Financial Aggregation and Index Number theory: Implications for Monetary Policy and Financial Modelling, A Survey

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July 21, 2008

1. Introduction

In this paper we survey the relevant theory and the most dramatic of the empirical evidence. We present the evidence in chronological order. The concerns, puzzles, and paradoxes in the literature have changed over time, in accordance with the evolving policy concerns. The first dramatic puzzle appeared in 1974, when it was purported that there had been a mysterious shift in both the demand and supply for money functions. We provide an overview of the issues and empirical evidence with emphasis on graphical displays. What has been happening in this literature and in policy is most clearly displayed graphically. We cite the original publications for those readers interested in the formal econometrics used in resolving the paradoxes.

We conclude with a discussion of the most recent research in this area, which introduces state space factor modeling into this literature. We also display the most recent puzzle regarding Federal Reserve data on nonborrowed reserves and show that the recent behavior of that data contradicts the definition of nonborrowed reserves: in short, is an oxymoron. Far from resolving the earlier data problems, the Federal Reserve's most recent data may be the most puzzling that the Federal Reserve has ever published.

There is a vast literature on the appropriateness of aggregating over monetary asset components using simple summation. Linear aggregation can be based on Hicksian aggregation (Hicks 1946), but that theory only holds under the unreasonable assumption that the user-cost prices of the services of individual money assets not change over time. This condition implies that each asset is a perfect substitute for the others within the set of components. Simple sum aggregation is an even more severe special case of that highly restrictive, linear aggregation, since simple summation requires that the coefficients of the linear aggregator function all be the same. This, in turn, implies that the constant user-cost prices among monetary assets be exactly equal to each other. Not only must the assets be perfect substitutes, but must be perfect one-for-one substitutes --- i.e., must be indistinguishable assets, with one unit of each asset being a perfect substitute for exactly one unit of each of the other assets.

In reality, financial assets provide different services, and each such asset yields its own particular rate of return. As a result, the user costs, which measure foregone interest and thereby opportunity cost, are not constant and are not equal across financial assets. The relative prices of U.S. monetary assets fluctuate considerably, and the interest rates paid on many monetary assets are not equal to the zero interest rate paid on currency. These observations have motivated serious concerns about the reliability of the simple sum aggregation method, which has been disreputable in the literature on index number theory and aggregation theory for over a century. In addition, an increasing number of imperfect substitute short-term financial assets have emerged in recent decades. Since monetary aggregates produced from simple summation do not accurately measure the quantities of monetary services chosen by optimizing agents, shifts in the series can be spurious, as those shifts do not necessarily reflect a change in the utility derived from money holdings.

Microeconomic aggregation theory offers an appealing alternative approach to the definition of money, compared to the atheoretical simple-sum method. The quantity index under the aggregation theoretic approach extracts and measures the income effects of changes in relative prices and is invariant to substitution effects, which do not alter utility and thereby do not alter perceived services received. The simple sum index, on the other hand, does not distinguish between income and substitution effects, if the aggregate's components are not perfect substitutes in identical ratios, and thereby the simple sum index confounds together substitution effects with actual services received. The aggregation-theoretic monetary aggregator function, which correctly internalizes substitution effects, can be tracked accurately by the Divisia quantity index, constructed by using expenditure shares as the component growth-rate weights. Barnett (1978,1980) derived the formula for the theoretical user-cost price of a monetary asset, needed in computation of the Divisia index's share weights, and thereby originated the Divisia monetary aggregates. The growth rate weights resulting from this approach are different across assets, depending on all of the quantities and interest rates in each share, and those weights can be time-varying at each point in time. For a detailed description of the theory underlying this construction, see Barnett (1982,1987).

It is important to understand that the direction in which an asset's growth-rate weight will change with an interest rate change is not predictable in advance. Consider Cobb-Douglas utility. Its shares are independent of relative prices, and hence of the interest rates within the component user cost prices. For other utility functions, the direction of the change in shares with a price change, or equivalently with an interest rate change, depends upon whether the own price elasticity of demand exceeds or is less than -1. In elementary microeconomic theory, this often overlooked phenomenon produces the famous "diamonds versus water paradox" and is the source of most of the misunderstandings of the Divisia monetary aggregates' weighting, as explained by Barnett (1983).

Several authors have studied the empirical properties of the Divisia index compared with the simple sum index. The earliest comparisons are in Barnett (1982) and Barnett, Offenbacher, and Spindt (1984). More recent examples include Belongia (1996), Belongia and Ireland (2006), and Schunk (2001), and the comprehensive survey found in Barnett and Serletis (2000). In particular, Belongia (1996) replicates some studies on the impact of money on economic activity and compares results acquired using a Divisia index instead of the originally used simple sum index, Schunk (2001) investigates the forecasting performance of the Divisia index compared with the simple sum aggregates, and Belongia and Ireland (2006) explore the policy implications in the dual space of aggregated user costs and interest rates. Barnett and Serletis (2000) collect together and reprint seminal journal articles from this literature.[1]

The most recent research in this area is Barnett, Chauvet, and Tierney (2008), who compare the different dynamics of simple sum monetary aggregates and the Divisia indexes, not only over time, but also over the business cycle and across high and low inflation and interest rate phases. The potential differences between the series can be economically very important. If one of the indexes corresponds to a better measure of money, its dynamical differences from the official simple sum aggregates increase the already considerable uncertainty regarding the effectiveness and appropriateness of current monetary policy. Barnett, Chauvet, and Tierney aim to find the nature of the differences and whether they occur during particular periods. Information about the state of monetary growth becomes particularly relevant for policymakers, when inflation enters a high growth phase or the economy begins to weaken. In fact Barnett (1997) has argued and documented the connection between the decline in the policy credibility of monetary aggregates and defects that are peculiar to simple sum aggregation.

Although traditional comparisons of the monetary aggregates series sometimes suggest that the series share similar long run dynamics, there are differences during certain important periods, such as around turning points. These differences cannot be evaluated by long run average behavior. The Barnett, Chauvet, and Tierney (2008) proposed approach offers several ways in which these differences can be analyzed. The approach separates out the common movements underlying the monetary aggregate indexes, summarized in the dynamic factor, from individual variations specific to each of the indexes, captured by the idiosyncratic terms. The idiosyncratic terms and the measurement errors reveal where the monetary indexes differ.[2] The idiosyncratic terms show the movements that are peculiar to each series,

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[1] Other overviews of published theoretical and empirical results in this literature are available in Barnett, Fisher, and Serletis (1992) and Serletis (2006).

[2] In aggregation theory measurement error refers to the tracking error in a nonparametric index number's approximation to the aggregator function of microeconomic theory, where the aggregator function is the subutility or subproduction function that is weakly separable within tastes or technology of an economic agent's complete utility or production function. Consequently, aggregator functions are increasing and concave and need to be estimated econometrically. On the other hand, state space models use the term measurement error to mean un-modeled noise, which is not captured by the state variable or idiosyncratic terms. In

whereas the measurement error captures the remaining noise inherent in the data. That is, the dynamic factor represents simultaneous downturn and upturn movements in money growth rate indexes. If only one of the indexes declines, this would be captured by its idiosyncratic term.

Barnett, Chauvet, and Tierney (2008) model both the common factor as well as the idiosyncratic terms for each index as following different Markov processes. Given that the idiosyncratic movements are peculiar to each index, the idiosyncratic terms' Markov processes are assumed to be independent of each other. In addition, Barnett, Chauvet, and Tierney allow the idiosyncratic terms to follow autoregressive processes. These assumptions entail a very flexible framework that can capture the dynamics of the differences across the indexes without imposing dependence between them.

Factor models with regime switching have been widely used to represent business cycles (see e.g., Chauvet (1998, 2001), Kim and Nelson (1998), among several others), but without relationship to aggregation theory. Barnett, Chauvet, and Tierney's proposed model differs from the literature in its complexity, as it includes estimation of the parameters of three independent Markov processes. In addition, the focus is not only on the estimated common factor, but on the idiosyncratic terms that reflect the divergences between the simple sum and Divisia monetary aggregate indexes in a manner relevant to aggregation theory.

To our knowledge, there is no parallel work in the literature that formally compares simple sum aggregate with the Divisia index directly, using a multivariate time-series framework to *estimate* the dynamical differences between these series in a manner extracting the idiosyncratic terms specific to each aggregate. Barnett, Chauvet, and Tierney's (2008) contribution goes beyond the simple comparison over time, as they also focus on major measurement errors that might have occurred during some periods, such as around the beginnings or ends of recessions or in transition times, as from low (high) to high (low) inflation or interest rate phases.

2. Monetary Aggregation Theory

2.1. Monetary Aggregation

Aggregation theory and index-number theory have been used to generate official governmental data since the 1920s. One exception still exists. The monetary quantity aggregates and interest rate aggregates

this paper, measurement error refers to this latter definition, which can be expected to be correlated with the former, when the behavior of the data process is consistent with microeconomic theory. But it should be acknowledged that neither concept of measurement error can be directly derived from the other. In fact the state space model concept of measurement error is more directly connected with the statistical ("atomistic") approach to index number theory than to the more recent "economic approach," which is at its best when data is not aggregated over economic agents.

supplied by many central banks are not based on index-number or aggregation theory, but rather are the simple unweighted sums of the component quantities and the quantity-weighted or arithmetic averages of interest rates. The predictable consequence has been induced instability of money demand and supply functions, and a series of 'puzzles' in the resulting applied literature. In contrast, the Divisia monetary aggregates, originated by Barnett (1980), are derived directly from economic index-number theory. Financial aggregation and index number theory was first rigorously connected with the literature on microeconomic aggregation and index number theory by Barnett (1980; 1987).

Data construction and measurement procedures imply the theory that can rationalize the aggregation procedure. The assumptions implicit in the data construction procedures must be consistent with the assumptions made in producing the models within which the data are to be used. Unless the theory is internally consistent, the data and its applications are incoherent. Without that coherence between aggregator function structure and the econometric models within which the aggregates are embedded, stable structure can appear to be unstable. This phenomenon has been called the 'Barnett critique' by Chrystal and MacDonald (1994).

2.2. Aggregation Theory versus Index Number Theory

The exact aggregates of microeconomic aggregation theory depend on unknown aggregator functions, which typically are utility, production, cost, or distance functions. Such functions must first be econometrically estimated. Hence the resulting exact quantity and price indexes become estimator- and specification-dependent. This dependency is troublesome to governmental agencies, which therefore view aggregation theory as a research tool rather than a data construction procedure.

Statistical index-number theory, on the other hand, provides indexes which are computable directly from quantity and price data, without estimation of unknown parameters. Within the literature on aggregation theory, such index numbers depend jointly on prices and quantities, but not on unknown parameters. In a sense, index number theory trades joint dependency on prices and quantities for dependence on unknown parameters. Examples of such statistical index numbers are the Laspeyres, Paasche, Divisia, Fisher ideal, and Törnqvist indexes.

The loose link between index number theory and aggregation theory was tightened, when Diewert (1976) defined the class of second-order 'superlative' index numbers, which track any unknown aggregator function up to the second order. Statistical index number theory became part of microeconomic theory, as economic aggregation theory had been for decades, with statistical index numbers judged by their non-parametric tracking ability to the aggregator functions of aggregation theory.

For decades, the link between statistical index number theory and microeconomic aggregation theory was weaker for aggregating over monetary quantities than for aggregating over other goods and asset quantities. Once monetary assets began yielding interest long ago, monetary assets became imperfect substitutes for each other, and the 'price' of monetary-asset services was no longer clearly defined. That problem was solved by Barnett (1978; 1980), who derived the formula for the user cost of demanded monetary services.[3]

Barnett's results on the user cost of the services of monetary assets set the stage for introducing index number theory into monetary economics.

2.3. The Economic Decision

Consider a decision problem over monetary assets. The decision problem will be defined in the simplest manner that renders the relevant literature on economic aggregation over goods immediately applicable.[4] Initially we shall assume perfect certainty.

Let $\mathbf{m}_t' = (m_{1t}, m_{2t}, \dots, m_{nt})$ be the vector of real balances of monetary assets during period t, let $\boldsymbol{r}_t$ be the vector of nominal holding-period yields for monetary assets during period t, and let R_t be the one period holding yield on the benchmark asset during period t. The benchmark asset is defined to be a pure investment that provides no services other than its yield, R_t, so that the asset is held solely to accumulate wealth. Thus, R_t is the maximum holding period yield in the economy in period t.

Let y_t be the real value of total budgeted expenditure on monetary services during period t. Under simplifying assumptions for data within one country, the conversion between nominal and real expenditure on the monetary services of one or more assets is accomplished using the true cost of living index, $p_t^* = p_t^*(\mathbf{p}_t)$, on consumer goods, where the vector of consumer goods prices is $\mathbf{p}_t$.[5] The optimal portfolio allocation decision is:

$$\text{maximize } u(\mathbf{m}_t) \qquad (1)$$

$$\text{subject to } \boldsymbol{\pi}_t' \mathbf{m}_t = y_t,$$

[3] Subsequently Barnett (1987) derived the formula for the user cost of supplied monetary services. A regulatory wedge can exist between the demand and supply-side user costs, if non-payment of interest on required reserves imposes an implicit tax on banks.
[4] Our research in this paper is not dependent upon this simple decision problem, as shown by Barnett (1987), who proved that the same aggregator function and index number theory applies, regardless of whether the initial model has money in the utility function, or money in a production function, or neither, so long as there is intertemporal separability of structure and certain assumptions are satisfied for aggregation over economic agents. The aggregator function is the derived function that has been shown in general equilibrium always to exist, if money has positive value in equilibrium, regardless of the motive for holding money.
[5] The multilateral open economy extension is available in Barnett (2007).

where $\boldsymbol{\pi}_t' = (\pi_{1t}, \ldots, \pi_{nt})$ is the vector of monetary-asset real user costs, with

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t}\,. \tag{2}$$

The function u is the decision maker's utility function, assumed to be monotonically increasing and strictly concave.[6] The user cost formula (2), derived by Barnett (1978; 1980), measures the forgone interest or opportunity cost of holding monetary asset i, when the higher yielding benchmark asset could have been held.

Let $\mathbf{m}_t^*$ be derived by solving decision (1). Under the assumption of linearly homogeneous utility, the exact monetary aggregate of economic theory is the utility level associated with holding the portfolio, and hence is the optimized value of the decision's objective function:

$$M_t = u(\mathbf{m}_t^*). \tag{3}$$

2.4. The Divisia Index

Although equation (3) is exactly correct, it depends upon the unknown function, u. Nevertheless, statistical index-number theory enables us to track M_t exactly without estimating the unknown function, u. In continuous time, the monetary aggregate, $M_t = u(\mathbf{m}_t^*)$, can be tracked exactly by the Divisia index, which solves the differential equation

$$\frac{d \log M_t}{dt} = \sum_{\mathrm{i}} s_{it} \frac{d \log m_{it}^*}{dt} \tag{4}$$

for M_t, where

$$s_{it} = \frac{\pi_{it} m_{it}^*}{y_t}$$

is the i'th asset's share in expenditure on the total portfolio's service flow.[7] The dual user cost price aggregate $\Pi_t = \Pi(\boldsymbol{\pi}_t)$, can be tracked exactly by the Divisia price index, which solves the differential equation

[6] To be an admissible quantity aggregator function, the function u must be weakly separable within the consumer's complete utility function over all goods and services. Producing a reliable test for weak separability is the subject of much intensive research, most recently by Barnett and Peretti (2008).

[7] In equation (4), it is understood that the result is in continuous time, so the time subscripts are a short hand for functions of time. We use t to be the time period in discrete time, but the instant of time in continuous time.

$$\frac{d\log \Pi_t}{dt} = \sum_{\mathrm{i}} s_{it} \frac{d\log \pi_{it}}{dt} . \quad (5)$$

The user cost dual satisfies Fisher's factor reversal in continuous time:

$$\Pi_t M_t = \boldsymbol{\pi}_t{}' \, \mathbf{m}_t. \quad (6)$$

As a formula for aggregating over quantities of perishable consumer goods, that index was first proposed by François Divisia (1925), with market prices of those goods inserted in place of the user costs in equation (4). In continuous time, the Divisia index, under conventional neoclassical assumptions, is exact. In discrete time, the Törnqvist approximation is:

$$\log M_t - \log M_{t\text{-}1} = \sum_{\mathrm{i}} \bar{s}_{it} (\log m_{it}^* - \log m_{i,t\text{-}1}^*) , \quad (7)$$

where

$$\bar{s}_{it} = \frac{1}{2}(s_{it} + s_{i,t\text{-}1}) .$$

In discrete time, we often call equation (7) simply the Divisia quantity index.[8] After the quantity index is computed from (7), the user cost aggregate most commonly is computed directly from equation (6).

2.5. Risk Adjustment

Extension of index number theory to the case of risk was introduced by Barnett, Liu and Jensen (2000), who derived the extended theory from Euler equations rather than from the perfect-certainty first-order conditions used in the earlier index number-theory literature. Since that extension is based upon the consumption capital-asset-pricing model (CCAPM), the extension is subject to the 'equity premium puzzle' of smaller-than-necessary adjustment for risk. We believe that the under-correction produced by CCAPM results from its assumption of intertemporal blockwise strong separability of goods and services within preferences. Barnett and Wu (2005) have extended Barnett, Liu, and Jensen's result to the case of risk aversion with intertemporally non-separable tastes.[9]

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[8] Diewert (1976) defines a 'superlative index number' to be one that is exactly correct for a quadratic approximation to the aggregator function. The discretization (7) to the Divisia index is in the superlative class, since it is exact for the quadratic translog specification to an aggregator function.

[9] The Federal Reserve Bank of St. Louis Divisia database, which we use in this paper, is not risk corrected. In addition, it is not adjusted for differences in marginal taxation rates on different asset returns or for sweeps, and its clustering of components into groups was not based upon tests of weak separability, but rather on the Federal Reserve's official clustering. The St. Louis Federal Reserve Bank is in the process of revising its MSI database, perhaps to incorporate some of those adjustments. Regarding sweep adjustment, see Jones, Dutkowsky, and Elger (2005). At the present stage of this research, we felt it was best to use data available from the Federal Reserve for purposes of replicability and comparability with the official simple sum data. As

2.6. Dual Space

User cost aggregates are duals to monetary quantity aggregates. Either implies the other uniquely. In addition, user-cost aggregates imply the corresponding interest-rate aggregates uniquely. The interest-rate aggregate r_t implied by the user-cost aggregate Π_t is the solution for r_t to the equation:

$$\frac{R_t - r_t}{1 + R_t} = \Pi_t.$$

Accordingly, any monetary policy that operates through the opportunity cost of money (that is, interest rates) has a dual policy operating through the monetary quantity aggregate, and vice versa. Aggregation theory implies no preference for either of the two dual policy procedures or for any other approach to policy, so long as the policy does not violate principles of aggregation theory. In their current state-space comparisons, Barnett, Chauvet, and Tierney model in quantity space rather than the user-cost-price or interest-rate dual spaces. Regarding policy in the dual space, see Barnett (1987) and Belongia and Ireland (2006).

2.7. Aggregation Error and Policy Slackness

Figure 1 displays the magnitude of the aggregation error and policy slackness produced by the use of the simple sum monetary aggregates. Suppose there are two monetary assets over which the central bank aggregates. The quantity of each of the two component assets is y_1 and y_2. Suppose that the central bank reports, as data, that the value of the simple sum monetary aggregates is M_{ss}. The information content of that reported variable level is contained in the fact that the two components must be somewhere along the Figure 1 hyperplane, $y_1 + y_2 = M_{ss}$, or more formally that the components are in the set A:

$$A = \{(y_1,y_2): y_1 + y_2 = M_{ss}\}.$$

But according to equation (3), the actual value of the service flow from those asset holdings is $u(y_1,y_2)$. Consequently the information content of the information set A regarding the monetary service flow is that the flow is in the set E:

$$E = \{u(y_1,y_2): (y_1,y_2) \in A\}.$$

Note that E is not a singleton. To see the magnitude of the slackness in that information, observe from Figure 1 that if the utility level (service flow) is M_{min}, then the indifference curve does touch the hyperplane, A, at its lower right corner. Hence that indifference curve cannot rule out the M_{ss} reported

a result, we did not modify the St. Louis Federal Reserve's MSI database or the Federal Reserve Board's simple sum data in any ways. This decision should not be interpreted to imply advocacy by us of the official choices.

value of the simple sum monetary aggregate, although a lower level of utility is ruled out, since indifference curves at lower utility levels cannot touch the hyperplane, *A*.

Now consider the higher utility level of M_{max} and its associated indifference curve in Figure 1. Observe that that indifference curve also does have a point in common with the hyperplane, *A*, at the tangency. But higher levels of utility are ruled out, since their indifference curves cannot touch the hyperplane, *A*. Hence the information about the monetary service flow, provided by the reported value of the simple sum aggregate, M_{ss}, is the interval

$$E = [M_{min}, M_{max}].$$

The supply side aggregation is analogous, but the lines of constant supplied service flow for financial firms are production possibility curves, not indifference curves, as shown by Barnett (1987). The resulting display of the information content of the simple sum aggregate is in Figure 2, with the analogous conclusions. To make the figure easy to understand, the same symbols are used as in Figure 1, with the exception of the replacement of the utility aggregator function, *u*, with production aggregator function, y_0.

Figure 1: Demand Side Aggregation Error Range

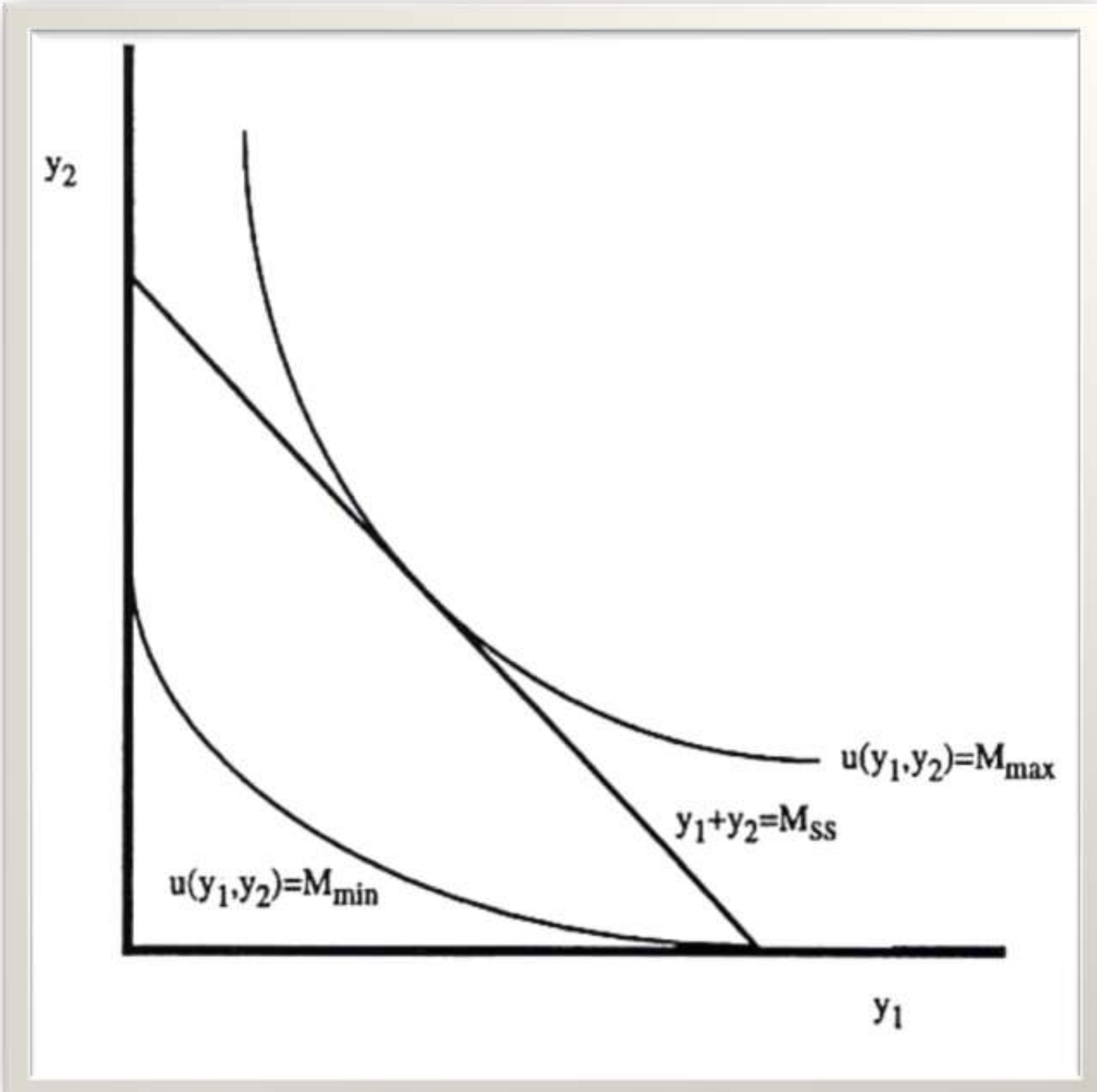
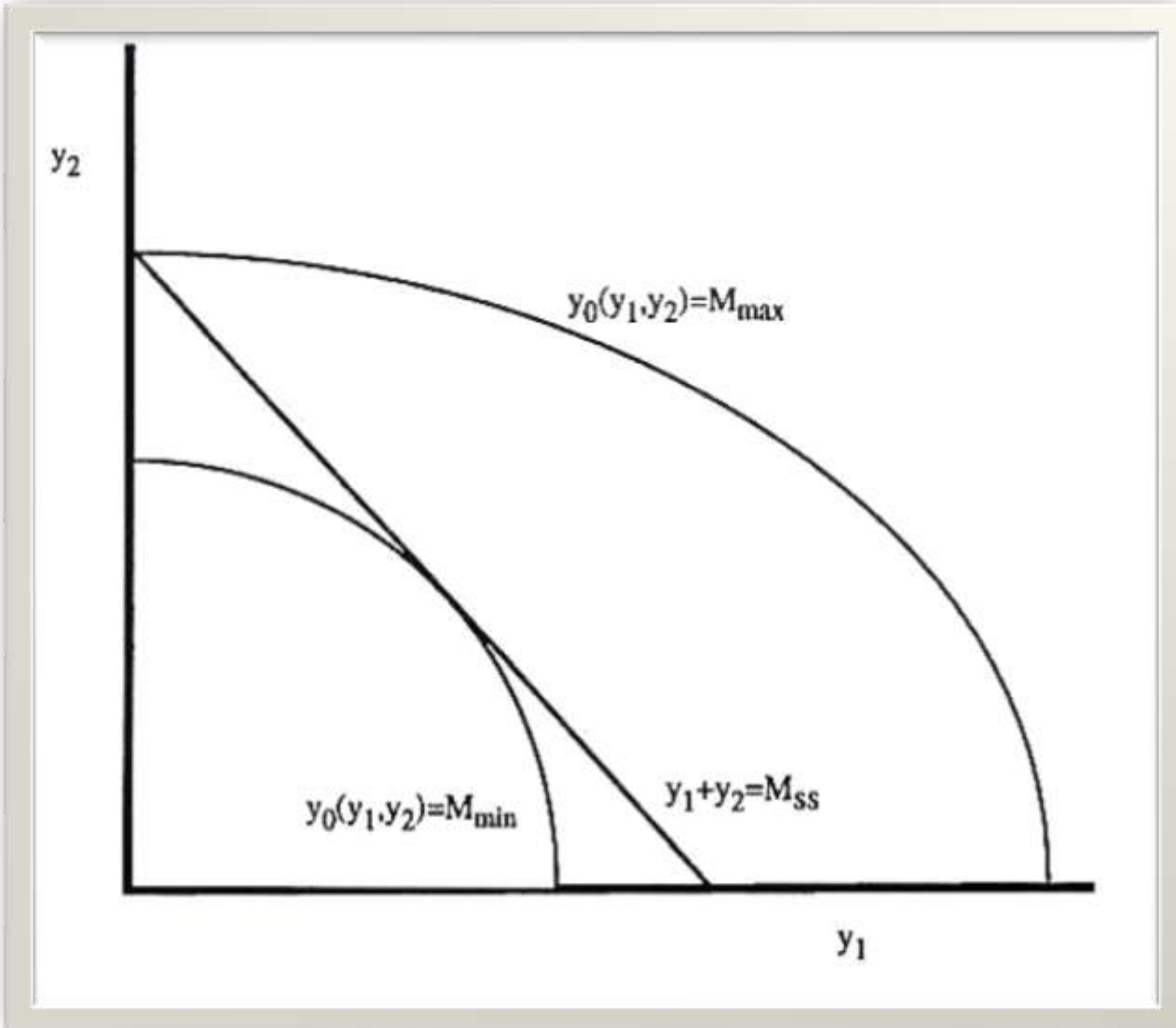


Figure 2: Supply Side Aggregation Error Range



3. The History of Thought on Monetary Aggregation

The field of aggregation and index number theory has a long history, going back for about a century. The first book to put together the properties of all of the available index numbers in a systematic manner was the famous book by Irving Fisher (1922). He made it clear in that book that the simple sum and arithmetic average indexes are the worst known indexes. On p. 29 of that book he wrote:

> "The simple arithmetic average is put first merely because it naturally comes first to the reader's mind, being the most common form of average. In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers, and if this book has

> no other effect than to lead to the total abandonment of the simple arithmetic type if index number, it will have served a useful purpose."

On p. 361 Fisher wrote:

> "The simple arithmetic should not be used under any circumstances, being always biased and usually freakish as well. Nor should the simple aggregative ever be used; in fact this is even less reliable."

Indeed data producing agencies and data producing newspapers switched to reputable index numbers, following the appearance of Fisher's book. But there was one exception: the world's central banks, which produced their monetary aggregates as simple sums. While the implicit assumption of perfect substitutability in identical ratios might have made sense during the first half of the 20th century, that assumption has made unreasonable since then, as interest bearing substitutes for currency became available from banks, including interest bearing checking accounts.

Nevertheless, the nature of the problem was understood by Friedman and Schwartz (1970, pp. 151-152), who wrote the following:

> "The [simple summation] procedure is a very special case of the more general approach. In brief, the general approach consists of regarding each asset as a joint product having different degrees of 'moneyness,' and defining the quantity of money as the weighted sum of the aggregated value of all assets…. We conjecture that this approach deserves and will get much more attention than it has so far received."

More recently, subsequent to the work of Diewert on superlative index number theory and Barnett's derivation of the user cost of monetary assets and the Divisia monetary aggregates, Lucas (2000, p. 270) wrote:

> "I share the widely held opinion that M1 is too narrow an aggregate for this period [the 1990s], and I think that the Divisia approach offers much the best prospects for resolving this difficulty."

In a related vane, Lucas (1999) stated in a published interview:

> "I am happy about the successes of general equilibrium theory in macro and sad about the de-emphasis on money that those successes have brought about."

In light of recent events, the following statement appeared in *The Economist Magazine, June 9-15, 2007, p. 88.*

> "Has the pendulum swung too far from monetarist overkill to monetary neglect? In a recent lecture, Mervyn King, the governor of Britain's inflation-targeting central bank, seemed to think so….Money, if not monetarism, is making a comeback in the way central bankers think about and carry out policy….Money still matters--as it always has done."

Even more recently, the following statement appeared in the *Wall Street Journal*, April 9, 2008. The title of the article was "Volcker's Demarche":

> "You don't have to predict it. We're in it." Thus did Paul Volcker respond to a question Tuesday about whether he still predicts a "dollar crisis" in the coming years …. The world has been staging a run on the greenback …. The present climate, Mr. Volcker told his audience, reminded him of nothing so much as the early 1970s."

4. The 1960s and 1970s

Having surveyed the theory and some of the relevant historical background, we now survey some key results. We organize them chronologically, to make the evolution of views clear. The source of the results in this section, along with further details, can be found in Barnett, Offenbacher, and Spindt (1984) and Barnett (1982).

Demand and supply of money functions were fundamental to macroeconomics and to central bank policy until the 1970s, when questions began to arise about the stability of those functions. It was common for general equilibrium models to determine real values and relative prices, and for the demand and supply for money to determine the price level and thereby nominal values. But it was believed that something went wrong in the 1970s. In Figure 3, observe the behavior of the velocity of M3 and M3+ (later called L), which were the two broad aggregates often emphasized in that literature. For the demand for money function to have the correct sign for its interest elasticity (better modeled as user-cost price elasticity), velocity must increase when nominal interest rates increase. In fact velocity should move in the same direction as nominal interest rates.

Figure 4 provides an interest rate during the same time period. Note that while nominal interest rates were increasing during the increasing inflation of that decade, the velocity of the simple sum monetary aggregates in Figure 3 were decreasing. While the source of concern is evident, note that the problem did not exist, when the data was produced from index number theory. The interest elasticity of velocity was positive for all three plots produced from index number theory.

Figure 3: Seasonally adjusted normalized velocity during the 1970s

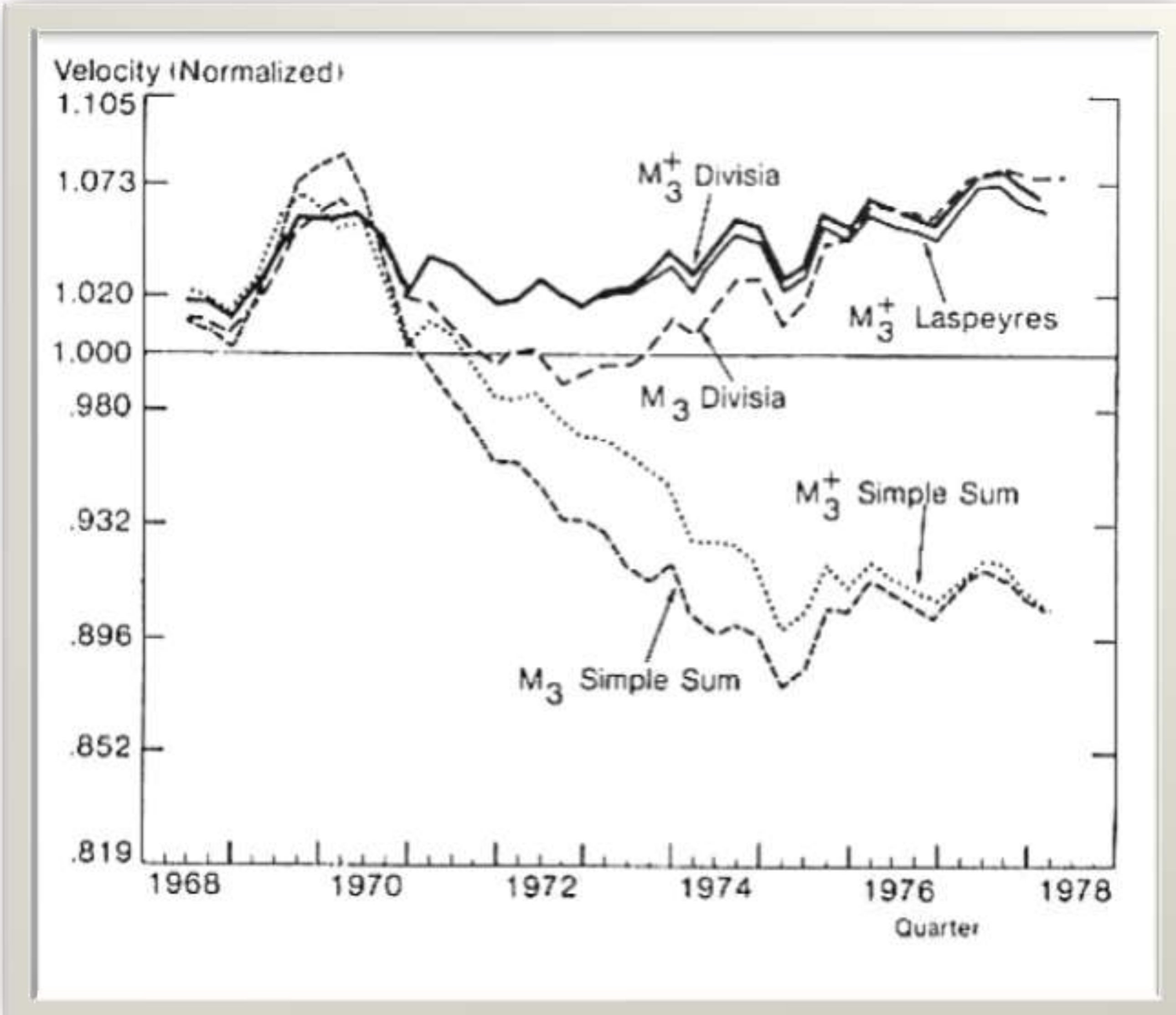
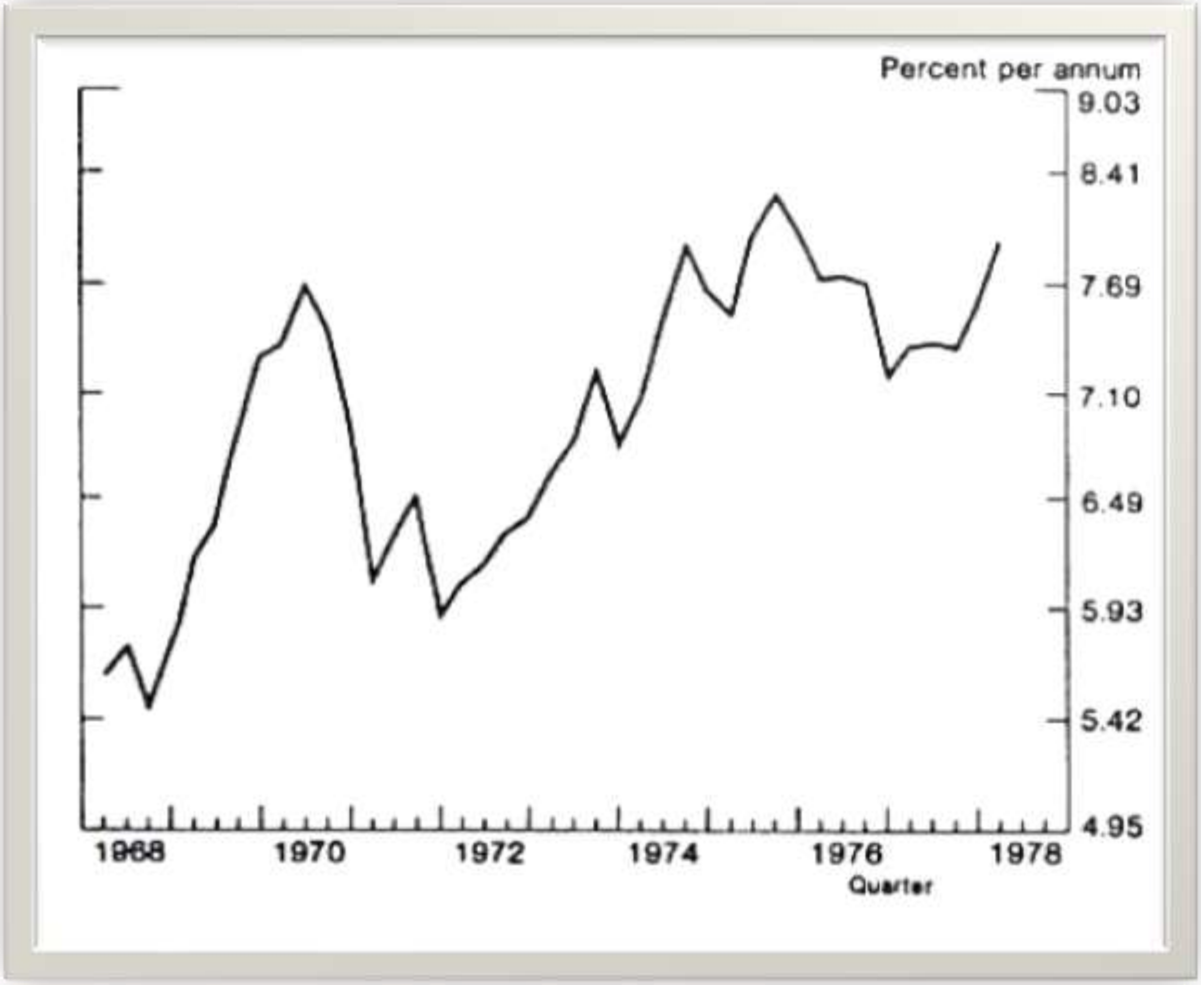


Figure 4: Interest Rates during the 1970s: 10 year government bond rate



Most of the concern in the 1970s was focused on 1974, when it was believed that there was a sharp structural shift in money markets. Figure 5 displays a source of that concern. In figure 5, we have plotted velocity against a bond rate, rather than against time, as in Figure 3. As is evident from Figure 5, there appears to be a dramatic shift downwards in that velocity function in 1974. But observe that this result was acquired using simple sum M3. Figure 6 displays the same cross plot of velocity against an interest rate, but with M3 computed as its Divisia index. Observe that velocity no longer is constant, either before or after 1974. But there is no structural shift.

There were analogous concerns about the supply side of money markets. The reason is evident from Figure 7, which plots the base multiplier against a bond rate's deviation from trend. The base multiplier is the ratio of a monetary aggregate to the monetary base. In this case, the monetary aggregate is again simple sum M3. Observe the dramatic structural shift. Prior to 1974, the function was a parabola. After 1974 the function is an intersecting straight line. But again this puzzle was produced by the simple sum monetary aggregate. In Figure 8, the same plot is provided, but with the monetary aggregate changed to Divisia M2. The structural shift is gone.

Figure 5: Simple Sum M3 Velocity versus Interest Rate: Moody's AAA corporate bond rate, quarterly, 1959.1-19980.3

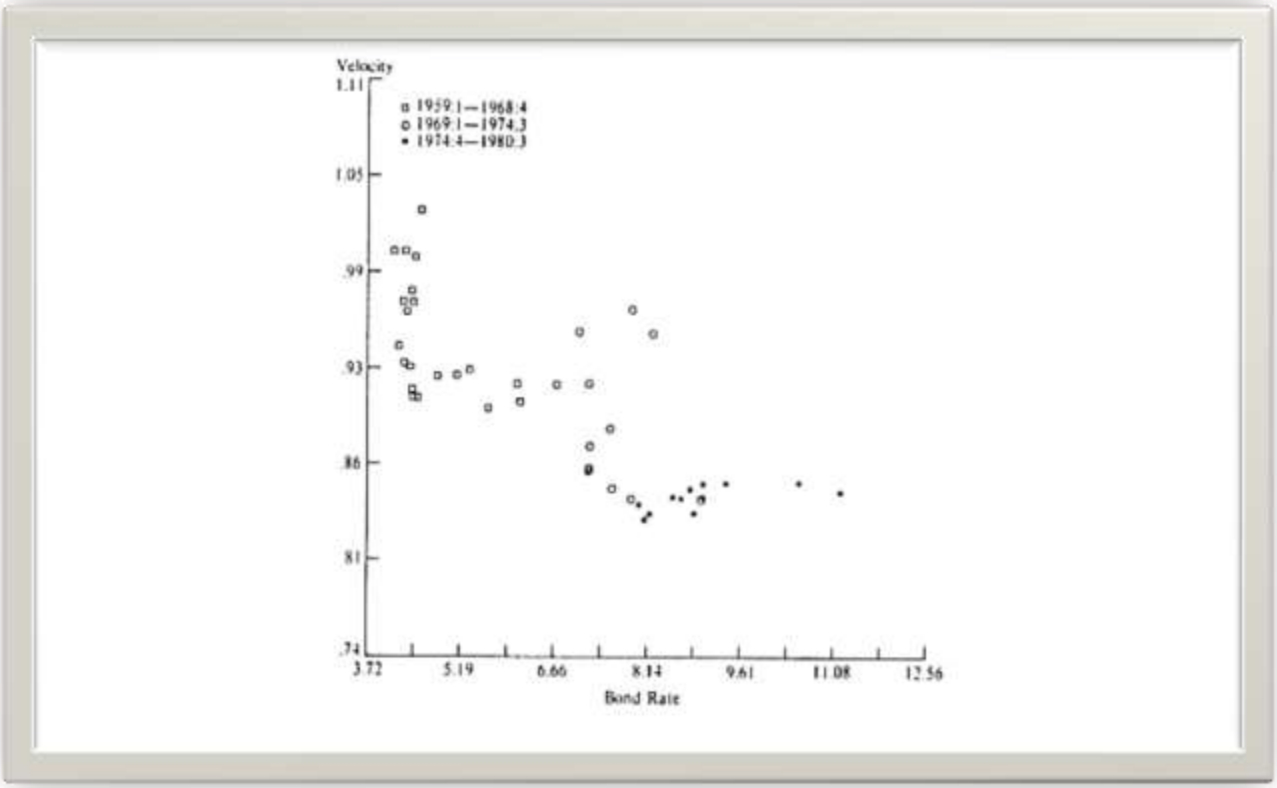


Figure 6: Divisia M3 Velocity versus Interest Rate: Moody's AAA corporate bond rate, quarterly, 1959.1-19980.3

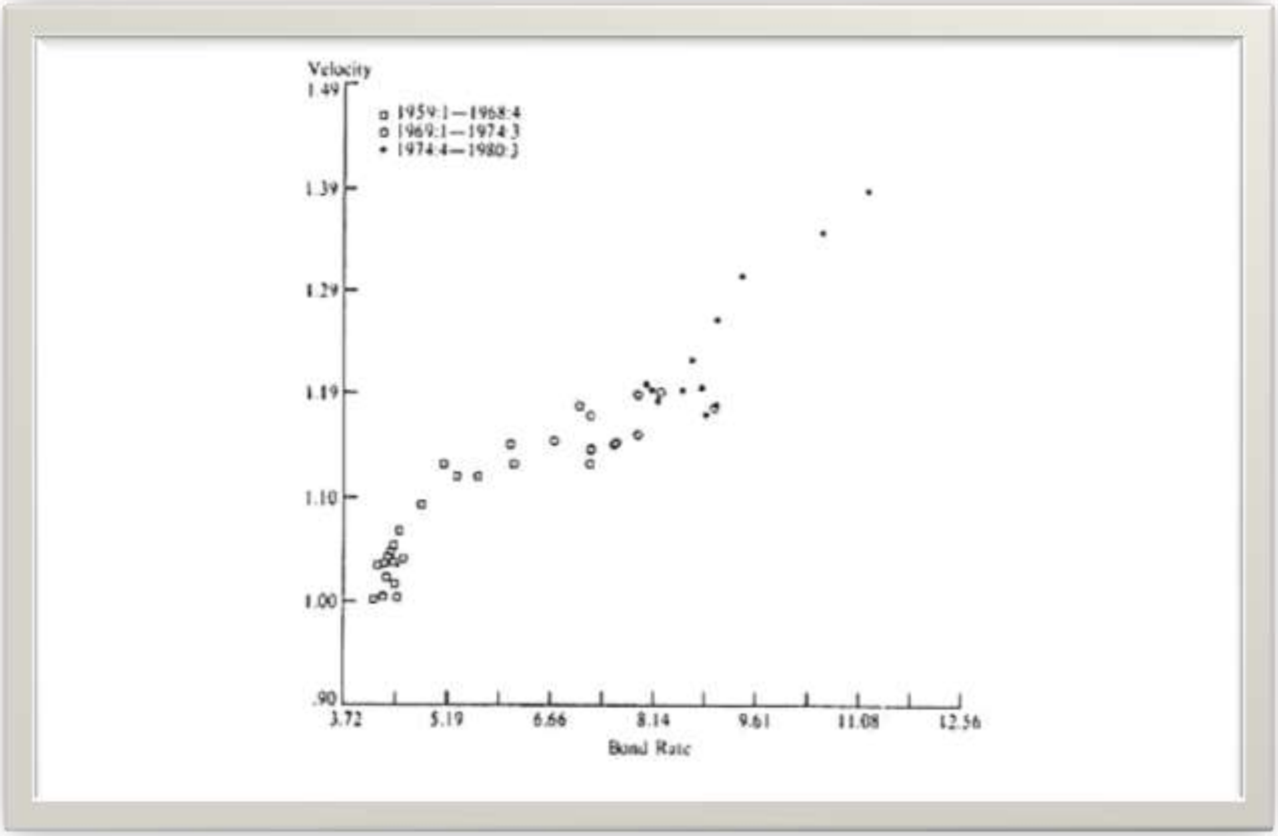


Figure 7: Simple Sum M3 Base Multiplier versus Interest Rate: deviation from time trend of Moody's Baa corporate bond rate, monthly 1969.1-1981.8.

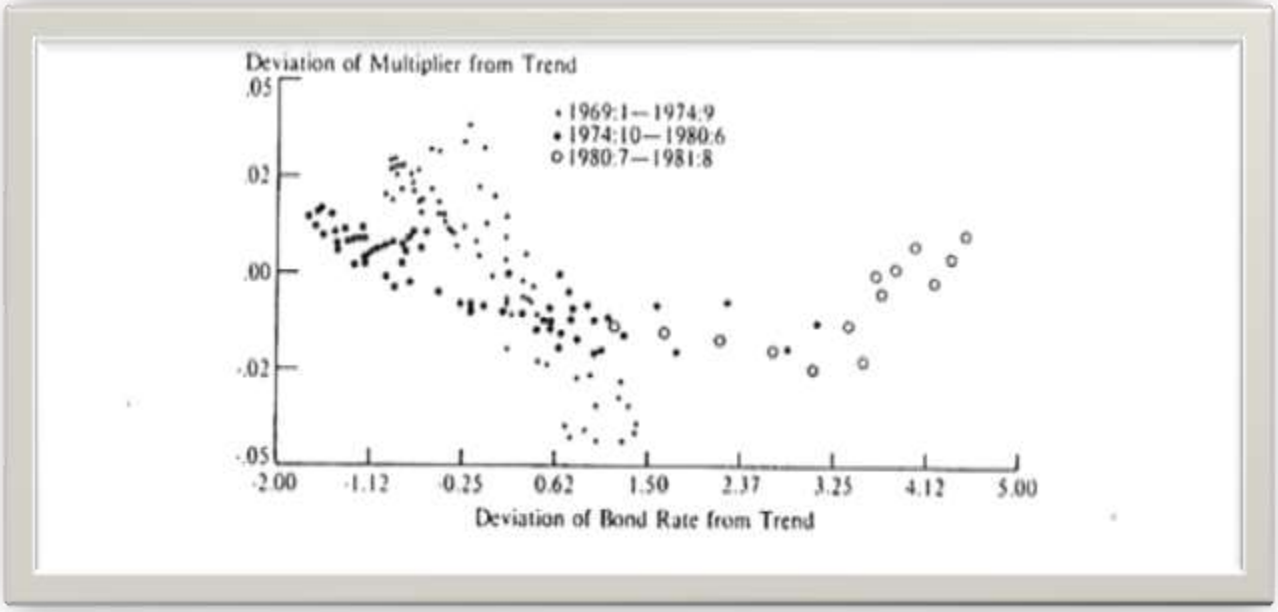
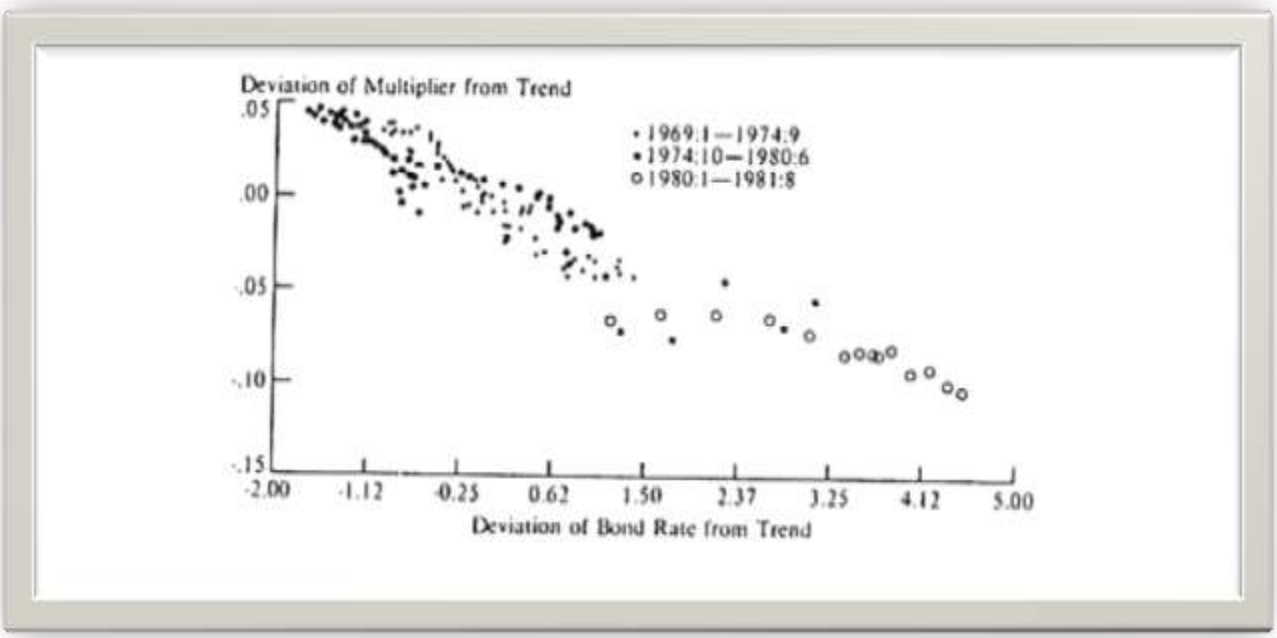


Figure 8: Divisia M3 Monetary Aggregate Base Multiplier versus Deviation from time trend of Moody's Baa corporate bond Interest Rate, monthly 1969.1-1981-8.



The most formal methods of investigating these concerns at the time were based on the use of the Goldfeld demand for money function, which was the standard specification used by the Federal Reserve System. The equation was originated by Steven Goldfeld at Princeton University. The equation was a linear function of a monetary aggregate on national income, a regulated interest rate, and an unregulated interest rate. It was widely believed that the function had become unstable in the 1970s. P. A. V. B. Swamy and Peter Tinsley, at the Federal Reserve Board in Washington, DC, had produced a stochastic coefficients approach to estimating a linear equation. The result was an estimated stochastic process for each coefficient. The approach permitted testing the null hypothesis that all of the stochastic processes are constant.

Swamy estimated the processes for the model's three coefficients at the Federal Reserve Board with quarterly data from 1959:2 – 1980:4, and the results were published by Barnett, Offenbacher, and Spindt. The realizations of the three coefficient processes are displayed in Figures 9, 10, and 11. The solid line is the process's realization, when money is measured by simple sum M2. The dotted line is the realization, when the monetary aggregate is measured by the Divisia index. The instability of the coefficient is very clear, when the monetary aggregate is simple sum, but the processes look like noise around a constant, when the monetary aggregate is Divisia. The statistical test could not reject constancy (i.e., stability of the demand for money function), when Divisia was used. But stability was rejected, when the monetary aggregate was simple sum.

Figure 9: Income Coefficient Time Path

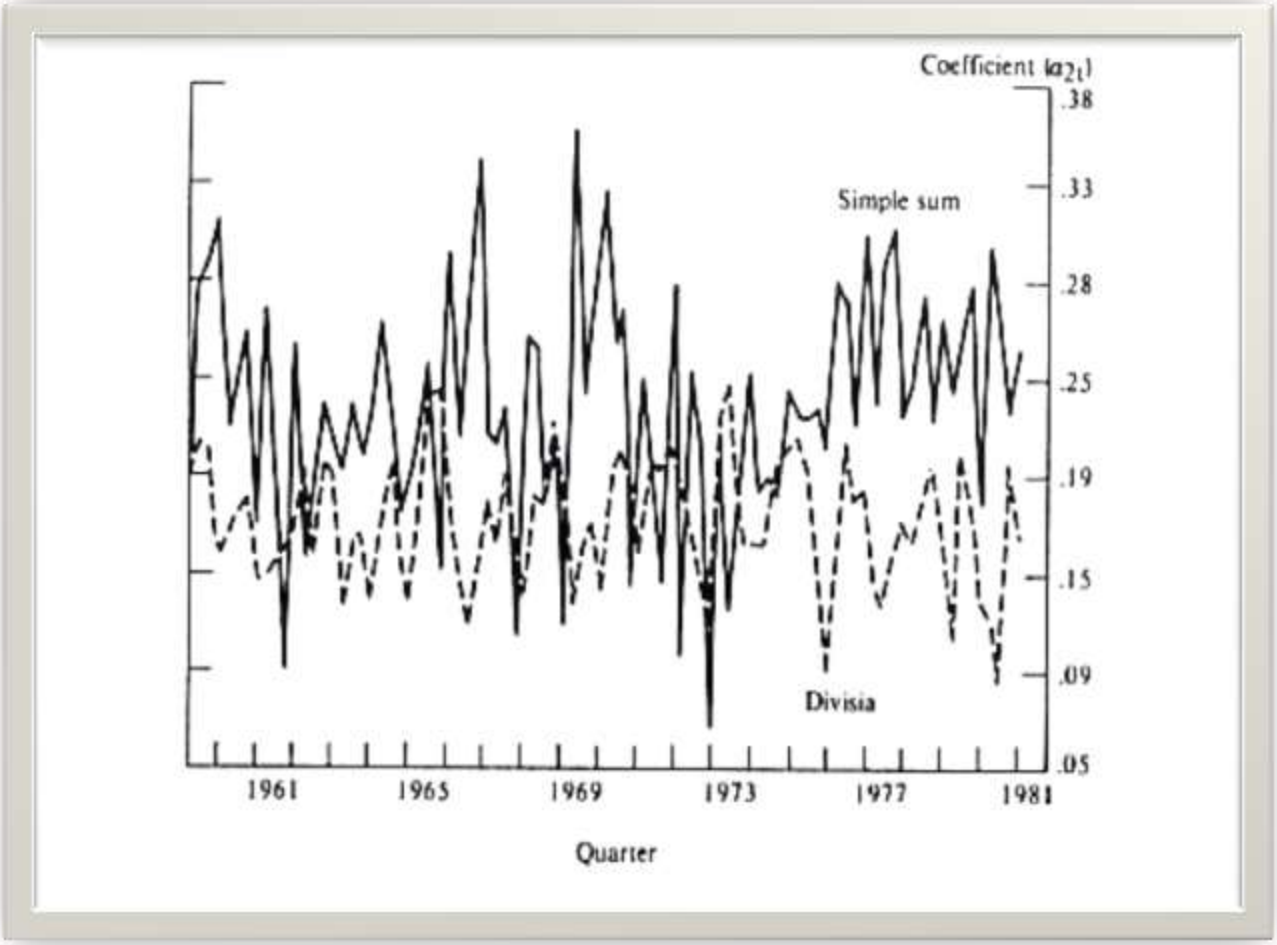


Figure 10: Market Interest Rate (commercial paper rate) Coefficient Time Path

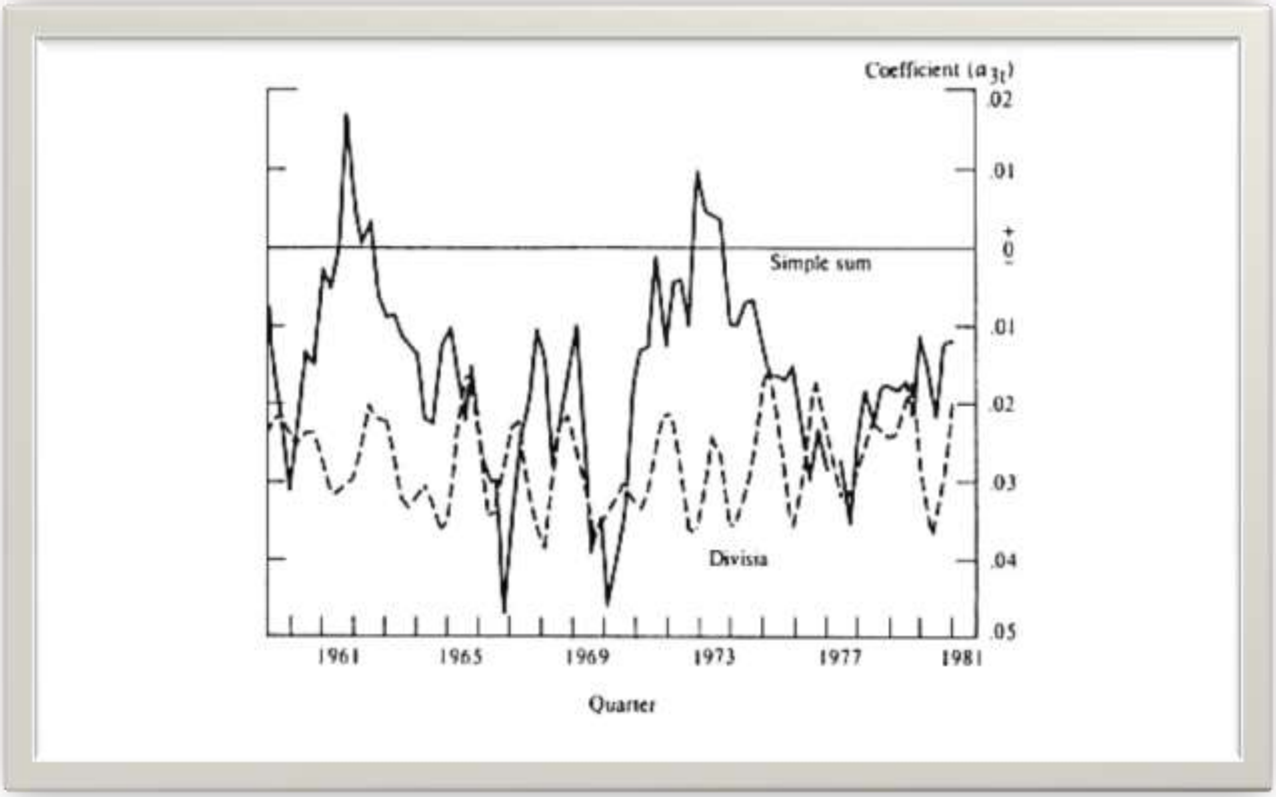
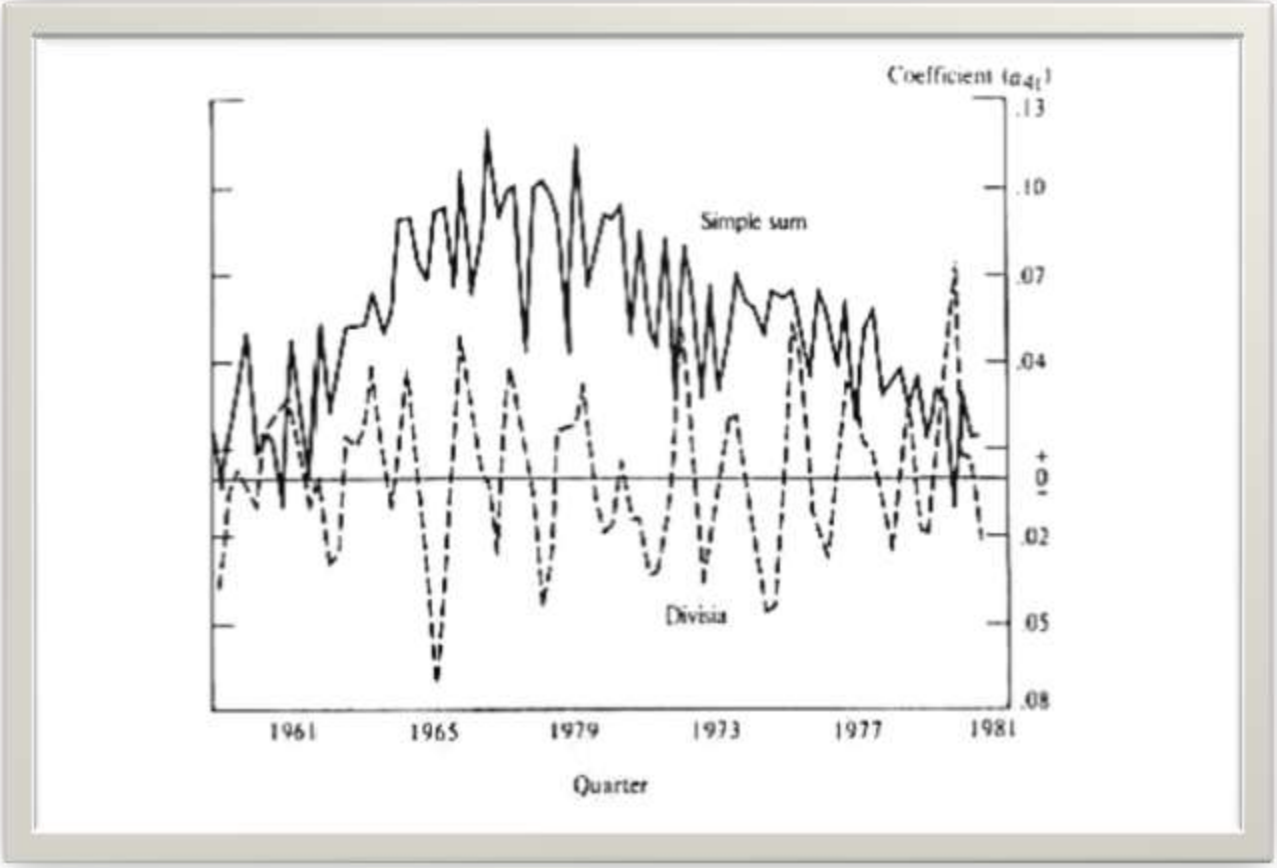


Figure 11: Regulated Interest Rate (passbook rate) Coefficient Time Path



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5. The Monetarist Experiment: November 1979 – November 1982.

Following the inflationary 1970s, Paul Volcker, as Chairman of the Federal Reserve Board, decided to bring inflation under control by decreasing the rate of growth of the money supply, with the instrument of policy being changed from the federal funds rate to nonborrowed reserves. The period, November 1979 – November 1982, during which that policy was applied was called the "Monetarist Experiment." The policy succeeded in ending the escalating inflation of the 1970s, but was followed by a recession. That recession was not intended. The Federal Reserve decided that the existence of widespread 3-year negotiated wage contracts precluded a sudden decrease in the money supply growth rate to the intended long run growth rate. The decision was to decrease from the high double-digit growth rates to about 10% per year and then gradually decrease towards the intended long run growth rate.

It was believed that a sudden drop to the intended long run growth rate would produce a recession. Figures 12 and 13 and Table 1 reveal the cause of the unintended recession. As is displayed in Figures 12 and 13, for the M2 and M3 levels of aggregation, the rate of growth of the Divisia monetary aggregates was less than the rate of growth of the official simple sum aggregate intermediate targets. As Table 1 summarizes, the simple sum aggregate growth rates were at the intended levels, but the Divisia growth rates were half as large, producing an unintended negative shock of substantially greater magnitude than intended. When a recession occurred, that unintended consequence was an embarrassment to monetarists, who subsequently denied that a monetarist policy actually had been in effect. But it is well known to those who were on the staff of the Federal Reserve Board at the time that the Federal Reserve was doing what it said it was doing.

Figure 12: Seasonally adjusted annual M2 Growth Rates. Solid line = Divisia, dashed line = simple sum. The last three observations to the right of the vertical line are post sample period.

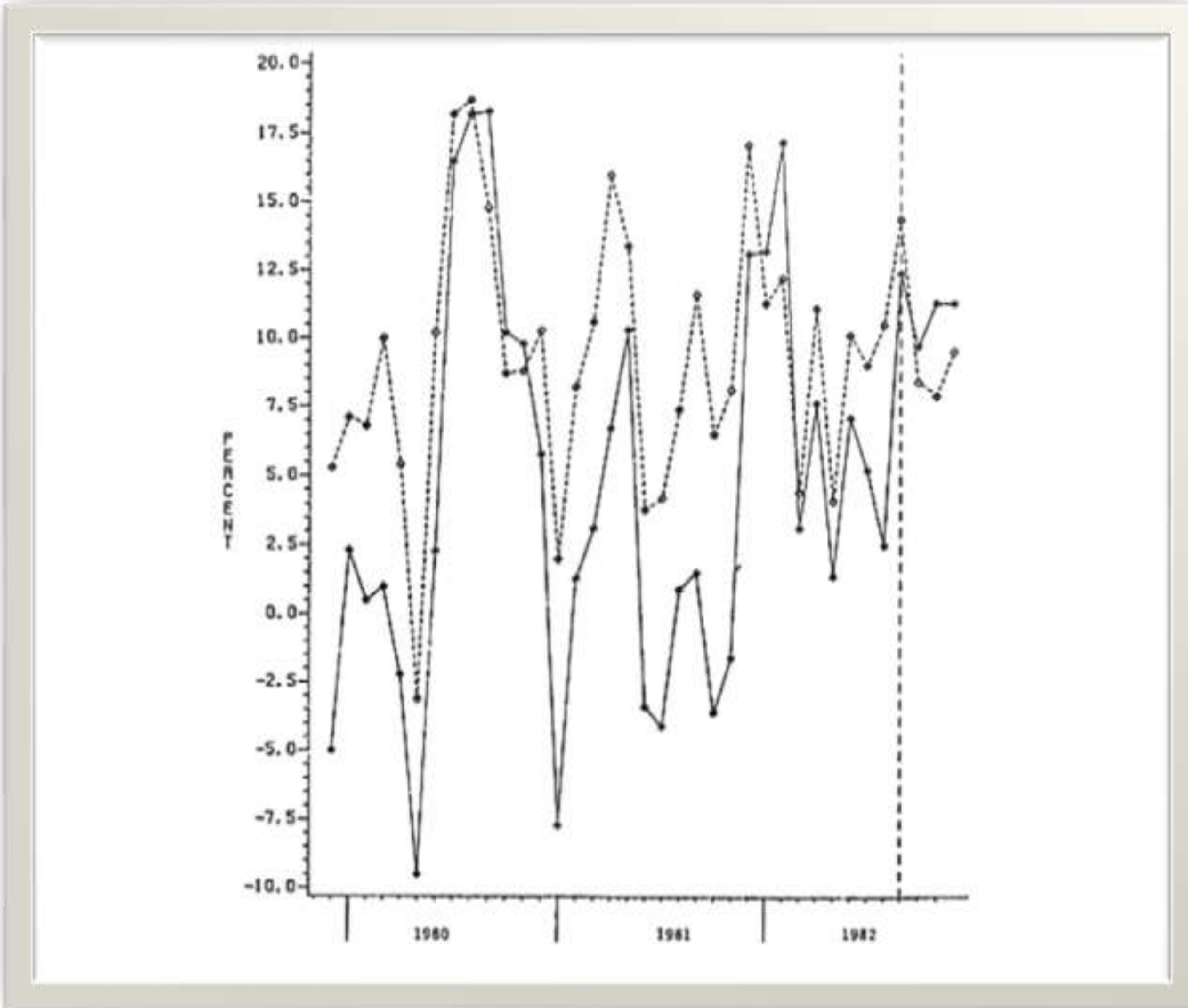


Figure 13: Seasonally adjusted annual M3 Growth Rates. Solid line = Divisia, dashed line = simple sum. The last three observations to the right of the vertical line are post sample period.

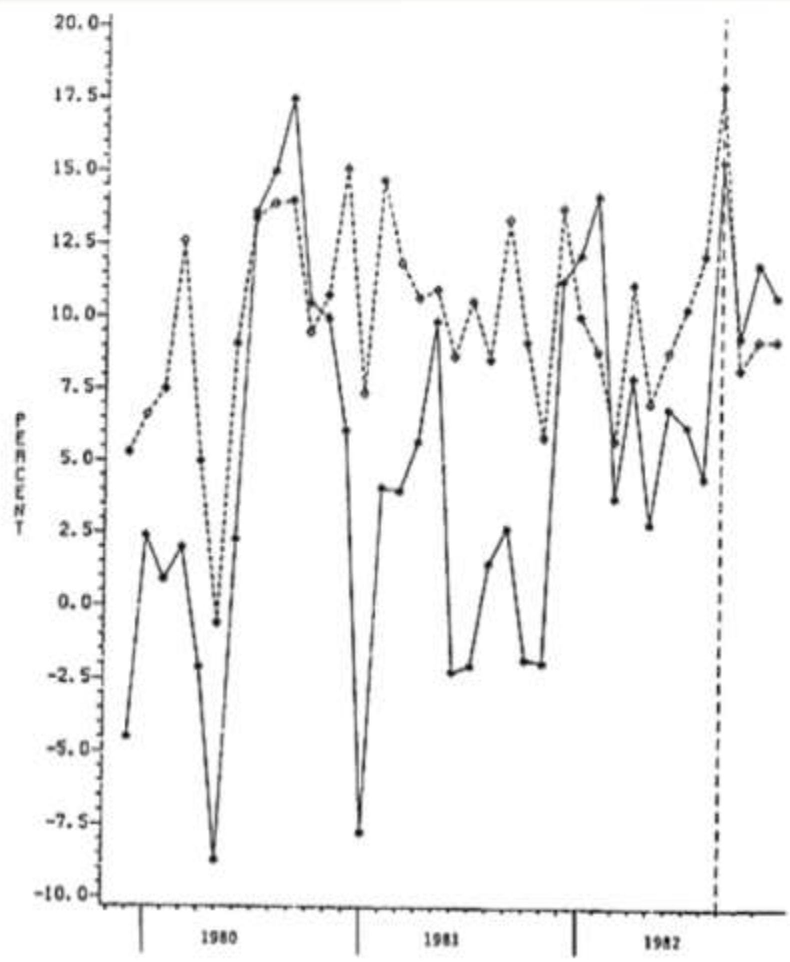


Table 1: Mean Growth Rates During the Period

Monetary Aggregate	Mean Growth Rate during Period
Divisia M2	4.5
Simple Sum M2	9.3
Divisia M3	4.8
Simple Sum M3	10.0

6. End of the Monetarist Experiment: 1983 - 1984

Following the end of the Monetarist Experiment and the unintended recession that followed, Milton Friedman became very vocal with his prediction that there had just been a huge surge in the growth rate of the money supply, and that surge would surely work its way through the economy and produce a new inflation. He further predicted that there would be an overreaction by the Federal Reserve, plunging the economy back down into a recession. He published this view repeatedly in the media in various magazines and newspapers, with the most visible being his *Newsweek* article that appeared on September 26, 1983. That article is provided below in Figure 14.

Figure 14: Milton Friedman, *Newsweek*, Sept 26, 1983

MILTON FRIEDMAN

A Case of Bad Good News

President Reagan, politicians of all political persuasions, journalists specializing in economics, Wall Street, the business community—all these and many more are hailing the recent economic statistics showing rapid growth in output and employment as very good news indeed.

One group of economists—the monetarists—are a conspicuous exception. We were also an exception to what I labeled in February as "misleading unanimity" among economic forecasters (NEWSWEEK, Feb. 7, 1983). Almost to a person economic forecasters were predicting a slow and sluggish recovery. We predicted that "1983 will be a year of rapid and vigorous economic growth."

Monetary Explosion: That judgment was based on two considerations: the length and severity of the recession that ended in December 1982, and the monetary explosion that was then in process—a rise in M_1 at the annual rate of 15 percent from July 1982 to January 1983. It seemed to us that "this monetary explosion assures vigorous economic growth in the coming months"—but also that "unfortunately, if it continues much longer, it will also produce a renewed acceleration of inflation and a sharp rise in interest rates." The monetary explosion did produce vigorous growth. Unfortunately, it also *did* continue—at the annual rate of more than 14 percent from January to July. Interest rates have already risen sharply. Inflation has not yet accelerated. That will come next year, since it generally takes about two years for monetary acceleration to work its way through to inflation.

The "good news," that output grew at the annual rate of nearly 9 percent in the second quarter of this year and may equal that record in the third quarter, is really bad news—the sign of an overheated economy headed for trouble. We do not need another sharp but brief expansion—like 1980 to 1981—followed by a relapse into recession. We need moderate growth at a rate that can be maintained for a long time along with continued reduction in inflation.

The only way to maintain anything like the recent hectic pace of real growth would be to keep the monetary explosion going. But even if the Fed has learned nothing from experience, the market has—as its recent reactions to money-growth figures demonstrate. If the monetary explosion continued, both interest rates and inflation would react much more promptly than in the past. Both would head toward the sky. That reaction would make it nearly impossible for the Fed to continue the monetary explosion. However reluctantly, it will have to step on the brakes—as it already has apparently started to do.

> The monetary explosion leaves no satisfactory way out of our present situation.

The monetary explosion from July 1982 to July 1983 leaves no satisfactory way out of our present situation. The Fed's stepping on the brakes will appear to have no immediate effect. Rapid recovery will continue under the impetus of earlier monetary growth. With its historical shortsightedness, the Fed will be tempted to step still harder on the brake—just as the failure of rapid monetary growth in late 1982 to generate immediate recovery led it to keep its collective foot on the accelerator much too long. The result is bound to be renewed stagflation—recession accompanied by rising inflation and high interest rates.

Recession and the Election: The only real uncertainty is when the recession will begin. That will depend partly on the pattern of monetary growth over coming months, partly on other developments that cannot now be foreseen. Monetarists have always emphasized that the time delay between monetary change and economic change is not only long but also highly variable. Indeed, that is why we are so skeptical about the kind of monetary "fine-tuning" that the Fed has engaged in and why we favor steady monetary growth.

The precise timing of the recession will largely determine the political climate of election year 1984. Both President Reagan's supporters and his opponents are acting as if they expect recent favorable economic developments to continue through 1984—as evidenced by the search on the part of both for other issues to stress. They will be right if the recession does not begin before the third quarter of 1984. But if the recession should begin in the first or second quarter of 1984—as is entirely possible—the situation will be very different. In that case the election campaign would be conducted in an environment of declining output, rising unemployment, rising inflation and high interest rates—hardly an environment favorable to an incumbent.

84 NEWSWEEK/SEPTEMBER 26, 1983

We have excerpted some of the sentences from that *Newsweek* article below:

> "The monetary explosion from July 1982 to July 1983 leaves no satisfactory way out of our present situation. The Fed's stepping on the brakes will appear to have no immediate effect. Rapid recovery will continue under the impetus of earlier monetary growth. With its historical shortsightedness, the Fed will be tempted to step still harder on the brake – just as the failure of rapid monetary growth in late 1982 to generate immediate recovery led it to keep its collective foot on the accelerator much too long. The result is bound to be renewed stagflation – recession accompanied by rising inflation and high interest rates... The only real uncertainty is when the recession will begin."

But on exactly the same day, September 26, 1983, William Barnett published a very different view in *Forbes* magazine. That article is reprinted below as Figure 15. The following is an excerpt of some of the sentences from that article:

> "people have been panicking unnecessarily about money supply growth this year. The new bank money funds and the super NOW accounts have been sucking in money that was formerly held in other forms, and other types of asset shuffling also have occurred. But the Divisia aggregates are rising at a rate not much different from last year's... the 'apparent explosion' can be viewed as a statistical blip."

Of course, Milton Friedman would not have taken such a strong position without reason. You can see the reason from Figure 16. The percentage growth rates in that figure are divided by 10, so should be multiplied by 10 to acquire the actual growth rates. Notice the large spike in growth rate, rising to near 30% per year. But that solid line is produced from simple sum M2, which was greatly overweighting the recent new availability of super NOW accounts and money market deposit accounts. There was no spike in the Divisia monetary aggregate, represented by the dashed line.

If indeed the huge surge in the money supply had happened, then inflation would surely have followed, unless money is extremely non-neutral, a view held by very few economists. But there was no inflationary surge and no subsequent recession.

Figure 15: William Barnett, *Forbes*, Sept 26, 1983

Faces
Behind The Figures

Edited by John R. Dorfman

What explosion?

If you were measuring the nation's vehicle supply, you wouldn't give equal weight to roller skates and locomotives. But that, in effect, is what the Federal Reserve Board does in measuring the money supply, says William Barnett, an economist at the University of Texas (Austin).

The Fed has realized, Barnett says, that M1 excludes too many forms of money, such as the new money market accounts at banks. But the higher aggregates like M2 and M3, he says, suffer greatly from the "skates and locomotives" problem. Some of their components are much more liquid than others.

How can the Fed control money if it can't adequately define it? You have heard the question before. Barnett thinks he has the answer: Divisia aggregates, named after the late French statistician François Divisia, who published a famous paper on the subject in 1925. In Barnett's Divisia formula, the changes in the supply of various forms of money are weighted according to how liquid each form is. Checking accounts, for example, get more weight than certificates of deposit. To get his weighting factors, Barnett looks at the opportunity cost of holding such low-yield assets as checking accounts and passbook savings accounts. When you give up yield, his model figures, you are probably getting liquidity in exchange.

Crunching Divisia numbers leads Barnett to two conclusions. One is that the Fed was much tighter than it intended to be during the period from late 1979 through mid-1982. For example, in 1982, M2, measured the usual way, was up 9.4%. But Divisia M2 was up only 7.8%.

The other conclusion is that people have been panicking unnecessarily about money supply growth this year. The new bank money funds and the super NOW accounts have been sucking in money that was formerly held in other forms, and other types of asset shuffling also have occurred. But the Divisia aggregates are rising at a rate not much different from last year's, Barnett says. Thus, the "apparent explosion" can be viewed as a statistical blip.

Economist William Barnett
A new set of "M" numbers?

Is anybody listening? Yes, says Barnett, who was a research economist for the Fed for eight years until 1981. Some Fed members, including Governor Henry Wallich, regularly peruse his numbers. Such economists as Paul Samuelson, James Tobin and Milton Friedman also receive the Divisia numbers monthly. It's only a matter of time, he figures, before the Fed will go to Divisia aggregates as its official guide. "They've run out of other alternatives," says Barnett, "and are looking at this very seriously."—J.R.D.

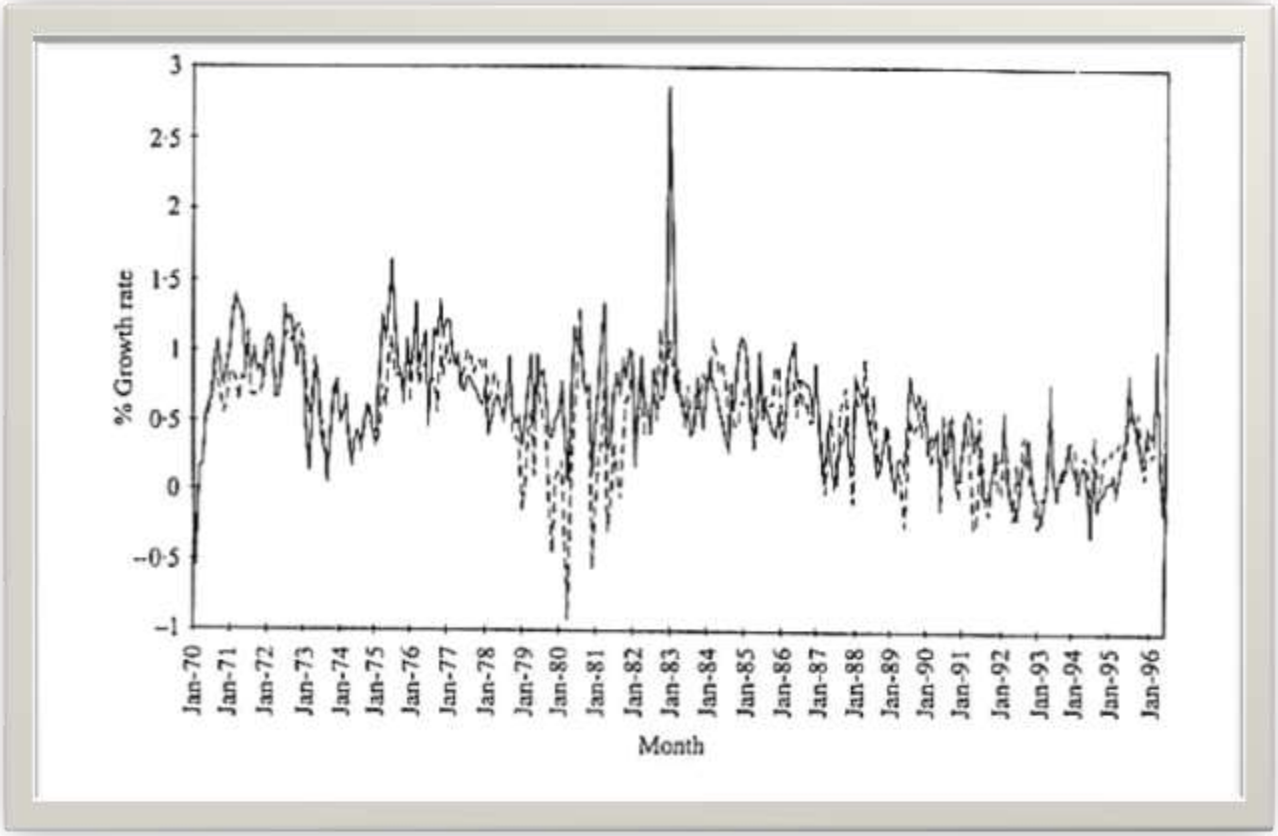
States' rights?

As the sovereign state of South Carolina sees it, there is a provision written into the Tax Equity & Fiscal Responsibility Act (TEFRA) of 1982 that amounts to an unconstitutional effort by Washington to abrogate what remains of state powers. TEFRA mandates that bonds sold by states after July 1, 1983 be issued in registered (not bearer) form. If a state breaks this rule, the interest on the bonds can be taxed by the IRS.

That requirement galls Grady L. Patterson Jr., 59, South Carolina's

South Carolina Treasurer Grady L. Patterson Jr.
Can one level of government tax another?

Figure 16: Monetary Growth Rates, 1970-1996, from St. Louis Federal Reserve's Database



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7. The Rise of Risk Adjustment Concerns: 1984 - 1993

The exact monetary quantity aggregator function $m_t = u(\mathbf{m}_t)$ can be tracked very accurately by the Divisia monetary aggregate, m_t^d, since that tracking ability is known under perfect certainty. However, when nominal interest rates are uncertain, the Divisia monetary aggregate's tracking ability is somewhat compromised. That compromise is eliminated by using the extended Divisia monetary aggregate derived by Barnett, Liu, and Jensen (1997) under risk. Let m_t^G denote the extended "generalized" Divisia monetary aggregate over the monetary assets. The only difference between m_t^G and m_t^d is the user cost formula used to compute the prices in the Divisia index formula.

Let π_{it}^G denote the generalized user cost of monetary asset *i*. Barnett, Liu, and Jensen (1997) prove that

$$\pi_{it}^{G} = \pi_{it}^{e} + \varphi_{it}$$

where

$$\pi_{it}^{e} = \frac{E_t(R_t - R_{it})}{E_t(1+R_t)}$$

and

$$\varphi_{it} = \frac{E_t(1+R_{it})}{E_t(1+R_t)} \frac{Cov(R_t, \frac{\partial T}{\partial C_{t+1}})}{\frac{\partial T}{\partial C_T}} - \frac{Cov(R_{it}, \frac{\partial T}{\partial C_{t+1}})}{\frac{\partial T}{\partial C_t}},$$

where

$$T = E_t \sum_{t=0}^{\infty} \beta^t F\left(c_t, m_t^G \right).$$

Barnett, Liu, and Jensen (1997) show that the values of π_{it} determine the risk premia in interest rates. Note that $\pi_{it}{}^{G}$ reduces to equation (2) under perfect certainty.

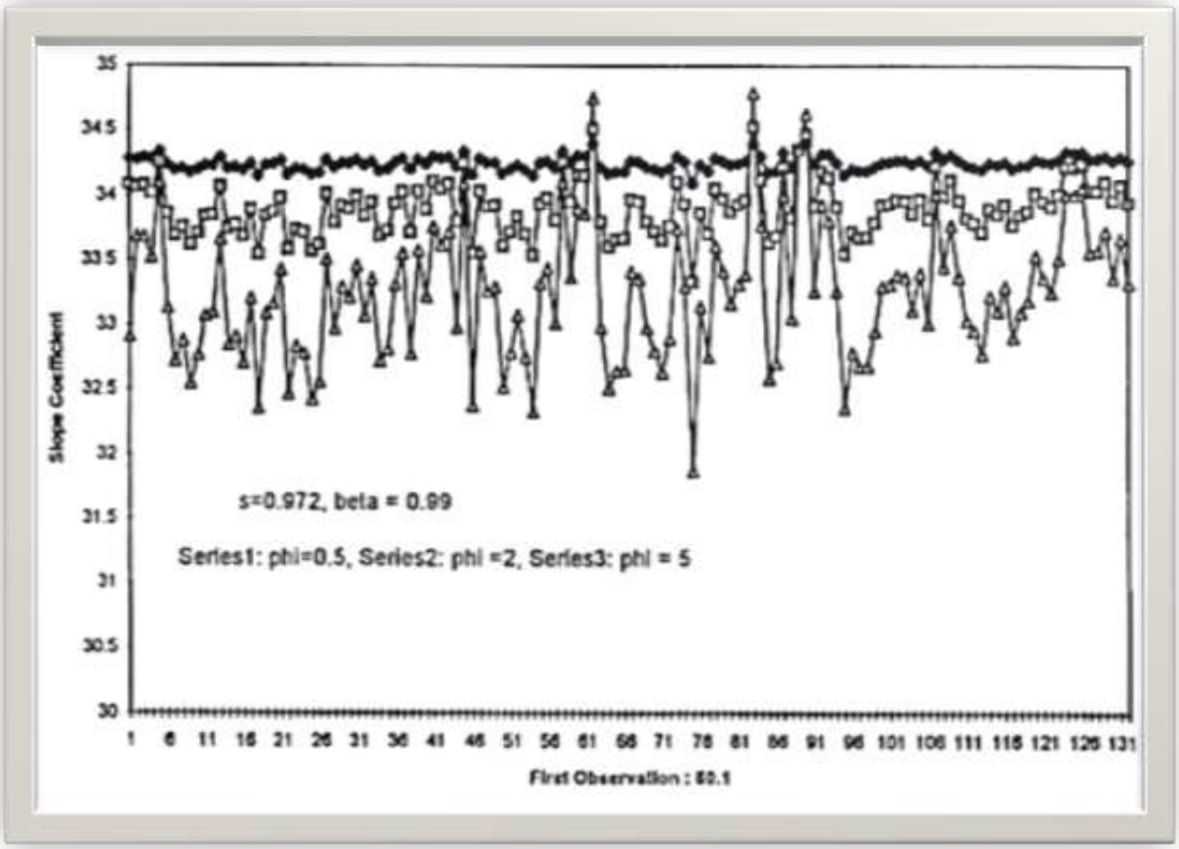
Using that extension, Barnett and Xu (1998) demonstrated that velocity will change, if the variance of an interest rate stochastic process changes. Hence the variation in the variance of an interest rate ARCH or GARCH stochastic process cannot be ignored in modelling monetary velocity. By calibrating a stochastic dynamic general equilibrium model, Barnett and Xu (1998) showed that the usual computation of velocity will appear to produce instability, when interest rates exhibit stochastic volatility. But when the CCAPM adjusted variables above are used, so that the variation in variance is not ignored, velocity is stabilized.

Figure 17 displays their simulated slope coefficient for the velocity function, treated as a function of the exact interest rate aggregate, but without risk adjustment. All functions in the model are stable, by construction. Series 1 was produced with the least stochastic volatility in the interest rate stochastic process, series 2 with greater variation in variance, and series 3 with even more stochastic volatility. Note that the velocity function slope appears to be increasingly unstable, as stochastic volatility increases. By the model's construction, the slope of the velocity function is constant, if the CCAPM risk adjustment is used.

In addition, with real economic data, Barnett and Xu (1998) showed that the evidence of velocity instability is partially explained by overlooking the variation in the variance of interest rates over time.

Subsequently Barnett and Wu (2005) found that the explanatory power of the risk adjustment increases, if the assumption of intertemporal separability of the intertemporal utility function, T, is weakened. The reason is the same as a source of the well known equity premium puzzle, by which CCAPM under intertemporal separability under-corrects for risk.

Figure 17: Simulated velocity slope coefficient with stochastic volatility of interest rates



The Divisia index tracks the aggregator function measuring service flow. But for some purposes, the economic capital stock, computed from the discounted expected future service flow, is relevant, especially when investigating wealth effects of policy. The economic stock of money (ESM), as defined by Barnett (2000) under perfect foresight, follows immediately from the manner in which monetary assets are found to enter the derived wealth constraint, (2.3). As a result, the formula for the economic stock of money under perfect foresight is

$$V_t = \sum_{s=t}^{\infty} \sum_{i=1}^{n} \left[\frac{p_s^*}{\rho_s} - \frac{p_s^*(1+r_{is})}{\rho_{s+1}} \right] m_{is} .$$

Where the true cost of living index on consumer goods is $p_s^* = p_s^*(\mathbf{p}_s)$, with the vector of consumer goods prices being $\mathbf{p}_s$, and where the discount rate for period s is

$$\rho_s = \begin{cases} 1 & for \quad s = t \\ \prod_{u=t}^{s-1} (1 + R_u) & for \quad s > t \end{cases} .$$

The CCAPM extension of the economic capital stock formula to risk is available from Barnett, Chae, and Keating (2006).

During the late 1980s and early 1990s, there was increasing concern about substitution of monetary assets within the monetary aggregates (especially money market mutual funds) with stock and bond mutual funds, which are not within the monetary aggregates. The Federal Reserve Board staff considered the possibility of incorporating stock and bond mutual funds into the monetary aggregates. Barnett and Zhou (1994a) used the formulas above to investigate the problem. They produced the figures that we reproduce below as Figures 19-21. The dotted line is the simple sum monetary aggregate, which Barnett (2000) proved is equal to the sum of economic capital stock of money, V_t, and the discounted expected investment return from the components.

Computation of V_t requires modeling expectations. In that early paper, Barnett and Zhou (1994a) used martingale expectations rather than the more recent approach of Barnett, Chae, and Keating, using VAR forecasting. When martingale expectations are used, the index is called CE. Since the economic capital stock of money, V_t, is what is relevant to macroeconomic theory, we should concentrate on the solid lines in those figures. Note that Figure 19 displays nearly parallel time paths, so that the growth rate is about the same in either. That Figure is for M2+, which was the Federal Reserve Board staff's proposed extended aggregate, adding stock and bond mutual funds to M2. But note that in Figure 18, the gap between the two graphs is decreasing, producing a slower rate of growth for the simple sum aggregate than for the economic stock of money.

The reason can be found in Figures 20 and 21, which use a solid line to display the monetary (i.e., liquidity) services from stock and bond mutual funds and a dotted line for simple sum measure of those funds, where again the simple sum measures a joint product: the discounted investment yield plus the discounted service flow. Hence the gap between the two lines is the amount motivated by investment yield. Clearly those gaps have been growing. But it is precisely that gap which does *not* measure monetary services. By adding the value of stock and bond mutual funds into Figure 18 to get Figure 19, the growth rate error of the simple sum aggregate is offset by adding in an increased amount of assets providing nonmonetary services. Rather than trying to stabilize the error gap by adding in more and more nonmonetary services, the correct solution would be to remove the entire error gap by using the solid line in Figure 18 (or Figure 19), which measures the actual capital stock of money.

Figure 18: M2 Joint Product and Economic Capital Stock of Money. M2=simple sum joint product; CEM2=economic capital stock part of the joint product.

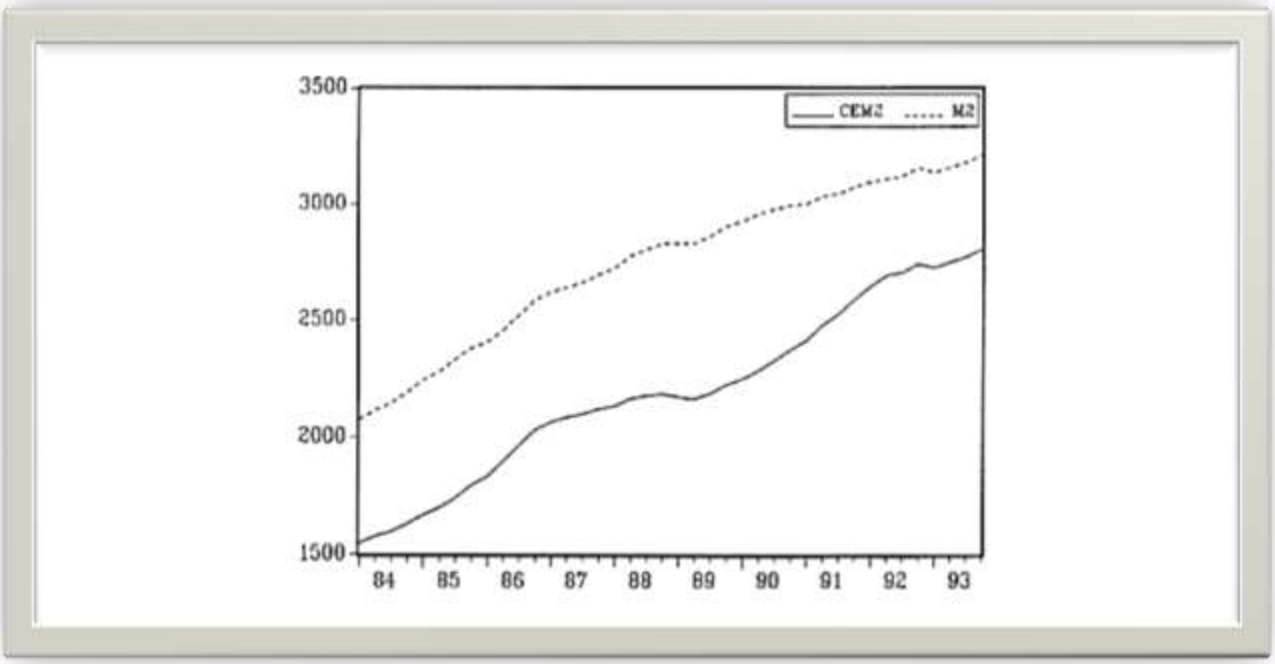


Figure 19: M2+ Joint Product and Economic Capital Stock of Money. M2+=simple sum joint product; CEM2+=economic capital stock part of the joint product.

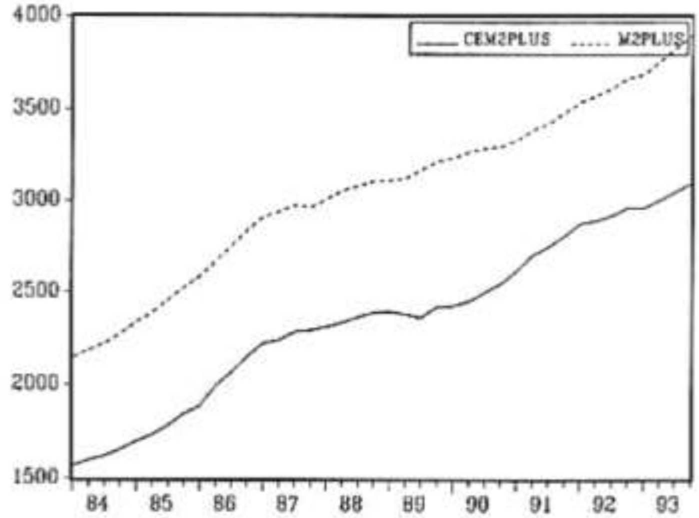


Figure 20: Common stock mutual funds joint product and their economic capital stock. StockQ=simple sum joint product; CEstock=economic capital stock part of the joint product.

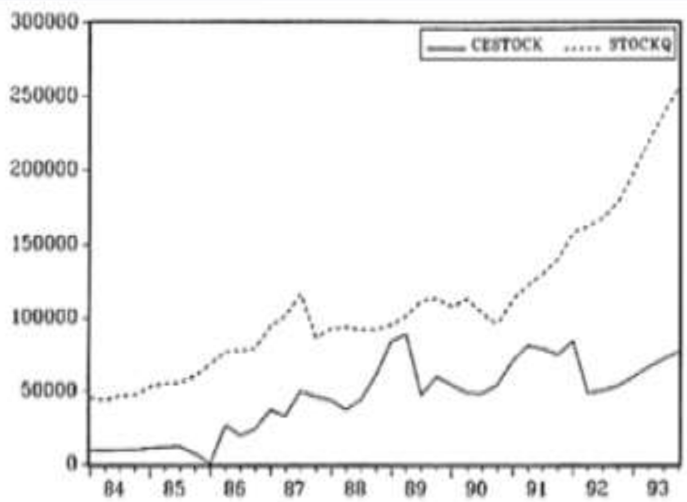
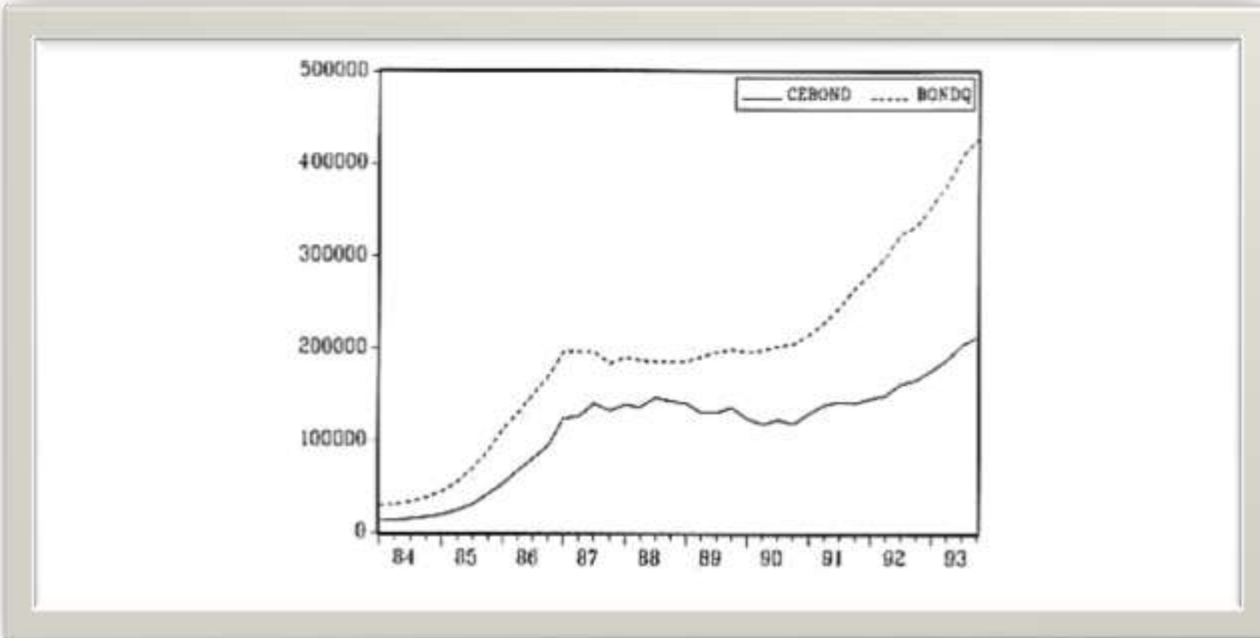


Figure 21: Bond mutual funds joint product and their economic capital stock. BondQ=simple sum joint product; CEbond=economic capital stock part of the joint product.



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8. The Y2K Computer Bug: 1999-2000

Following the problems of risk that increased in importance in the 1990s with the increasing substitution of monetary assets into stock and bond mutual funds, the next major concern about monetary aggregates and monetary policy was at the end of 1999. In particular, the financial press became highly critical of the Federal Reserve for what was perceived by those commentators to be a large, inflationary surge in the monetary base. The reason is clear from Figure 22. But in fact there was no valid reason for concern, since the cause was again a problem with the data.

The monetary base is the sum of currency plus bank reserves. Currency is dollar for dollar pure money, while reserves back deposits in an amount that is a multiple of the reserves. Hence as a measure of monetary services, the monetary base is severely defective, even though it is a correct measure of "outside money." At the end of 1999, there was the so-called Y2K computer bug, which was expected to cause temporary problems with computers throughout the world, including at banks. Consequently many depositors withdrew funds from their checking accounts and moved them into cash. While the decrease in deposits thereby produced an equal increase in currency demand, the decrease in deposits produced a smaller decline in reserves, because of the multiplier from reserves to deposits. The result was a surge in the monetary base, even though the cause was a temporary dollar-for-dollar transfer of funds from demand deposits to cash, having little effect on economic liquidity. Once the computer bug was resolved, people put the withdrawn cash back into deposits, as is seen from Figure 23.

Figure 22: Monetary Base Surge

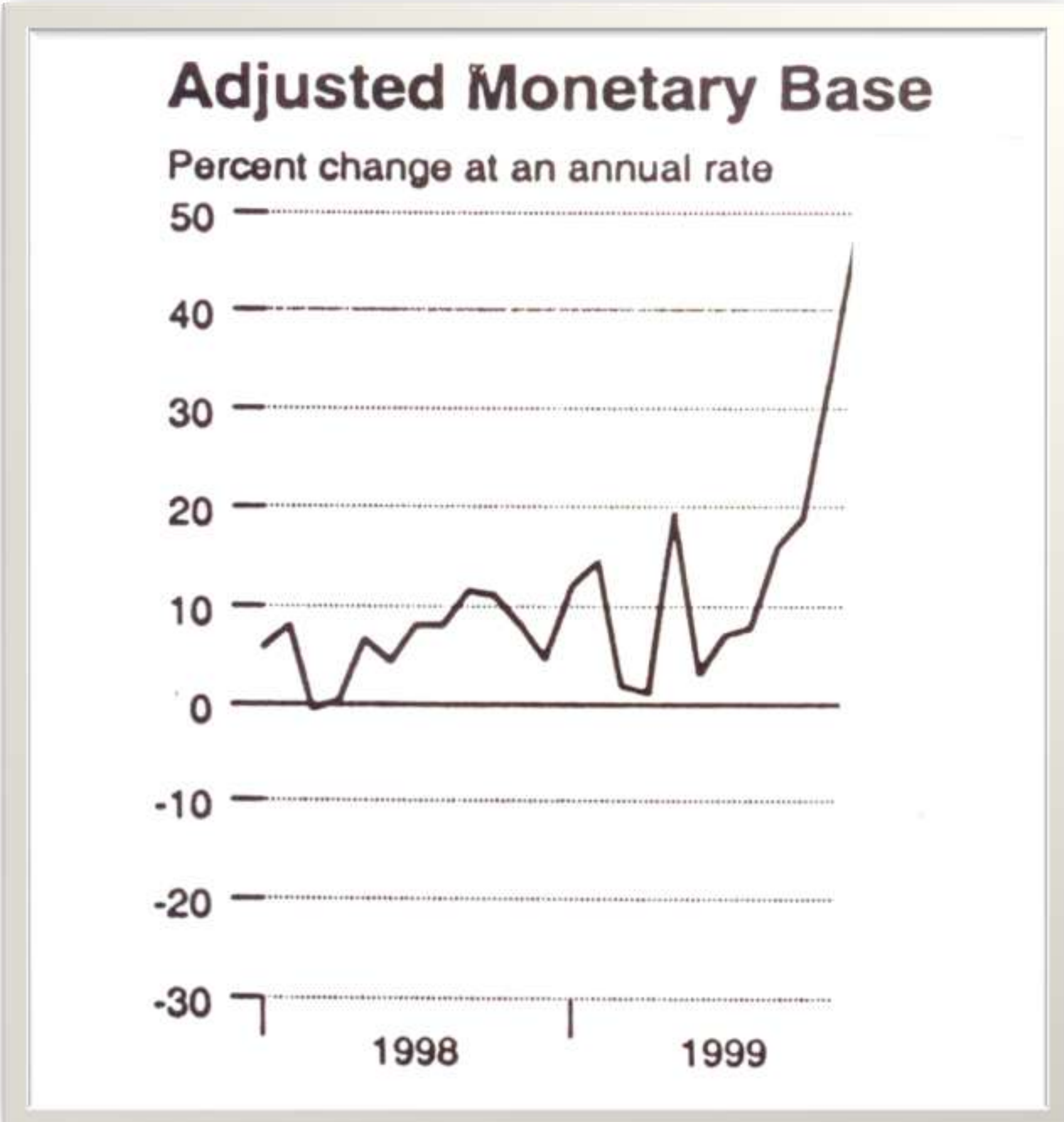
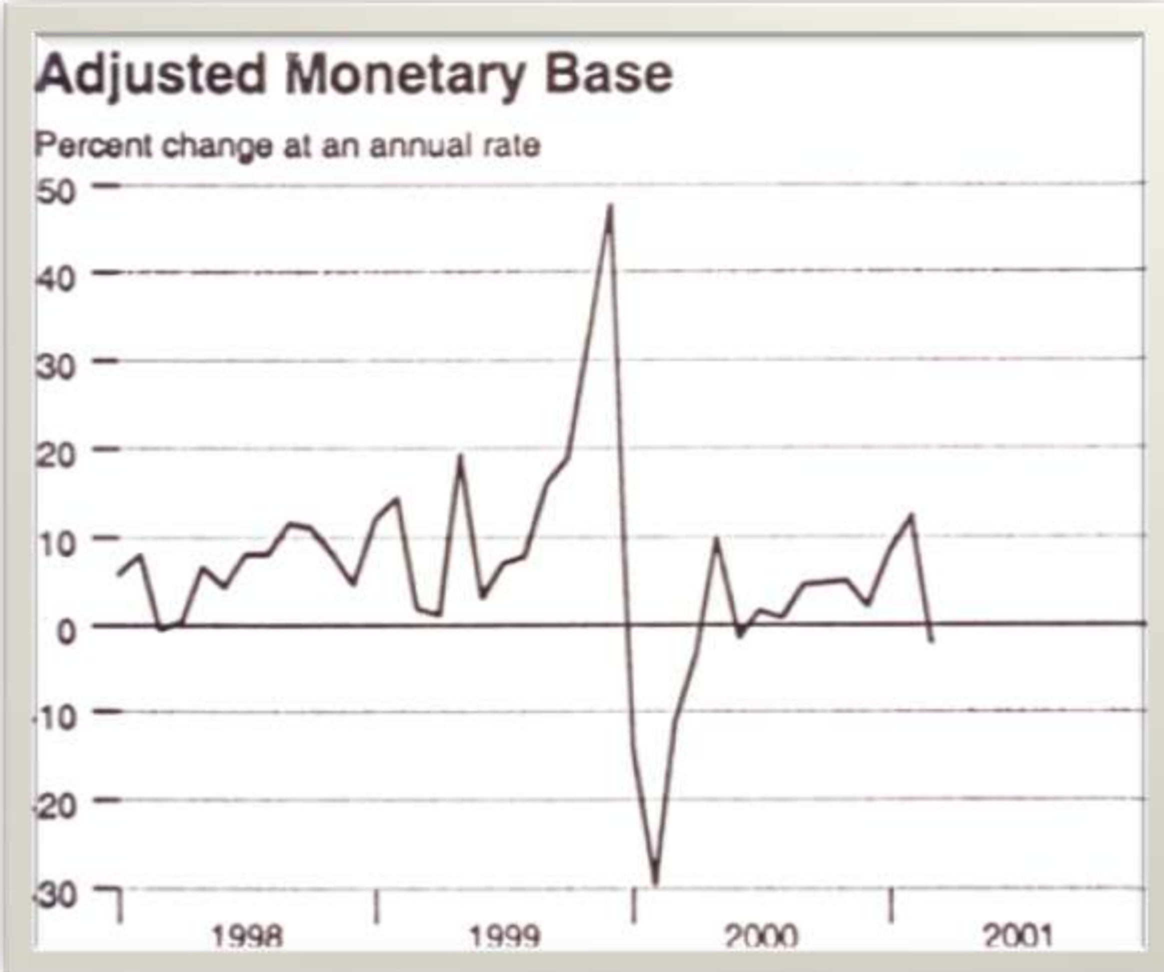


Figure 23: Y2K Computer Bug

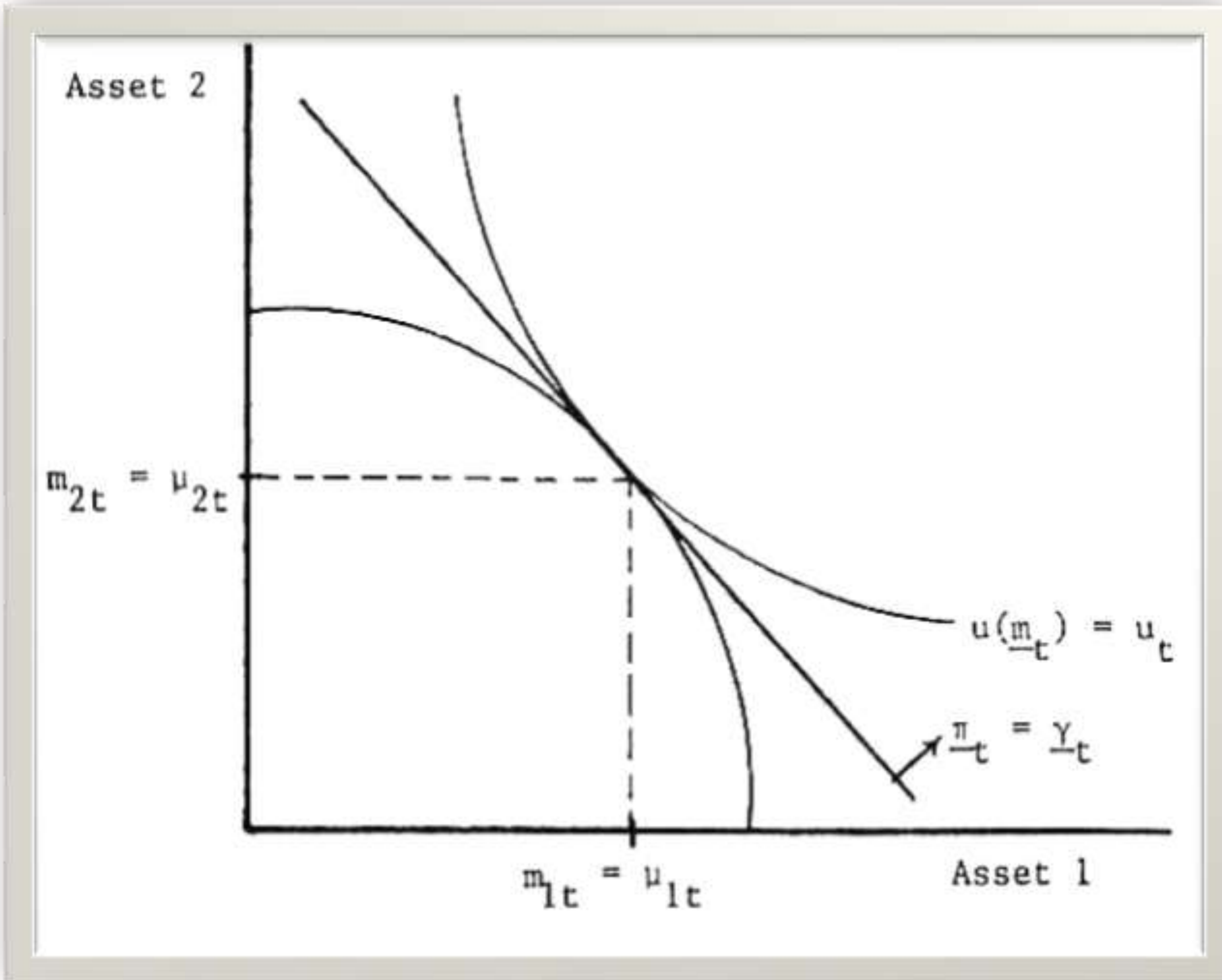


9. The Supply Side

While much of the concern in this literature has been about the demand for money, there is a parallel literature about the supply of money by financial intermediaries. Regarding the aggregation theoretic approach, see Barnett and Hahm (1994) and Barnett and Zhou (1994). It should be observed that the demand-side Divisia monetary aggregate, measuring perceived service flows received by financial asset holders, can be slightly different from the supply side Divisia monetary aggregate, measuring service flows produced by financial intermediaries. The reason is the regulatory wedge resulting from non-interest-bearing required reserves. That wedge produces a difference between demand side and supply side user cost prices and thereby can produce a small difference between the demand side and supply side Divisia aggregates.

When there are no required reserves and hence no regulatory wedge, the general equilibrium looks like Figure 24, with the usual separating hyperplane determining the user cost prices, which are the same on both sides of the market. The production possibility surface between deposit types 1 and 2 is for a financial intermediary, as in Figure 2, while the indifference curve is for a depositor allocating funds over the two asset types, as in Figure 1. But now the separating hyperplane can have any slope and is not the same as the simple sum hyperplane displayed in Figures 1 and 2.

Figure 24: Financial General Equilibrium without Required Reserves



While this separating-hyperplane general-equilibrium diagram is elementary, it assumes that the same prices and user costs are seen on both sides of the market. But when noninterest-bearing required reserves exist, the foregone investment return to banks is an implicit tax on banks and produces a regulatory wedge between the demand and supply side. It was shown by Barnett (1987) that under those circumstances, the user cost of supplied financial services by banks is not equal to the demand price, (2), but rather is

$$\gamma_{it} = \frac{(1-k_{it})R_t - r_{it}}{1+R_t},$$

where k_{it} is the required reserve ratio for account type i. Note that this supply-side user cost is equal to the demand-side formula, (2), when $k_{it} = 0$.

The resulting general equilibrium diagram, with the regulatory wedge, is displayed in Figure 25. Notice that one tangency determines the supply-side prices, while the other tangency produces the demand-side prices, with the angle between the two straight lines being the "regulatory wedge." Observe that the demand equals the supply for each of the component assets, 1 and 2.

Although the component demands and supplies are equal to each other, the failure of tangency between the production possibility curve and the indifference curve at the equilibrium results in a wedge between the growth rates of aggregate demand and supply services, as reflected in the fact that the user cost prices in the Divisia index are not the same in the demand and the supply side aggregates. To determine whether this wedge might provide a reason to compute and track the Divisia monetary supply aggregate as well as the more common demand-side Divisia monetary aggregate, Barnett, Hinich, and Weber (1986) conducted a detailed spectral analysis in the frequency domain. Some of the results are reprinted in Figures 26 – 28.

Figure 26 displays the supply and demand side Divisia M2 power spectrum. The demand side spectrum is displayed only at its estimated value, while the supply side spectrum is displayed as its confidence region. Notice that the supply side confidence region contains the demand side at all frequency. Hence the null hypothesis that the supply and demand side aggregates are the same cannot be rejected at any frequency, even though the computed aggregates are not exactly the same on the demand and supply sides. Figure 27 displays the squared coherence between the demand and supply side Divisia monetary aggregates, where coherence measures correlation as a function of frequency. The figure provides those plots at three levels of aggregation. Note that the correlation usually exceeds 95% for all three levels of aggregation at all frequencies, but the coherence begins to decline at very high frequencies (i.e., very short cycle periods in months). Hence the difference between the demand and supply side monetary aggregates is relevant only in modelling very short run phenomena.

But to put this into context, that paper also displays plots in the time domain for simple sum M3, the supply side M3 Divisia index (SDM3), and the demand side M3 Divisia index (DDM3) over the same time period used in producing the frequency domain comparisons. See Figure 28 for those reprinted plots. Notice that it takes over a decade for the difference between the demand side and supply side Divisia index to get wider than a pencil point, but the divergence between simple sum and either Divisia aggregate begins immediately and is cumulative. In short, the error in using the simple sum monetary aggregates is overwhelmingly greater than the usually entirely negligible difference between the demand and supply side Divisia monetary aggregates. Furthermore, in recent years reserve requirements have been low and largely offset by sweeps, and it is the intent of the Federal Reserve eventually to begin paying interest on required reserves, so the difference between the demand and supply side Divisia monetary aggregates now is much smaller than during the time period displayed in Figure 28, and eventually will be a zero difference, when interest is paid on required reserves in the future.

Figure 25: Financial Equilibrium with Positive Required Reserves

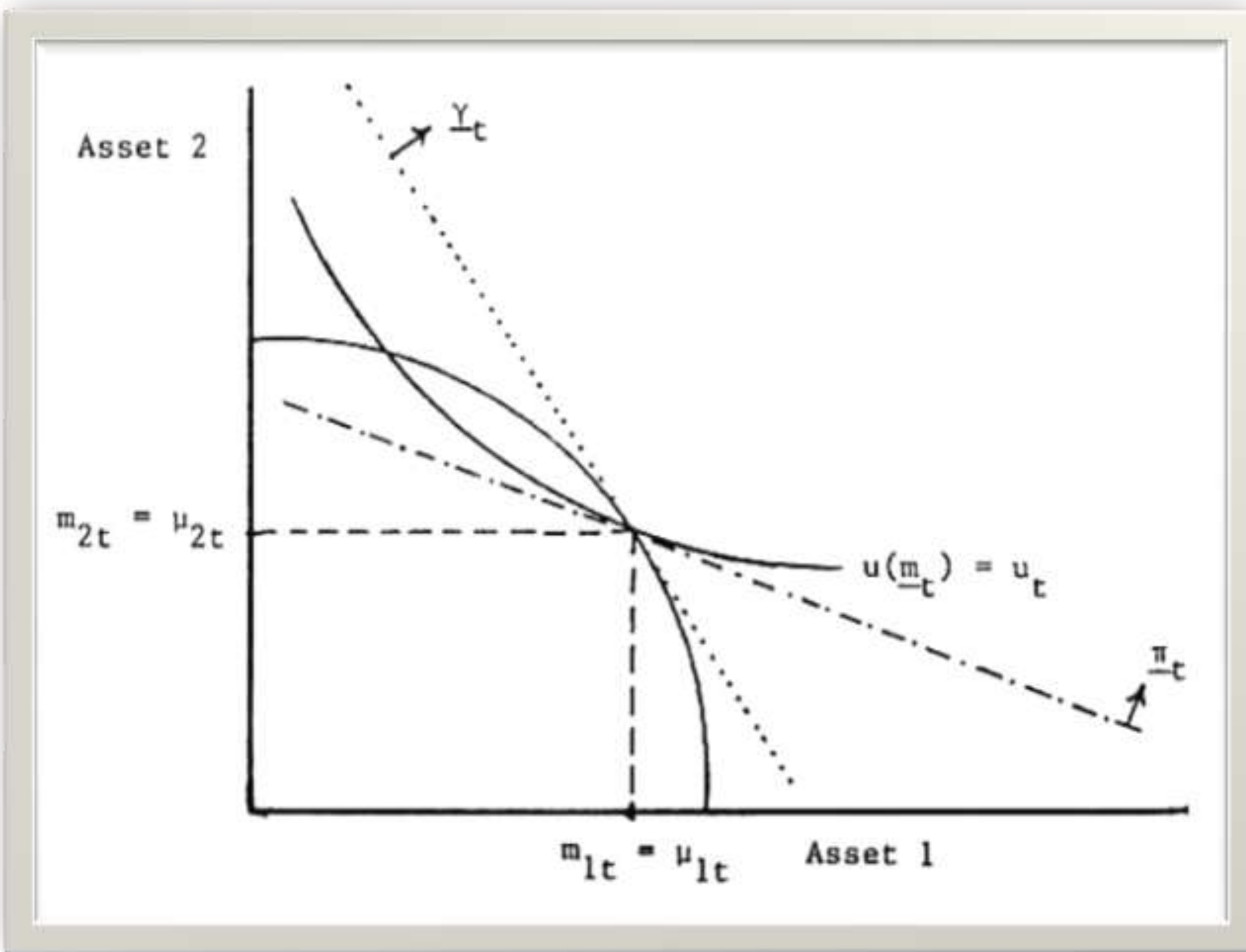


Figure 26: Confidence Region for Supply Side Spectrum: log spectrum of DDM2 and confidence band for log spectrum of SDM2

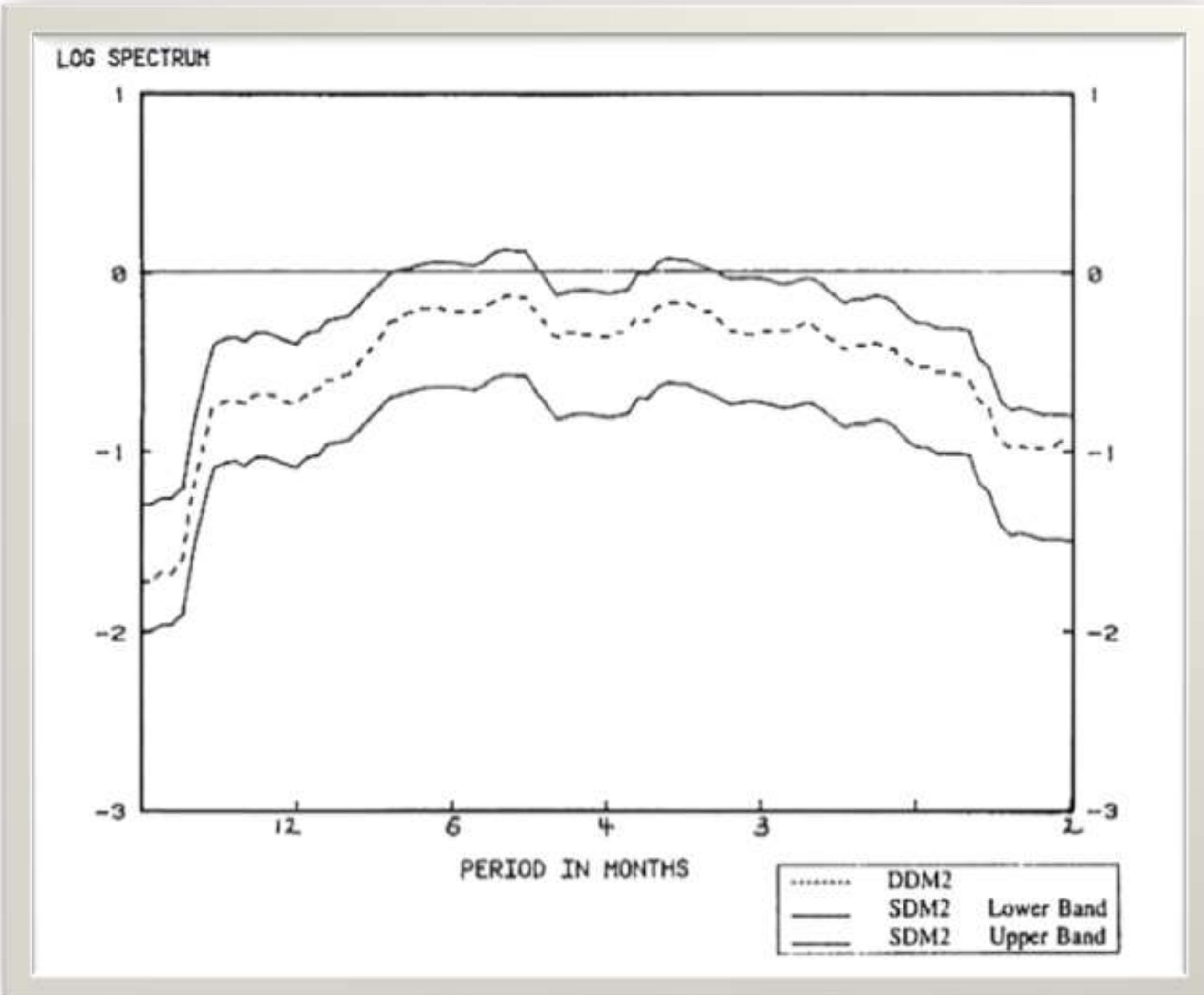


Figure 27: Squared Coherence between Divisia Demand and Supply Side Divisia

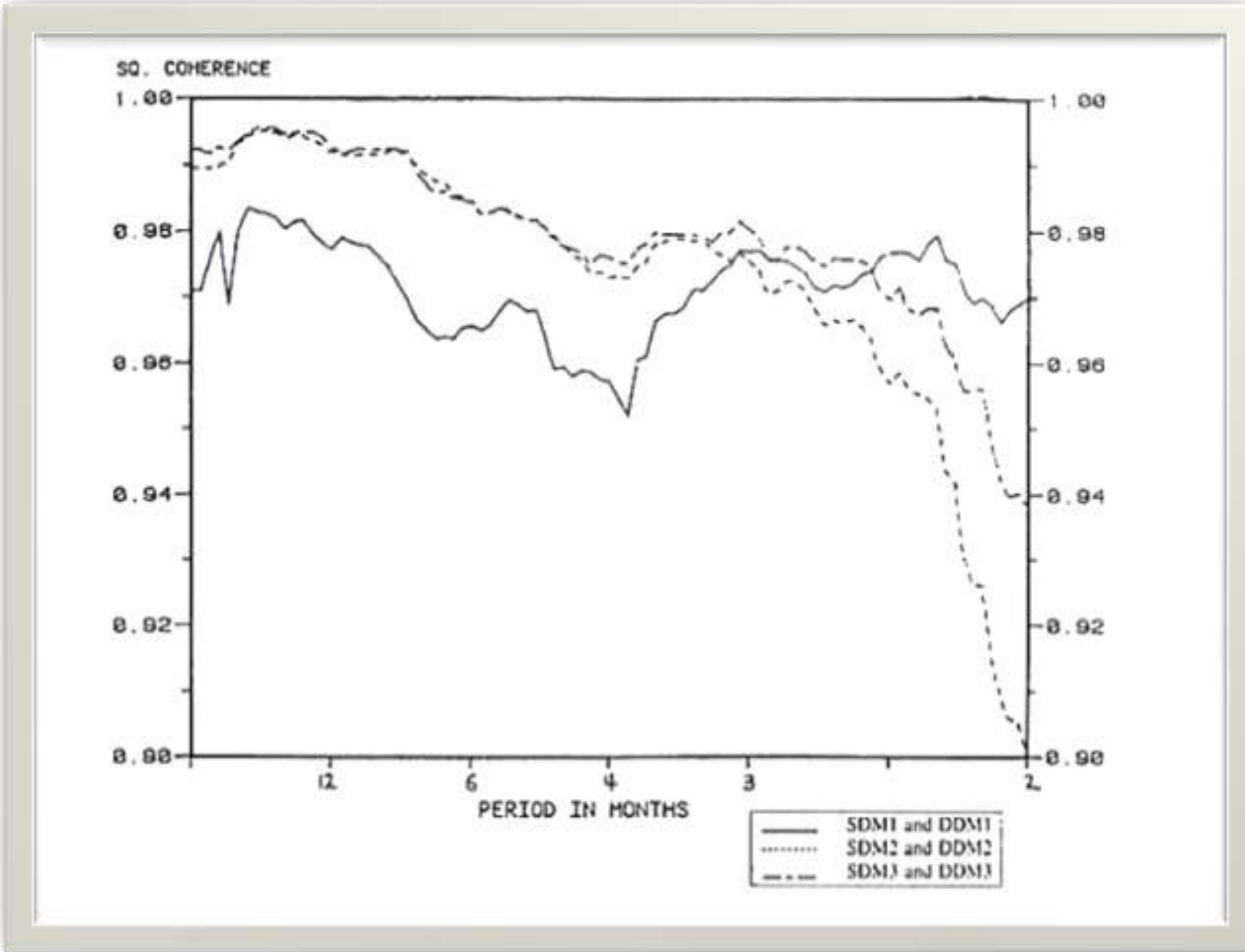
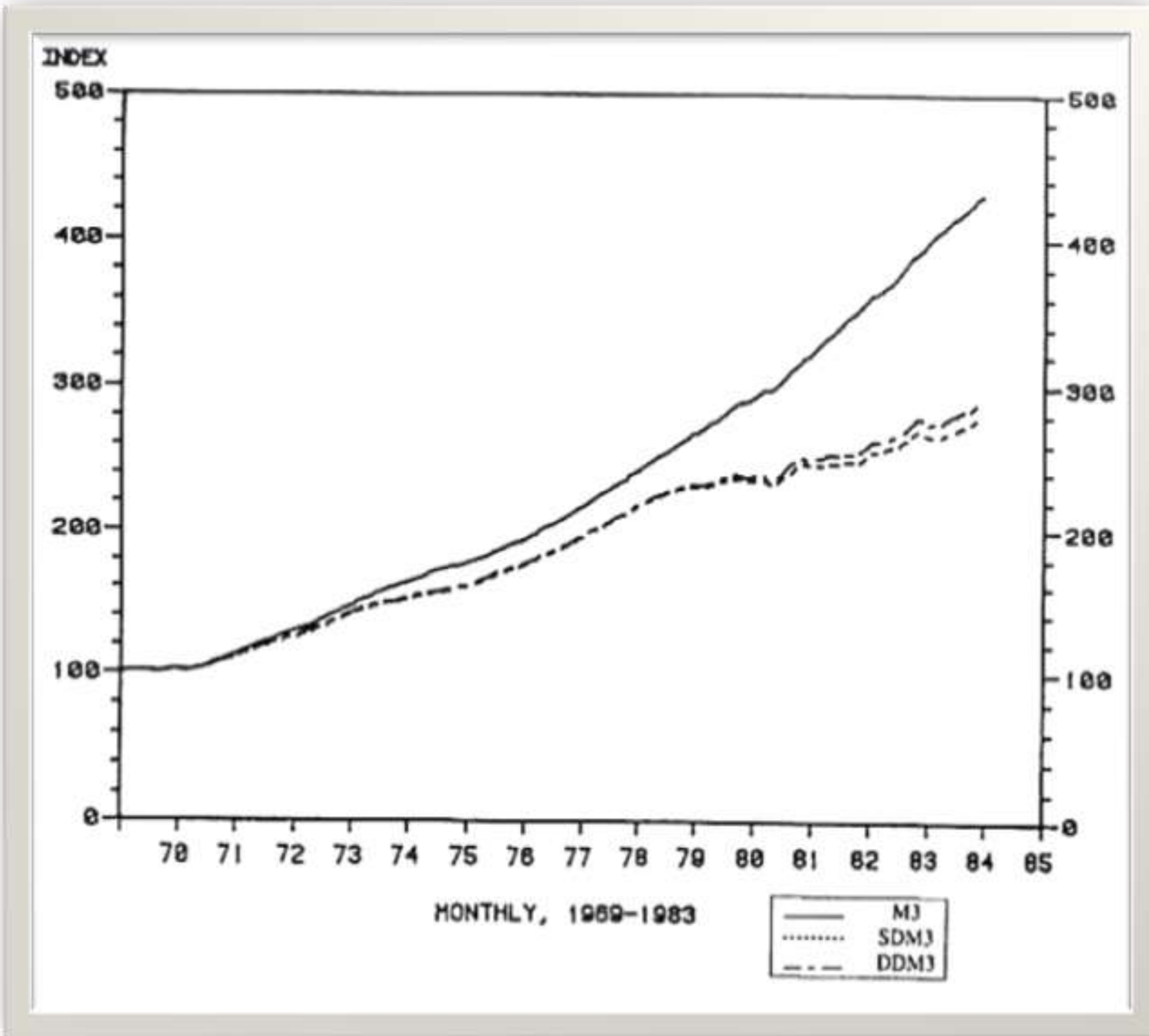


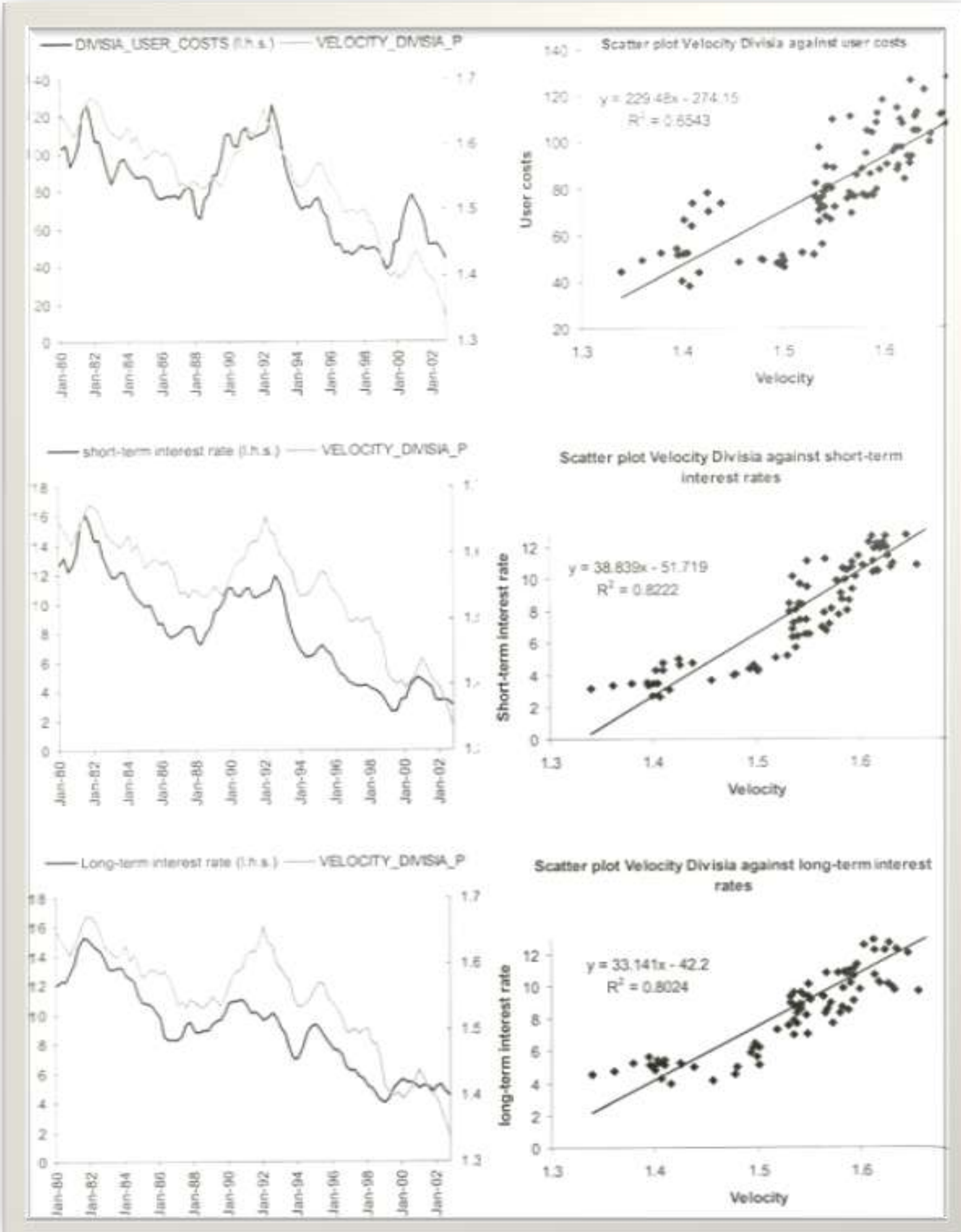
Figure 28: Simple sum, Divisia demand, and Divisia supply



10. European ECB Data

This survey concentrates on the experience in the US, where the theory of monetary aggregation originated. But the Bank of England and the European Central Bank now also have Divisia monetary aggregates. While the Bank of England makes its Divisia monetary aggregates public, the ECB does not. However an economist on the staff of the ECB in Frankfurt provided to William Barnett the plots displayed in Figure 29 (without any implicit or explicit authorization of publication from the ECB). The date was based upon the multilateral Divisia monetary aggregation theory produced for the ECB by Barnett (2007). While there is nothing "official" about Figure 29, the cross plots of velocity against an interest rate in that figure are interesting. Note the stable relationship, closely resembling that for the US displayed in Figure 6.

Figure 29: ECB Mondetary Velocity



11. The Most Recent Data: Would You Believe This?

The most recent research on this subject is Barnett, Chauvet, and Tierney (2008) described in our introductory section 1 above. It is a latent factor Markov switching approach that separates out common dynamics from idiosyncratic terms. The dynamic factor measures the common cyclical movements underlying the observable variables. The idiosyncratic term captures movements peculiar to each index. The approach is used to provide pairwise comparisons of Divisia versus simple-sum monetary aggregates quarterly from 1960:2 to 2005:4. In that paper, they introduced the connection between the state-space time-series approach to assessing measurement error and the aggregation theoretic concept, with emphasis upon the relevancy to monetary aggregation and monetary policy.

We have provided below as Figure 30 one of the figures from that paper. The figure displays the idiosyncratic terms specific to Divisia M3 and simple sum M3. Compare Divisia M3's idiosyncratic downward spikes in figure 30 with simple sum M3's idiosyncratic behavior and then compare the relative predictive ability of the two extracted idiosyncratic terms with respect to NBER recessions. Figure 30 speaks for itself. Divisia is much to be preferred to simple sum.

Considering this most recent results along with the many others surveyed in this paper, and the relevant theory, based solidly on microeconomic aggregation theory, you might find it to be worthwhile comparing the results in this literature with the most recent behavior of the Taylor rule, which does not use money at all. Figure 31 is reproduced from the St. Louis Federal Reserve Bank's publication, *Monetary Trends*. That figure displays the range of the target for the federal funds rate produced from the Taylor rule along with the actual interest rate over that time period, where the actual funds rate is the dark solid line. Notice that the actual interest rate was off target for more than three successive years. Perhaps we now have a real paradox: the evident instability of the Taylor rule.

As documented in this survey, monetary policy and monetary research has been plagued by extremely bad monetary aggregates data, resulting from simple sum aggregation, that has been disreputable to professional aggregation and index number theorist for over a half century. In addition, we have shown that the puzzles that have arisen since the early 1970s were produced by simple sum aggregation and would go away, if reputable index number formulas were used, such as Divisia. With so much history and evidence and so much research documenting the data problems, it might be assumed that central banks would now be taking much care to provide high quality data that is consistent with economic theory.

If that is what you would expect, then look at Figure 32, which was downloaded from the St. Louis Federal Reserve Bank web site and is produced from official Federal Reserve Board data in Washington, DC. Recall that during Volcker's "Monetarist Experiment" period, the instrument of policy was nonborrowed reserves. Also recall from his quotation in section 3 above, that he recently has been highly critical of Federal Reserve policy. Hence it is interesting to look at Figure 32, which displays official recent data on nonborrowed reserves from the Federal Reserve Board.

Total reserves are the sum of borrowed reserves and nonborrowed reserves. Nonborrowed reserves are those reserves that were not borrowed, while borrowed reserves are those reserves that were borrowed. Clearly everything included in borrowed reserves must be reserves, and everything contained in nonborrowed reserves must be reserves. Hence it is impossible for either borrowed reserves or nonborrowed reserves to exceed total reserves. A negative value for either borrowed reserves or nonborrowed reserves would be an oxymoron.

Now look at Figure 32. Observe that nonborrowed reserves recently have crashed to about minus 50 billion dollars. The Federal Reserve's explanation is that they are including the new auction borrowing from the Federal Reserve in nonborrowed reserves, even though they need not be reserves at all. Hence according to this terrible "data," the instrument of monetary policy during Volcker's Monetarist Experiment period now has been driven to a very negative value, as is impossible by the definition of nonborrowed reserves.

Figure 30: Idiosyncratic terms for M3 (red) and Divisia M3 growth (blue), High Interest Rate Phases (green), High Inflation Phases (black), and NBER Recessions (shaded area).

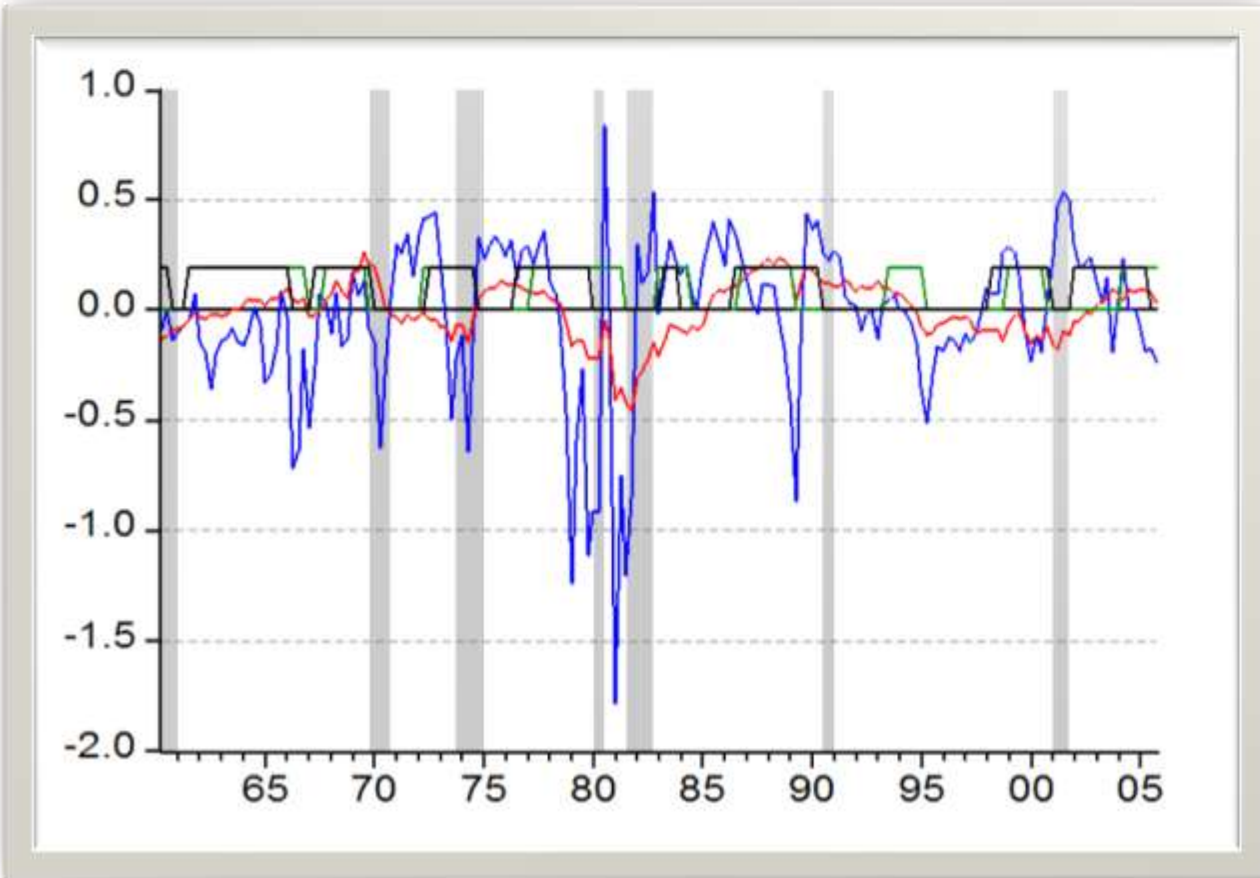


Figure 31: Taylor Rule Federal Funds Rate

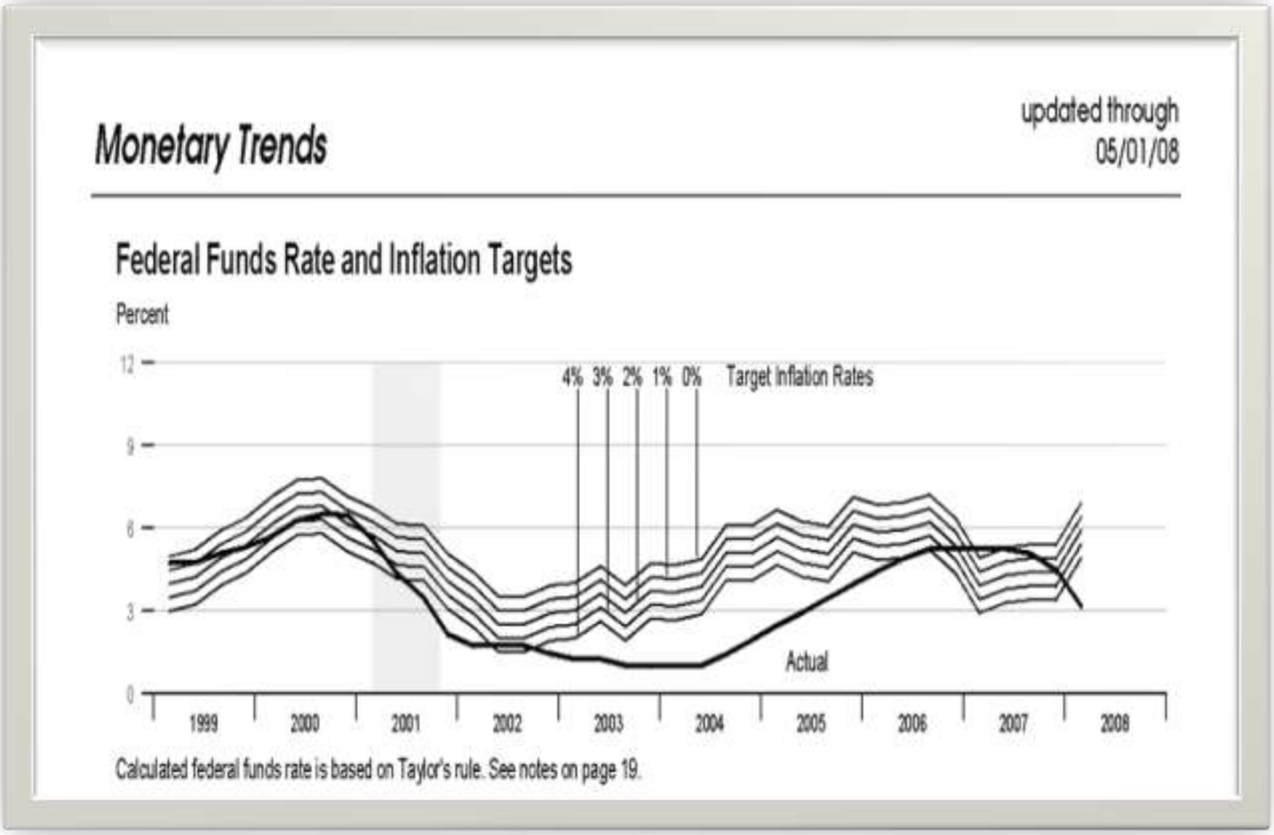
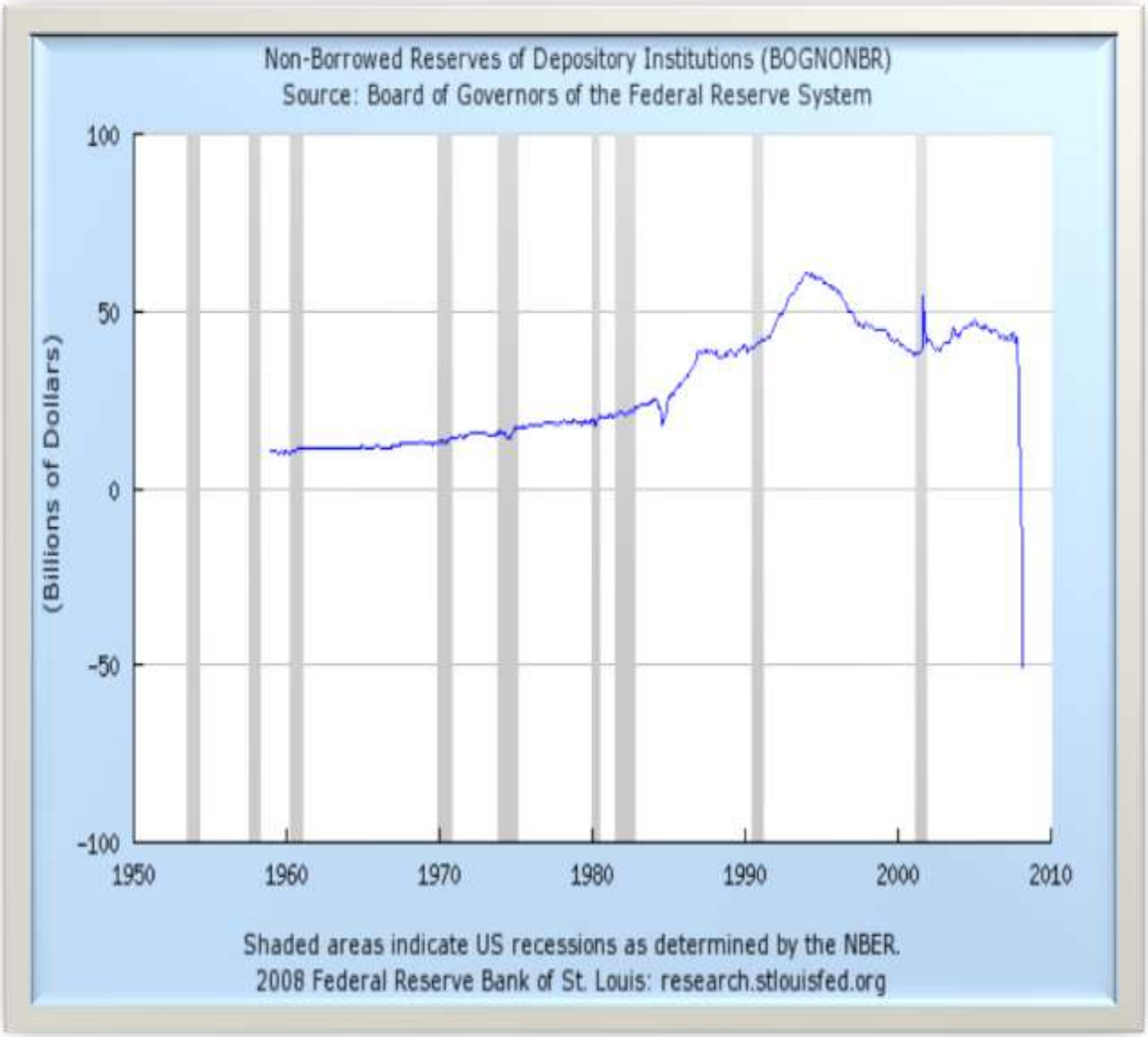


Figure 32: Nonborrowed Reserves



12. Conclusion

In this paper we have surveyed the modern literature on financial aggregation and index number theory. We have shown that all of the puzzles and paradoxes that have evolved in the monetary economics literature were produced by the disreputable simple sum monetary aggregates used by central banks, and are resolved by use of aggregation theoretic monetary aggregates. We argue that Federal Reserve data has not gotten better. But it may have gotten worse.

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