})\right\|^2.$$

Given the parameterized Euler equations, we use bisection methods to solve the assignment of workers and jobs for any period $t$. Once allocations have been determined, we update the expectations by re-estimating the parameters $\{\overrightarrow{\gamma}, \overrightarrow{\boldsymbol{\eta}}\}$ until convergence is achieved. In order to guarantee that the no aggregate uncertainty condition is satisfied, once mobility have been determined, we update the distribution of the labor force as in (15) using a transition matrix $\Phi(\cdot, x)$ approximated from the first-order autoregressive process for productivity. Finally, in the rare event in which wage expectations predict a positive effect of employment on wages, we impose a bound in the estimated coefficient given by $-b/(1-\alpha)$ which is the value in contemporary wages, see equation (16).

**Sensitivity analysis.** The following are additional results when some central parameters of the model are changed. The response in the labor market is as one expected to be. An increase in $b$ reduces the volatility of unemployment and vacancies as well as the volatility of aggregate output. This also increases the persistence of vacancies but it reduces the slope of the Beveridge curve. Changes in the volatility of productivity shocks increase the volatility of the labor market and the volatility of output. More volatile idiosyncratic shocks also generate a positively-sloped Beveridge curve sometimes. The same is also the case for a low elasticity of substitution between workers and jobs in production. This is the case because workers and jobs behave closer to the case in which they are in fixed proportions and hence they move in the same direction. Other changes in the predictions of the model are similar to what one might expect from the real business cycle literature, i.e., a lower elasticity of intertemporal substitution makes output and the labor market more volatile.

Table A1. Summary statistics and simulations for the labor market.

| | Standard deviation | | | Quarterly autocorrelation | | Beveridge | Corr. |
|---|---|---|---|---|---|---|---|
| | $U$ | $V$ | $Y$ | $U$ | $V$ | curve | $U^{search}$, $V$ |
| $b = 0.30$ | 0.2562 | 0.0482 | 0.0183 | -0.1272 | 0.8142 | -0.0575 | -0.2732 |
| | (0.0266) | (0.0026) | (0.0000) | (0.0166) | (0.0233) | (0.0314) | (0.0177) |
| $\sigma_{\boldsymbol{\varepsilon}}= 0.083$ | 0.4729 | 0.3060 | 0.1726 | -0.0259 | 0.5380 | -0.2549 | -0.4637 |
| | (0.0108) | (0.0297) | (0.0000) | (0.0313) | (0.0465) | (0.0362) | (0.0423) |
| $\sigma_{\phi}= 0.083$ | 0.5244 | 0.4100 | 0.0662 | 0.0716 | 0.7885 | 0.1553 | -0.1046 |
| | (0.0119) | (0.0254) | (0.0000) | (0.0240) | (0.0142) | (0.0348) | (0.0225) |
| $\boldsymbol{\sigma}= 0.5$ | 0.0627 | 0.0469 | 0.0172 | 0.1650 | 0.8019 | 0.1804 | -0.4682 |
| | (0.0316) | (0.0227) | (0.0000) | (0.0423) | (0.0279) | (0.0644) | (0.0517) |
| $\varphi = 0.5$ | 0.4219 | 0.1857 | 0.0344 | -0.1061 | 0.5644 | -0.2781 | -0.5611 |
| | (0.0202) | (0.0115) | (0.0000) | (0.0230) | (0.0417) | (0.0322) | (0.0247) |
| $\delta = 0.10$ | 0.2497 | 0.1030 | 0.0211 | -0.0544 | 0.7491 | -0.1295 | -0.4560 |
| | (0.0667) | (0.0280) | (0.0000) | (0.0282) | (0.0553) | (0.0370) | (0.0368) |

Note: Each sample is based on 2500 periods. Cross-sample moments from 10 samples and only one parameter is changed at each time. The data has been filtered by the Hodrick-Prescott filter. The case of $\boldsymbol{\sigma}= 0.5$ is based on a CES production function of the form $f(k,e) = (\alpha k^{(\boldsymbol{\sigma}-1)/\boldsymbol{\sigma}} + (1 - \alpha - b)e^{(\boldsymbol{\sigma}-1)/\boldsymbol{\sigma}} + b)^{\boldsymbol{\sigma}/(\boldsymbol{\sigma}-1)}$.

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