From: CARNAP BROUGHT HOME,
S. Awodey & C. Klein, eds.,
Open Court, 2004

From Frege and Russell to Carnap: Logic and Logicism in the 1920s

Erich H. Reck

Rudolf Carnap was a remarkably productive thinker and writer, making contributions on a variety of philosophical topics. As a result, he is a different philosopher for different audiences: For the general public (insofar as it knows him), he is the author of *Scheinprobleme in der Philosophie* (1928a), *Wissenschaftliche Weltauffassung* (Hahn, Neurath & Carnap 1929), and "Überwindung der Metaphysik durch logische Analyse der Sprache" (1932), thus a prototypical positivist and anti-metaphysician; for philosophers of science he is the author of "Testability and Meaning" (1936–37), *Logical Foundations of Probability* (1950a), and *Philosophical Foundations of Physics* (1966), so a main contributor to the conceptual foundations of science and inductive logic; for philosophers of language, in turn, he is the author of *Introduction to Semantics* (1942), *Meaning and Necessity* (1947), and "Empiricism, Semantics, and Ontology" (1950b), an early and main supporter of Tarskian semantics, as well as a pioneer in the study of intensional and modal logic; for historians of analytic philosophy, finally, he is the author of *Der Raum* (1922a), *Der logische Aufbau der Welt* (1928b), and *Logische Syntax der Sprache* (1934), someone initially influenced by neo-Kantianism, then enthusiastically embracing the new logical-constructionist approach of Russell, and instrumental in establishing scientific or analytic philosophy as a separate school, first in Vienna and then in the US.[1]

In this paper I want to draw attention to yet another side of Carnap: I want to talk about Carnap the *logician*, where "logic" is meant in the narrow sense of deductive and mathematical logic. In this area, too, he published a number of writings over the years: from *Abriss der Logistik* (1929a), through *Foundations*

1. I have used the original titles and publication dates for Carnap's works, both here and below.

of Logic and Mathematics (1939) and *Formalization of Logic* (1943), to *Einführung in die symbolische Logik* (1954). Moreover, several of the texts mentioned above are directly connected with deductive logic as well, especially *Logische Syntax der Sprache* and *Introduction to Semantics*. Nevertheless, in the recent literature on Carnap his contributions to logic are typically not the center of attention. This has several reasons, some of which I will mention later. But there are also good reasons to reexamine this side of Carnap's work, as I will indicate. To keep things manageable, I will not try to cover Carnap's contributions to logic throughout his career. Instead, I will restrict myself to the period from Carnap's student years in Jena, 1911-1914, to around 1930, when his research turned in a new direction. In other words, my topic will be Carnap's views on logic before the (later, mature) shape they took in *Logische Syntax der Sprache* (1934).[2]

In reexamining Carnap's contributions to logic from this period, I have both a historical and a philosophical motive. In terms of history, I want to establish in what way exactly he was influenced, early on in his career, by several of the main logicians of his time: above all by Frege and Russell, then also by Ramsey, Hilbert, Fraenkel, and others. As will become evident, this influence was direct, profound, and rich. Moreover, it led Carnap to a logical research project of his own, in the second half of the 1920s, that deserves further attention. On the philosophical side, I want to analyze this research project with respect to its central goals. This will allow me to draw some conclusions about Carnap's position in logic and the philosophy of mathematics during that period, in particular about the sense in which the research project mentioned constitutes, or was meant as, a contribution to logicism.

. . .

It is, of course, well known from Carnap's intellectual autobiography (Carnap 1963) that he was strongly influenced by Frege and Russell. Let me start by reviewing some of the things he remarks in this connection in his later reflections.

In Part I, §1, of his autobiography, entitled "My Student Years", Carnap recalls the beginnings of his relation to Frege as follows:

> In 1909 we moved to Jena. From 1910 to 1914 I studied at the Universities of Jena and Freiburg i. Br. First I concentrated on philosophy and mathematics; later, physics and philosophy were my major fields. . . .Within the field of philosophy, I was mainly interested in the theory of knowledge and in the philosophy of science. On the other hand, in the field of logic, lecture courses and books by philosophers appeared to me dull and entirely obsolete after I had become acquainted with a genuine logic through Frege's lectures. (Carnap 1963, pp. 3-4)

2. Carnap's work in logic from this early period has not been discussed much so far. My own interest in it was stirred up by (Awodey & Carus 2001). The present paper continues work done in that paper, as well as in (Awodey & Reck 2002a) and (Awodey & Reck 2002b). For discussions of Carnap's views in *Logische Syntax der Sprache*, see (Goldfarb 1996), (Friedman 1999), and the references in them.

Half a page later, he comes back to Frege's lectures in more detail:

> [T]he most fruitful inspiration I received from university lectures did not come from those in the field of philosophy proper or mathematics proper, but rather from the lectures of Frege on the borderlands between those fields, namely, symbolic logic and the foundations of mathematics.
>
> Gottlob Frege (1848-1925) was at that time, although past 60, only Professor Extraordinarius (Associate Professor) of mathematics in Jena. His work was practically unknown in Germany; neither mathematicians nor philosophers paid any attention to it. It was obvious that Frege was deeply disappointed and sometimes bitter about this dead silence. No publishing house was willing to bring out his main work, the two volumes of *Grundgesetze der Arithmetik*; he had it printed at his own expense. In addition, there was the disappointment over Russell's discovery of the famous antinomy which occurs both in Frege's system and in Cantor's set theory. I do not remember that he ever discussed in his lectures the problem of this antinomy and the question of possible modifications of his system in order to eliminate it. But from the Appendix of the second volume it is clear that he was confident that a satisfactory way for overcoming the difficulty could be found. He did not share the pessimism with respect to the "foundation crisis" of mathematics sometimes expressed by other authors.
>
> In the fall of 1910, I attended Frege's course "Begriffsschrift" (conceptual notation, ideography), out of curiosity, not knowing anything either of the man or the subject except for a friend's remark that somebody had found it interesting. We found a very small number of other students there. Frege looked old beyond his years. He was of small stature, rather shy, extremely introverted. He seldom looked at the audience. Ordinarily we saw only his back, while he drew the strange diagrams of his symbolism on the blackboard and explained them. Never did a student ask a question or make a remark, whether during the lecture or afterwards. The possibility of a discussion seemed to be out of the question.
>
> Towards the end of the semester Frege indicated that the new logic to which he had introduced us could serve for the construction of the whole of mathematics. This remark aroused our curiosity. In the summer semester of 1913, my friend and I decided to attend Frege's course "Begriffsschrift II." This time the entire class consisted of the two of us and a retired major of the army who studied

> some of the new ideas in mathematics as a hobby. . . . In this small group Frege felt more at ease and thawed out a bit more. There were still no questions or discussions. But Frege occasionally made critical remarks about other conceptions, sometimes with irony and even sarcasm. In particular he attacked the formalists, those who declared that numbers were mere symbols. . . .
>
> In the advanced course on *Begriffsschrift*, Frege explained various applications, among them some which are not contained in his publications, e.g., a definition of the continuity of a function, and of the limit of a function, the distinction between ordinary convergence and uniform convergence. All these concepts were expressible with the help of the quantifiers, which appear in his system of logic for the first time. He gave also a demonstration of the logical mistake in the ontological proof for the existence of God.
>
> Although Frege gave quite a number of examples of interesting applications of his symbolism in mathematics, he usually did not discuss general philosophical problems. It is evident from his works that he saw the great philosophical importance of the new instrument which he had created, but he did not convey a clear impression of this to his students. Thus, although I was intensely interested in his system of logic, I was not aware at that time of its great philosophical significance. Only much later, after the first world war, when I read Frege's and Russell's books with greater attention, did I recognize the value of Frege's work not only for the foundations of mathematics, but for philosophy in general.
>
> In the summer semester of 1914 I attended Frege's course, *Logik in der Mathematik*. Here he examined critically some of the customary conceptions and formulations in mathematics. He deplored the fact that mathematicians did not even seem to aim at the construction of a unified, well-founded *system* of mathematics, and therefore showed a lack of interest in foundations. He pointed out a certain looseness in the customary formulation of axioms, definitions, and proofs, even in works of the more prominent mathematicians. (Ibid., pp. 4–6)

As indicated by Carnap, he was not only influenced by Frege through his lectures, but also through studying his published writings. Here is what he adds, a few pages later, about immersing himself in both Frege's and Russell's main works after World War I:

> After the war, I lived for a while in Jena, and then in Buchenbach near Freiburg i. Br. In this period, I first passed my examinations, and then I began my own research in philosophy, first in relative isolation, but later in contact with Reichenbach and others who worked in a similar direction. This period ended in 1926, when I went to Vienna and joined the Vienna Circle. . . .
>
> Around 1919 I studied the great work *Principia Mathematica* by Whitehead and Russell, to which Frege had sometimes referred in his lectures. I was strongly impressed by the development of the theory of relations in this work. The beginnings of a symbolic logic of relations were also in Frege's system, but in *P.M.* the theory was developed in a very comprehensive way and represented by a much more convenient notation. I began to apply symbolic notation, now more frequently in the *Principia* form than in Frege's, in my own thinking about philosophical problems or in the formulation of axiom systems. When I considered a concept or a proposition occurring in a scientific or philosophical discussion, I thought that I understood it clearly only if I felt that I could express it, if I wanted to, in symbolic language. I performed the actual symbolization, of course, only in special cases where it seemed necessary or useful. . . .
>
> [Frege's] main work, *Die Grundgesetze der Arithmetik* (2 vols., 1893 and 1903), I studied in 1920. From Frege I learned carefulness and clarity in the analysis of concepts and linguistic expressions, the distinction between expressions and what they stand for, and concerning the latter between what he called "*Bedeutung*" (denotation or *nominatum*) and what he called "*Sinn*" (sense of *significatum*). From his analysis I gained the conviction that knowledge in mathematics is analytic in the general sense that it has essentially the same nature as knowledge of logic. I shall later explain how this view became more radical and precise, chiefly through the influence of Wittgenstein. Furthermore the following conception, which derives essentially from Frege, seemed to me of paramount importance: It is the task of logic and of mathematics within the total system of knowledge to supply the forms of concepts, statements, and inferences, forms which are then applicable everywhere, hence also to non-logical knowledge. It follows from these considerations that the nature of logic and mathematics can be clearly understood only if close attention is given to their application in non-logical fields, especially in empirical science. (Ibid., pp. 10–12)

Carnap goes on to describe the strong impact that reading Russell's book *Our Knowledge of the External World* (1914) had, in 1921, on his general philosophical perspective. After that, he comes back to logic:

> I also continued to occupy myself with symbolic logic. Since the *Principia Mathematica* was not easily accessible, I began work on a textbook in symbolic logic. There was no copy of the *Principia* in the University Library in Freiburg. The price of a new copy was out of reach because of the inflation in Germany. Since my efforts to find a

secondhand copy in England were unsuccessful, I asked Russell whether he could help me in finding one. Instead, he sent me a long list containing all the most important definitions of *Principia*, handwritten by himself, on 35 pages, which I still cherish as a priceless possession. In 1924 I wrote the first version of the later book, *Abriss der Logistik* [1929a]. It was based on *Principia*. Its main purpose was to give not only a system of symbolic logic, but also to show its application for the analysis of concepts and the construction of deductive systems. (Ibid., p. 14)

Besides his own studies in logic, Carnap also began to exchange ideas about it with like-minded philosophers. As one result, he, Reichenbach, and others soon organized a conference on logic and its applications. This conference took place in March 1923, in Erlangen. In his autobiography, Carnap recalls it briefly as follows:[3]

> Among the participants were Heinrich Behmann, Paul Hertz, and Kurt Lewin. There were addresses on pure logic, e.g., a new symbolism, the decision problem, relational structures, and on applied logic, e.g., the relation between physical objects and sense-data, a theory of knowledge without metaphysics, a comparative theory of science, the topology of time, and the use of the axiomatic method in physics. (Ibid., p. 14)

Finally, Carnap describes his continued involvement with logic after moving to Vienna in 1926:

> All members of the Vienna Circle had studied at least the elementary parts of *Principia Mathematica*. For students of mathematics Hahn gave a lecture course and a seminar on the foundations of mathematics, based on the *Principia*. When I came to Vienna I continued these courses for students both of mathematics and of philosophy. (Ibid., p. 28)
>
> During my time in Vienna two earlier works of mine were completed and published: *Der logische Aufbau der Welt* [1928b] and the *Abriss der Logistik* [1929a]. Schlick urged me to prepare the *Logistik* for publication because he felt the need for an introduction to symbolic logic which emphasized its applications in non-logical fields and which thus could also be used in our philosophical work. . . . I used in my book a hierarchy of types, like Russell and Whitehead's *Principia Mathematica*, but in a simplified form. A system with type distinctions seemed to me a more natural form for the total language of science. However, I was also interested in a form of logic without type distinctions. In 1927 I had planned a logic system of this form, to be based on the Zermelo-Fraenkel axiom system of set theory, but restricted in the sense of a constructivist method. However, I did not find the time to carry out this plan. In my later book, *Logical Syntax*, I again used a language with types. (Ibid., p. 32)

3. For further background on the Erlangen conference, see (Thiel 1993).

. . .

Many points are quite interesting in these reflections by Carnap for our purposes, especially the following three: First, there is the coincidence that Carnap, as one of a few people, found his way into Frege's lectures on logic "out of curiosity, not knowing anything either of the man or the subject except a friend's remark that somebody had found it interesting." As we saw, Carnap attended three of Frege's courses while a student at Jena: "Begriffsschrift I" in 1911, "Begriffsschrift II" in 1913, and "Logik in der Mathematik" in 1914. He was, thus, exposed to modern mathematical logic long before any textbooks on it became available – one of the first such textbooks would be his own *Abriss der Logistik* (Carnap 1929a) – and right around the time when *Principia Mathematica*, the most influential text in twentieth century logic, appeared in print (Whitehead & Russell 1910-13). Later, in 1919-1920, he returned to studying Frege's and Russell's published writings carefully and in detail.[4] Second, it is remarkable that Carnap, intent on working his way through Whitehead & Russell's *Principia* but unable to obtain a copy, wrote to Russell directly in 1922. Moreover, Russell responded, with a handwritten 35-page letter in which he simply summarized, or excerpted, his *magnus opum* for him. As a third point, note that, after making personal contact with both Frege and Russell and after studying their main works thoroughly, Carnap initiated further discussions about logic with other like-minded thinkers, including organizing the conference in Erlangen in 1923. He also soon embarked on writing a textbook on logic himself: *Abriss der Logistik* (first version 1924). Finally, with respect to the main logical topics of interest to Carnap during this period he lists the following: simple type theory, seen as a logical framework not only for mathematics, but for "the total system of knowledge"; the decision problem; relational structures; and the uses of the axiomatic method.

But there is more below the surface, beyond what Carnap, in his reflections 40 years later, chooses to mention. Namely, sometime during the 1920s he formed the plan to write a second book on logic, in addition to *Abriss*. This second book was intended to be more than a textbook – it was to be a research monograph, containing Carnap's own, original contributions to logic. As such it was meant to be a successor to both Frege's *Grundgesetze* and Whitehead &

4. Carnap's Nachlaß in the Archives for Scientific Philosophy, University of Pittsburgh, contains copies of Frege's three main works: *Begriffsschrift* (1879), *Die Grundlagen der Arithmetik* (1884), and *Grundgesetze der Arithmetik* (1893/1903). There are marginal notes by Carnap scattered through all of them. Moreover, Carnap's copy of *Begriffsschrift* (item ASP/RC 111-C-58) contains the following handwritten remark: "Respectfully presented by the author [Ehrerbietigst überreicht vom Verfasser]." It seems, thus, that he asked Frege to autograph his copy.

Russell's *Principia*, at least in certain respects. The working title for this ambitious project was *Untersuchungen zur Allgemeinen Axiomatik*; sometimes Carnap also used the title *Metalogik* for it.

Carnap finished an almost complete draft of Part I of *Allgemeine Axiomatik* around 1928, which he then circulated among friends; he also made various notes and outlines for Part II (Carnap 1928c). However, he never completed the book; the draft and his further notes disappeared in a drawer. They were only recently edited and published, by Thomas Bonk and Jesus Mosterin (Carnap 2000). The only related texts Carnap himself chose to publish towards the end of the decade were a few short articles: "Eigentliche und Uneigentliche Begriffe" (Carnap 1927), "Bericht über Untersuchungen zur allgemeinen Axiomatik" (Carnap 1930a), and "Die alte und die neue Logik" (Carnap 1930b); a few years later, with his student Friedrich Bachmann, he also published "Über Extremalaxiome" (Carnap & Bachmann 1936). But basically, Carnap came to see his logic project from the 1920s as a failure, and he abandoned it around 1930. This is, no doubt, why the work on *Allgemeine Axiomatik* is not even mentioned in his autobiography.

There certainly are problems with *Allgemeine Axiomatik*, thus good reasons for Carnap not to publish it, as we will see later. Nevertheless, it is unfortunate, I think, that he didn't mention the corresponding research project *at all* in his autobiographical reflections. His silence had the effect that until quite recently this side of Carnap's work was largely ignored, or even completely unknown. This is regrettable because Carnap was addressing interesting issues, and in a way that put him at the cutting edge of research in logic at the time. The issues addressed are closely connected with those mentioned by Carnap, in some asides, in his autobiography: the simple theory of types, the decision problem, relational structures, and the axiomatic method.

. . .

Let us go back to Carnap's relation to Frege and Russell for a moment, since this will help to put Carnap's logic project from the 1920s into context. I want to complement Carnap's own recounting of this relation by reporting on additional material from his Nachlaß. Besides bringing to light further historical details which merit attention in themselves, this will allow us to determine more fully how Carnap's work on logic is related to Frege's and Russell's.

First again to Frege's lectures on logic, which Carnap attended in 1911-14. What was it that Frege covered in these lectures? Basically, "Begriffsschrift I," the first course Carnap attended, provided an introduction to Frege's logic, i.e., to its basic notions, its two-dimensional notation, etc. In "Begriffsschrift II," Frege developed this apparatus further and then applied it to various concepts in mathematics, for example to those of continuity and uniform continuity, as mentioned by Carnap in his autobiography. And in the third course, "Logik in der Mathematik," Frege added a discussion of various approaches to the foundations of mathematics.

Now, it turns out that Carnap took detailed *notes* in all three courses. These notes were kept in his Nachlaß, in the form of college notebooks. For a long time they just lay there, unstudied and unedited. One reason why they were overlooked for so long is, no doubt, that they were written in a shorthand notation few people are able to read nowadays.[5] It was only in the 1990s that these notes were transcribed into ordinary German and edited by Gottfried Gabriel and his co-workers. The first two sets of them, for "Begriffsschrift I" and "Begriffsschrift II," were published in German in 1995 (Frege 1995). The third set of notes, for "Logik in der Mathematik," has not been published yet, mainly because a closely related text is available, under the same title, in Frege's *Nachgelassene Schriften* (Frege 1914). All three sets of notes are currently being prepared for publication in English (Frege forthcoming).

Several aspects make Carnap's notes from Frege's lectures interesting and important: To begin with, they show how Frege, the founder of modern logic, presented his path-breaking insights into relational and quantificational logic to students in his classes. Being the only evidence we have in this connection, they allow us to see, thus, how Frege's presentation compares to and contrasts with contemporary ones. In addition, the notes present us with Frege's views on logic after he was informed of Russell's antinomy (in 1902), and from a period during which he published little. Finally and crucially for present purposes, they permit us to determine exactly what it was that Carnap learned from Frege as a student in Jena.

Without being able to go into great depth here, the most important observations in the latter connection are the following:[6] First, the logic Frege presented to Carnap was not first-order, but higher-order logic, essentially in the form of a simple theory of types. Second, Frege's theory of extensions of concepts or, more generally, of value ranges – the notorious part that leads to Russell's antinomy – was simply left out in the lectures. (This is why Carnap can say in his autobiography: "I do not remember that [Frege] ever discussed in his lectures the problem of [Russell's] antinomy.") Third, while numbers are still treated as objects by Frege, in a few asides, he does not provide a systematic theory in this connection; in particular, he does not show how to do so without bringing extensions back in. Fourth, right along with the technical, symbolic material and the application of his logic to mathematics, Frege introduces the Sinn-Bedeutung distinctions and related issues. Finally, in the third course attended by Carnap Frege discusses and criticizes various approaches to the foundations of mathematics besides his own, including crude formalist views, but especially Hilbert's axiomatic approach to geometry. In that connection, Frege makes his well-known point that what Hilbert's axioms really define (not implicitly, but explicitly) is a higher-order concept, namely that of a three-

5. The shorthand notation is called *Stolze-Schrey*; see (Frege 1996), pp. xii-xvi, for further information.
6. For more, see (Frege 1996) and (Frege forthcoming), including both introductions by the editors.

dimensional Euclidean space. Overall, what Carnap was presented with in the lectures was the consistent, unproblematic part of Frege's logic, together with typical Fregean reflections in the philosophy of logic and of mathematics and with a still relatively optimistic attitude about the prospects for logicism. (The latter is what allows Carnap to say in his autobiography: "[Frege] did not share the pessimism with respect to the 'foundational crisis' of mathematics sometimes expressed by other authors.")

. . .

While Frege's influence on Carnap with respect to logic was early, direct, and strong, the influence of Russell was equally important and almost as direct. We have already been told that Carnap had heard about *Principia Mathematica* in Frege's lectures; also that his serious study of it started in 1919; and that, after moving to Freiburg i. Br. that year, the problem for him was that the university library did not own a copy of *Principia*. This is the point at which he wrote to Russell.

As it turns out, contained in Carnap's Nachlaß there is a whole series of letters to Russell from the 1920s. The first, dated Nov. 17, 1921, is the cover letter for a package in which Carnap sent Russell his book *Der Raum*. The letter itself starts as follows:

> Dear Sir:
>
> In sending you a copy of my treatise *Der Raum*, I am pleased to have an opportunity to thank you for the rich stimulation I have received in my studies of the philosophy of the exact sciences from your works. Besides Frege (whose personal student I had the good fortune to be) and Couturat, I would name you as the person to whom I owe the most in this area. . . . (Carnap 1921, p. 1, my translation)[7]

Half a year later, on June 13, 1922, Carnap sent another letter to Russell. Let me quote it in full, since it is both relevant and remarkable:

> Dear Sir:
>
> I would like to ask you for a favor: Would it be possible to obtain, with your help, the first volume of "Principia Mathematica" at a reduced price?

7. In a remark that gives a sense both of the times and of his political convictions, Carnap adds in the same letter: "I am particularly happy that it is you, as the first Englishman, with whom I can join hands in the scientific field, since already during the time of the war you stood up so openly against the intellectual enslavement resulting from hatred between peoples and in favor of a humane and pure way of thinking. When I remember that the same conviction was also held by Couturat, who unfortunately died so prematurely, I ask myself: Is it mere coincidence that the people who achieve the greatest clarity in the most abstract area of mathematical logic are also those who oppose, clearly and forcefully, the narrowing of the human spirit by means of affect and prejudice in the area of human relations?" (Ibid., p. 2, my translation)

> Because of inflation it is customary in scientific circles in Germany, if one has connections to an author, to buy his book that way for the so-called "author's price," i.e., for a reduced price which the publisher charges the author. For us, foreign books are even harder, often impossible, to get (1 Shilling = approx. 60 Mark). This is why I hope I can ask you for this favor, even if we are not personally acquainted. I do have a relation to your work, however: It was brought to my attention when I was a student of Frege's in Jena. At that time I studied "Principia Mathematica," which was available in the library there, and made excerpts from it that have often been of use to me. I have also reproduced selections from them so as to make them the basis for discussions of related topics, since I have found the variety of logistic systems and notations often inconvenient in such discussions. The enclosed copy shows, of course, nothing but my interest in the area and in your work, most likely also some mistakes. For that reason all the more, I wish to own the book; it is not available in the university library in Freiburg which I am currently using, and I would like to work my way through it more thoroughly. It is my belief that I can make logistic fruitful, especially the theory of relations, in the area of the theory of knowledge as well (more precisely: the structural theory of the object of knowledge), an area in which I currently work (although in a somewhat different direction than your book "Our Knowledge . . . ," which I do own and value).
>
> If I may trouble you in this way, please send me a bank account and the amount (in Shillings) to be wired for a paperback copy of Volume I, including postage and other expenses.
>
> (As long as, unfortunately, no international language has been introduced in general, I follow the usual custom that each of us writes in his own language.)
>
> Most sincerely,
>
> R.C. (Carnap 1922b, my translation)

Apparently Russell responded quickly. He was willing to help, although not exactly in the way requested; instead, he offered to send Carnap *excerpts* from *Principia*.

Carnap's answer to that offer can be found in a letter from July 29, 1922:

> Dear Sir:
>
> For your kind offer to send me the definitions and main theorems of vol. I of "Principia", I want to thank you sincerely. This will most certainly be of use to me. And after using your excerpts myself, I would like to pass them on to other people who are in the same situation. In addition, I am playing with the idea of publishing such a

collection, with annotations, as "Introduction to Logistic" (or: Introduction to Mathematical Logic). Not to function as a construction of mathematics on a logical basis, but as a general tool for logicians and epistemologists, applicable in various areas. What would you think of such a plan? I believe that today such an instrument would be adopted and employed gladly in philosophical circles; many will probably be in the same situation as I: happy to be able to refer to such a collection, so that it becomes unnecessary to preface every application of the symbolism (especially that of the theory of relations) with an extensive explanation. There is a question, however, whether in the current economic situation a publisher can be found who is willing to take the risk of publication. Perhaps another manner of reproduction should be considered, instead of the expensive process of printing in the case of a small edition, e.g., autographical printing. . . . (Carnap 1922c, pp. 1–2, my translation)

We can see here, among other things, that the seed for Carnap's book *Abriss der Logistik* had been planted as early as 1922.

Two months later, on September 29, 1922, Carnap sent another letter to Russell. By that time he had received Russell's handwritten excerpts. His new letter starts as follows:

> Dear Sir:
>
> After coming back from my trip, I have found your package. I am extremely thankful for all the effort you have put into it. I am already looking forward to starting to study this work, after having taken care of some other things, and I hope that something fruitful will result from it. Later I will also gladly pass on your notes to other workers in the same field; I have in mind in particular: Gerhards, Aachen; Behmann, Göttingen; Wilke, Weisswasser; Reichenbach, Stuttgart. . . . (Carnap 1922d, p. 1, my translation)

Next, in a letter from February 20, 1923, Carnap writes:

> Dear Sir:
>
> Although a publication is still not forthcoming, as the next fruit of studying your system and related problems there will be several discussions of it. Concerning the corresponding conference . . . , see the attached announcement. There will only be a small number of participants. . . . Only now does the thought occur to me (perhaps too late) that your participation might be possible, too. I blame myself for not having thought about that earlier. But these days other countries seem, in normal circumstances, so unreachably far away that one forgets that the road from outside in is often easier. . . . I will take the collection of definitions etc., on which you spent such an effort and for which I am quite thankful, with me to the conference so as to let its influence on me deepen, also so as to give it to some of the other participants. . . . (Carnap 1923a, p. 1, my translation)

The conference Carnap speaks of here, as in his autobiography, is the one in Erlangen, which Russell did not attend in the end.

Carnap's interactions with Russell continued, including his attempts to incorporate Russell in his and his co-workers' activities. For example, in a letter from September 9, 1923, Carnap asks Russell to be on the editorial board of a newly planned journal. As he explains, that journal is to cover epistemology, philosophy of science, and logic, including "basic issues concerning axiomatics and applications of the axiomatic method." Referring back to their earlier correspondence, he adds: "[My] introduction to logic (*Der Abriss der Logistik*)–which I want to finish this winter, perhaps in collaboration with Behmann, Göttingen–will probably be published in that journal, too" (Carnap 1923b, p. 1, my translation). Carnap's correspondence with Russell continues throughout the 1920s; but let me mention only two more details from it, in order to round off our discussion so far. First, in 1924 Carnap did, after all, obtain a copy of *Principia Mathematica* itself; and he obtained it again via Russell (see Carnap 1926a and 1926b). Second, in 1929 Carnap, in turn, sent a copy of *Abriss der Logistik* to Russell, with the comment: "Now it is finally finished (after completing the ms. already in the fall of 1927)" (Carnap 1929c).

As is evident both from this extended correspondence and from Carnap's autobiography, he was extremely grateful to Russell for his support–starting with Russell sending him the collection of main definitions and theorems in *Principia Mathematica* in 1922. In retrospect that initial episode alone is, as a matter of fact, rather extraordinary: Here is Carnap, a young, completely unknown German, writing to Russell, the famous co-author of *Principia Mathematica* (Whitehead & Russell 1910–13), author of *Principles of Mathematics* (1903) and of various other well-known works, asking him for help in obtaining a copy of *Principia*. And in response, Russell, presumably not able to obtain such a copy at the time, sends Carnap back a handwritten 35-page summary of the book![8]

. . .

Throughout his career Carnap emphasized the strong influence Russell had on him, including his adoption of a Russellian (and Fregean) logicism in the philosophy of mathematics and logic. What the letters to Russell from the 1920s show, more particularly, is that Carnap's *Abriss der Logistik* was a direct outcome of studying *Principia Mathematica*; also, that plans for Carnap's book started to take shape as early as 1922; that a first draft of it was written in 1924; that for a while Heinrich Behmann was supposed to be a co-author; and that the

8. This handwritten summary is also available in the Carnap Nachlaß in Pittsburgh (Russell 1922). It is still awaiting further study and, perhaps, publication.

manuscript for the book was essentially finished in 1927, although it was only published in 1929. In *Abriss* itself, Carnap gives generous credit to Russell and his works in remarks such as the following:

> The establishment of modern logic and the theory of relations is, above all, due to B. Russell and A.N. Whitehead. My own presentation is based on their work *Principia Mathematica*. Everyone who uses logistic tools owes a debt of thanks to them. I owe an additional debt to Mr. Russell for his extraordinarily helpful and generous support of my early studies in this area. (Carnap 1929a, pp. iii–iv, my translation; cf. pp. 2, 107, etc.)

The basic purpose of Carnap's *Abriss der Logistik* is, clearly, to make the logical system from *Principia* available to a broader audience. For our purposes two other observations are also important, however: First, according to its author the material in *Abriss* is "not to function as a construction of mathematics on a logical basis, but as a *general tool* for logicians and epistemologists" (Carnap 1922c, emphasis added); more specifically, it is to function as a tool for dealing with "basic issues concerning *axiomatics*" (Carnap 1923b, emphasis again added). Second, while generally speaking Carnap does work with a theory of types in his research during the 1920s, it is not Russell's ramified theory, as presented both in *Principia Mathematica* and in Russell's 35-page summary, but a version of the *simple* theory of types that he employs. The first of these two observations leads to the question whether Carnap should really, or unequivocally, be seen as a logicist in the tradition of Russell and Frege. Both observations together lead to a further, more specific question: What was Carnap trying to do within a simple theory of types, used as a "general tool for logicians;" what, more specifically, did he mean by "basic issues concerning axiomatics"? I will come back to Carnap's relation to logicism shortly. Before that, let me fill in some background on the history and use of the simple theory of types.

If we consider the reception of *Principia Mathematica* in the 1910s and 20s, it is clear that there were questions and doubts early on about certain aspects of its general framework: ramified type theory. They usually focused on the following three axioms in it: the axiom of reducibility, the axioms of infinity, and the axiom of choice. In fact, Russell himself began to express doubts about these as axioms of *logic*, as his introduction to the second edition of *Principia* illustrates (Whitehead & Russell 1925). Frank Ramsey's article "The Foundations of Mathematics" (Ramsey 1925) is usually considered to be the main source of the simple theory of types, a theory proposed, subsequently, as a way of solving or avoiding the corresponding problems.

Now, that article by Ramsey, together with the second edition of *Principia*, is exactly the source mentioned by Carnap in connection with not using a Russellian ramified type theory, including the axiom of reducibility, in *Abriss der Logistik*. As he remarks:

> According to a more recent conception (Russell, PM I², S. XIV; Ramsey [Found.] 275ff.), the ramified hierarchy and, thus, the axiom of reducibility is dispensable. Consequently it will not be discussed in what follows. The ramified theory of types was constructed in the belief that certain antinomies of a special kind could not otherwise be avoided. . . . Closer inspection seems to show, however, that these antinomies are not of a logical, but of a semantic kind; i.e., they only occur because of a defect in ordinary language. . . . Then again, the problem of the antinomies has not been solved completely. It is tied up with the problem of the thesis of extensionality. (Carnap 1929a, p. 21, my translation)

Similarly, in a review of *Principia Mathematica* for the journal *Erkenntnis*, published in 1931 (Carnap 1931b), Carnap declares:

> A problem that still awaits solution is the axiom of reducibility. It has to be rejected for philosophical reasons. . . . A charitable attempt to solve the difficulties has been made by Ramsey [in "The Foundations of Mathematics"], partly following Wittgenstein. He doesn't just drop the axiom of reducibility, but also the ramified theory of types. His overall conception has not found much sympathy; but various of his results remain correct and important. (Carnap 1931b, p. 74)

Going back to *Abriss*, Carnap goes on to say that a more general rejection of type theory on the basis of a rejection of the axiom of reducibility is not justified, since one can avoid that axiom by using the simple theory of types.

While Ramsey's article is undoubtedly important, also for Carnap, it should be added that Ramsey was not the only person to suggest a simplified version of type theory in the early 1920s. The theory seems to have been in the air at the time. For example, the Polish logician Leon Chwistek proposed related ideas independently.[9] More importantly for us, Ramsey does not present a systematic, technical development of simple type theory in his "The Foundations of Mathematics"; he only formulates, as Carnap puts it, a number of "philosophical reasons" for considering it. It is sometimes assumed that the first systematic presentation of simple type theory occurs (in 1928) in Hilbert & Ackermann's *Grundzüge der theoretischen Logik*. However, the first edition of that book (Hilbert & Ackermann 1928) still centers around ramified type theory, along the lines of *Principia Mathematica*. It is only in the second edition (of 1938) that the two authors switch to simple type theory as the preferred logical framework. In Carnap's *Abriss der Logistik* (Carnap 1929a), on the other hand, the framework is immediately the simple theory of types. Moreover, Carnap's text was essentially finished in 1927, as we have seen, and then circulated among a number of logicians.

9. Compare (Church 1956), p. 355, (Grattan-Guinness 1979), pp. 278–79, and the references in them.

One conclusion concerning the history of logic to be drawn at this point is this: Carnap should be given some credit for the development and, especially, the dissemination of simple type theory in the 1920s. Note here that exactly this theory (not first-order logic) was subsequently the theory underlying seminal works by Gödel, Tarski, and others, in the late 1920s and early 30s. In particular, a version of the simple theory of types was the framework for Gödel's famous "Über formal unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme I" (Gödel 1931).[10] The theory reached its canonical form a few years later, in Alonzo Church's "A Formulation of the Simple Theory of Types" (Church 1940). Then again, at that time it was already in the process of being replaced by first-order logic as the preferred framework for work in logic. Looking in the opposite direction, we should also note that Frege's work – including the lectures Carnap attended in 1911-14 – already contains at its core a simple theory of types. Rather than following Russell, Carnap can thus be seen as going back to Frege with respect to type theory. Or at the very least, what he learned from Frege as a student made him see very quickly the usefulness and significance of simple type theory, as suggested most prominently by Ramsey.[11]

...

To get the proper perspective on Carnap's general goals for his logic project from the 1920s, beyond disseminating Russellian logic or the simple theory of types, it is important to recognize two further influences on him, in addition to Frege, Russell, and Ramsey. First, there is the influence of Hilbert and his school; second, we have to consider Carnap's exchange of ideas with Abraham Fraenkel during the 1920s.

In Carnap's *Abriss der Logistik*, Hilbert & Ackermann's *Grundzüge der theoretischen Logik* (Hilbert & Ackermann 1928) is listed in the bibliography, but with the following parenthetical remark added: "Not used for this text" (Carnap 1929a, p. 106). It seems that Carnap received their book only when his own was already basically finished. Together with the observations that Carnap is usually rather generous in his references, that he started to work on *Abriss* early in the 1920s, and that he had finished a manuscript for it by 1927, this suggests strongly that he composed his own logic text independently from Hilbert's. As far as I know, Carnap also didn't have any personal contact with Hilbert during the 1920s. On the other hand, he certainly knew several of Hilbert's and his collaborators' earlier publications well; and he was in direct, sustained contact with at least some of the members of the Hilbert school. Concerning published works, Carnap often refers to Hilbert's *Grundlagen der Geometrie* (Hilbert 1899) and "Axiomatisches Denken" (Hilbert 1918) in his writings; he also lists Hermann Weyl's *Philosophie der Mathematik und Naturwissenschaften* (Weyl 1926), which was at least partly a text in the Hilbert school, in the bibliography of *Abriss* (ibid., p. 107). Concerning personal contact, another member of the Hilbert school has to be seen as central: Heinrich Behmann.[12]

10. Moreover, there is evidence that Gödel not only learned simple type theory directly from Carnap, but was led to some of his specific results that way; compare (Awodey & Carus 2001, pp. 163-64).

11. Frege's published works are mentioned in the list of suggested readings in *Abriss der Logistik* under the rubric "Older texts . . . that still contain valuable material for current research" (Carnap 1929a, p. 107).

Today Behmann's work is not very well known, clearly not as well as that of other members of the Hilbert school (Ackermann, Bernays, Gentzen, etc.). But in the 1920s, logicians like Carnap were familiar with texts such as his "Beiträge zur Algebra der Logik, insbesondere zum Entscheidungsproblem" (Behmann 1922) and *Mathematik und Logik* (Behmann 1927), as the references in *Abriss der Logistik* show (pp. 102-4). Moreover, Carnap's letters to Russell make clear that he and Behmann were frequent interlocutors and close collaborators – not only was Behmann one of the participants of the Erlangen conference and someone Carnap mentions to Russell several times by name, he even considered writing *Abriss der Logistik* in collaboration with Behmann at some point. Finally, in the preface to the published version of *Abriss* Carnap thanks Behmann explicitly and prominently (besides Hahn and Waismann) for his advice and help (Carnap 1929a, p. iv). It is reasonable to assume, then, that Carnap was well aware of basic developments in the Hilbert school through Behmann. And it is not surprising that two of the topics mentioned by Carnap repeatedly during the 1920s were absolutely central to that school: the use of the axiomatic method, and the question of the decidability of logic, i.e., the "Entscheidungsproblem."[13]

A second main contact for Carnap during the 1920s, also mentioned in his letters to Russell, was Abraham Fraenkel. As Fraenkel, too, is not usually recognized as a central figure in the development of logic (as opposed to set theory), let me again provide some background information in this connection. If we start, once more, with the historical and bibliographical references in *Abriss der Logistik*, Fraenkel's name comes up several times. These references concern partly set theory, as one would expect, but they also occur in connection with the axiomatic method (e.g., p. 72 and p. 128, fn. 1). This is no accident – together with Hilbert's and Weyl's works, Fraenkel's *Einleitung in die Mengenlehre* (Fraenkel 1928) is one of the first and main texts to discuss the use of this method systematically. More specifically, Fraenkel discusses axiomatics in a relatively objective, nonpartisan way, an aspect that must have appealed to Carnap. Actually, in the first edition of *Einleitung in die Mengenlehre* (published

12. Behmann finished a dissertation under Hilbert in 1918, on a topic closely related to Whitehead & Russell's *Principia Mathematica*. From that time until 1925, when he left Göttingen, he was an active participant in the discussions on logic and the foundations of mathematics there. For more on Behmann's dissertation and his role in these discussions in Göttingen, see (Mancosu 1999); for Hilbert's school more generally, compare (Sieg 1999).

13. Besides in Hilbert's own works, these two topics play a central role in (Behmann 1922), (Weyl 1926), (Behmann 1927), and (Hilbert & Ackermann 1928).

in 1919) this topic doesn't play a major role yet; no special chapter is reserved for it. It is in the second edition (1923) and, especially, in the third (1928) that things change. It is also in the third edition that Fraenkel refers explicitly to Carnap, both in general and in this very connection: in the preface of the book, Fraenkel thanks him for his help overall; and in the chapters on axiomatics he not only refers to "Eigentliche und Uneigentliche Begriffe" (Carnap 1927), but also acknowledges "deeper still unpublished work by the same author" as important for his discussions (Fraenkel 1928, p. x and p. 353, note 3).[14]

The parts of Fraenkel's book in which the more specific references to Carnap occur contain an early consideration of what we would call *metamathematical* questions about axiomatic systems. Among others, Fraenkel considers three requirements (many) such axiom systems are meant to fulfill: *consistency*, *completeness*, and mutual *independence* of the axioms. The crucial notion for our purposes is the second requirement, that of completeness. It is precisely this notion that is at the center of Carnap's own research project on logic in the 1920s. Fraenkel actually distinguishes three different notions of completeness for a system of axioms, as does Carnap in "Eigentliche und Uneigentliche Begriffe." Besides the question of how precisely to define each of these three notions, the main problem raised by Fraenkel is how they are related to each other.

. . .

Now, finally, the stage is set to present the core of Carnap's research project in logic from the 1920s, the project that was meant to culminate in *Untersuchungen zur Allgemeinen Axiomatik*. Its overall topic is "general axiomatics," as indicated by the title. In "Bericht über Untersuchungen zur allgemeinen Axiomatik," Carnap's short and programmatic report that made it into print, he characterizes general axiomatics as "the theory of axiomatic systems of arbitrary form" (Carnap 1930a, p. 303, my translation). *Allgemeine Axiomatik* itself starts with a related, but more detailed explanation along the same lines:

> In the course of recent investigations into general properties of axiomatic systems such as: completeness, monomorphy (categoricity), decidability, consistency, etc., and into the problems of determining criteria for and the mutual relations between these properties, one thing has become increasingly clear: that the main difficulty with respect to these problems lies in the insufficient precision of the concepts used. The most important requirement for a fruitful treatment of them is: on the one hand, to *establish explicitly the logical basis to be used* in each case, as is usually not done with

14. The interactions between Carnap and Fraenkel on this and related topics is noted in (Awodey & Carus 2001), and discussed further in (Awodey & Reck 2002a). Compare also (Carnap 1929b) and, for some biographical background, (Fraenkel 1967), pp. 151–55.

> enough precision; and on the other hand, to *give precise definitions for the concepts used* on that basis. In what follows, my aim will be to satisfy those two requirements and, subsequently, to establish the fruitfulness of the established foundation by *deriving a number of theorems of general axiomatics*. By "general axiomatics," I mean the theory of general, logical-formal properties of axiomatic systems, as opposed to a "special axiomatics" that deals with certain particular systems of axioms (e.g., an axiom system for Euclidean geometry, for set theory, etc.). (Carnap 2000, p. 59, my translation, original emphases)

There are, thus, three main goals for Carnap: "to establish explicitly the logical basis to be used"; "to give precise definitions for the concepts used on that basis"; and to "derive a number of theorems of general axiomatics."

Now, what is the "logical basis" – or the "basic discipline [Grunddisziplin]" as he also calls it elsewhere (ibid., p. 60) – that Carnap has in mind? It is, not surprisingly at this point, the simple theory of types (see especially pp. 68–70). On its basis, or within this framework, the notions to be "defined precisely" include: the notion of model for an axiom system; logical consequence, independence, and consistency; isomorphism between models; and, especially, completeness. Actually, instead of one notion of completeness Carnap introduces three distinct notions – as suggested by Fraenkel – namely: "monomorphy [Monomorphie]," "non-forkability [Nicht-Gabelbarkeit]," and "decidability [Entscheidungsdefinitheit]." If reconstructed charitably, these notions amount to the following:[15] a) A theory T in simple type theory is *monomorphic* if all of its models are isomorphic, i.e., if the theory is categorical. b) T is *non-forkable* if there is no formula j in simple type theory such that both $T \cup \{\varphi\}$ and $T \cup \{\neg\varphi\}$ are satisfiable. And c), T is *decidable* if for all formulas j in simple type theory either φ or $\neg\varphi$ is a logical consequence of T. Finally, the two most important "theorems of general axiomatics" Carnap formulates in terms of these notions are: *Theorem 1*: Being monomorphic is equivalent to being non-forkable. *Theorem 2*: Being non-forkable is equivalent to being decidable. (See Carnap 2000, chapter 3; compare also Carnap 1930a, pp. 305–7.) If true, it would follow, of course, that all three notions of completeness introduced by Carnap were equivalent.

What I have described so far should give a good first sense of Part I of *Untersuchungen zur Allgemeinen Axiomatik*, the part on which I want to

15. There are a number of subtle differences between what follows and Carnap's actual definitions, most of them connected with three issues: (i) Carnap uses the background of simple type theory in a peculiar and problematic way; (ii) he doesn't separate object-language and meta-language carefully, or as we have learned to do since; (iii) he doesn't define the notion of logical consequence in a way that is appropriate for his own purposes, especially by not distinguishing appropriately between syntactic and semantic consequence. For more details, see (Bonk & Mosterin 2000), (Awodey & Carus 2001), (Awodey & Reck 2002a).

focus.[16] There are, of course, various problems with Carnap's treatment of the issues just mentioned, including some of his definitions (especially the actual, unreconstructed ones). Before remarking further on these problems, I want to make three more general observations. First, note that Carnap is studying axiomatic systems in general, and in a way that we would again call *metamathematic* or *model-theoretic*. Second, unlike in (most) contemporary model theory he is not working with first-order logic and set-theory as the background theories, but with the simple theory of types. Third, if looked at in more detail his approach is only partly metamathematical and model-theoretic, since he does not separate precisely between object-language and meta-language, and since he does not yet have a full understanding of the model-theoretic notion of an interpretation for a formal language at his disposal.

What, beyond the third point just mentioned, is problematic about Carnap's definitions and proofs? The difficulties and background issues involved are subtle, and I cannot go into much detail here. But very briefly, there are two main problems: First, in the proof of *Theorem 1* (stating that being "monomorphic" is equivalent to being "non-forkable" in simple type theory) Carnap makes an implicit assumption that is not justified, and indeed not true in general.[17] As a result, his proof does not establish what it is supposed to establish, only a weaker, partial result. Second and more basically, in his definition of "decidable"–formulated above, vaguely and ambiguously, in terms of "logical consequence"–Carnap is not sufficiently aware of the difference between the notions of semantic and syntactic consequence, and he does not succeed in formulating either notion adequately.[18] As a consequence, while his *Theorem 2* (stating that being "non-forkable" is equivalent to being "decidable") is true as presented by him (basically, involving the notion of semantic consequence), it does not provide an answer to the question that Carnap – and Fraenkel before him – really wanted to have answered (involving syntactic consequence instead).[19] More generally, Carnap's definitions of several of his central notions are not given with "sufficient precision," contrary to his stated aims.

Carnap himself became aware of these problems when discussing his ideas with a number of logicians in the late 1920s. In particular, it was conversations with Gödel and Tarski around and shortly after 1930 that made him see the source and the depth of the difficulties involved. Actually, several parts of what Carnap did, or tried to do, can be salvaged by using contemporary tools.[20] But apparently he himself either lost confidence in the project completely, just after submitting "Bericht über Untersuchungen zur allgemeinen Axiomatik" (Carnap 1930a) for publication, or he concluded that the amount of what could be salvaged was not enough to merit publication of his research monograph.

Let me make one further comment in this connection. It seems to me that a more detailed investigation of what Carnap tried to do in his logic project, including of the ways in which he failed, could be quite interesting, not only from a logical, but also from a philosophical perspective. Note, for example, the following: With his investigations into "general axiomatics," Carnap stands right at the threshold between an older *universalist* conception of logic, in the tradition of Frege and Russell, and a more recent *model-theoretic* conception, in the tradition of Hilbert, Tarski, and others.[21] Like Frege and Russell, Carnap works within a theory of types as the fixed, universal background theory, and in a way that treats this theory as always already interpreted. But neither Frege nor Russell was much interested in "general axiomatics," or in what Hilbert later called "formal axiomatics." It has even been argued that the metamathematical or model-theoretic point of view required for an adequate treatment of the corresponding issues was fundamentally alien to their philosophical orientations.[22] What makes Carnap's logic project from the 1920s so interesting is that he is *both* directly and deeply influenced by Frege, Russell, and (at least certain aspects of) their universalist conception of logic *and* very intent on doing "general axiomatics," thus (at least to some degree) metamathematics and model theory. It could be quite illuminating, then, to analyze further the extent to which combining these two aspects was possible for him, and the extent to which it wasn't.[23]

16. Part II of the book was, among other things, supposed to investigate the relation between completeness and "extremal axioms," such as Hilbert's "Axiom der Vollständigkeit" for geometry and the axiom of induction for arithmetic. See (Carnap & Bachmann 1936) for a later partial exposition; compare also the discussions and further references in (Bonk & Mosterin 2000), (Awodey & Carus 2001), (Awodey & Reck 2002a), and (Awodey & Reck 2002b).

17. Carnap implicitly assumes that, for any model M, "being isomorphic to M" is expressible in the simple theory of types. This assumes that any model M is definable in simple type theory, which is not true as became clear after Carnap's work. See (Lindenbaum & Tarski 1935) for an explicit statement of the correct result; compare also the corresponding remarks in (Awodey & Carus 2001) and (Awodey & Reck 2002a).

18. In some late additions to (Carnap 1928c), he distinguishes between "decidable [entscheidungsdefinit]" and "k-decidable [k-entscheidungsdefinit]," where the latter, but not the former, involves provability in a formal system. However, various problems remain with these two notions. For further discussions and some clarifications in this connection see again (Bonk & Mosterin 2000) and (Awodey & Carus 2001).

19. If formulated adequately in terms of syntactic consequence, Theorem 2 is, of course, false, as Gödel established in his famous Incompleteness Theorems. If formulated in terms of semantic consequence, Theorem 2 asserts the true but trivial equivalence between two versions of what may be called "semantic completeness"; compare (Awodey & Reck, 2002a), in particular Definitions 3.2 and 3.4.

20. Compare (Awodey & Reck 2002a) and (Awodey & Reck 2002b).

21. See (van Heijenoort 1967), (Goldfarb 1979), and (Hintikka 1988) for the distinction between a universalist and a model-theoretic conception of logic.

22. This has been argued especially in connection with Frege; see (Ricketts 1986), (Goldfarb 2001), and the further references in them. For a different perspective, compare (Tappenden 1997).

23. In (Hintikka 1991) and (Hintikka 1992), Carnap's work on logic is discussed in this light. However, Hintikka sees him too much, or too unequivocally, as a universalist, I think. What needs to be investigated further in this connection are the details of Carnap's treatment in *Allgemeine Axiomatik*.

Reserving further considerations of this issue for future work, I want to close the present paper with a different suggestion. It concerns the standard assumption that, with respect to his basic position in the philosophy of mathematics and logic, Carnap was a *logicist*, in the tradition of Frege and Russell. Taking account of his logic project from the 1920s puts this view into a new light – and partly into question.

. . .

Logicism is usually understood to be the thesis that all, or at least large parts, of mathematics can be reduced to logic. This thesis has two sides: (i) that all mathematical *concepts* can be defined in terms of basic logical concepts; (ii) that all mathematical *theorems* can be deduced from basic logical truths. Considering both of these sides together, Carnap writes in *Abriss der Logistik*: "All of mathematics becomes, then, part of logic" (Carnap 1929a, p. 2, my translation). To distinguish logicism further from other positions in the philosophy of mathematics and logic, in particular from formalism, three additional, more specific aspects should be mentioned: (iii) According to standard logicism, there is *one logical system* that is fundamental for all investigation, both mathematical and empirical. (iv) Within this one system, all terms, including all mathematical terms, are to be given a *definite meaning*. And (v), doing so is meant to allow mathematical and nonmathematical statements to be deductively *connected* in a systematic way.

It is well know that Carnap expressed support for points (i) and (ii) – for the theses that all mathematical concepts and all mathematical theorems can be reduced to logical concepts and truths – throughout his career.[24] He also explicitly traced this support back to what he learned from Frege and Russell. We have already encountered the following remark, in the early parts of Carnap's autobiography, concerning the influence of Frege's lectures and publications: "From his analysis I gained the conviction that knowledge in mathematics is analytic in the general sense that it has essentially the same nature as knowledge of logic" (Carnap 1963, p. 12). In a later part of the autobiography, entitled "The Foundations of Mathematics," he elaborates on this point:

> I had learned from Frege that all mathematical concepts can be defined on the basis of the concepts of logic and that the theorems of mathematics can be deduced from the principles of logic. Thus the truths of mathematics are analytic in the general sense of truths based on logic alone. (Ibid., p. 46)

In addition, from the 1920s on Carnap keeps referring to Whitehead & Russell's *Principia Mathematica* as the most systematic and convincing defense of (i) and (ii), in spite of the fact that he also acknowledges some remaining problems, or at least open questions, not just involving their axiom of reducibility, but also the axioms of infinity and choice.[25] As to (iii), at least until around 1930–31 Carnap's basic procedure is to work with one overarching logical system as the "basic discipline," generally the simple theory of types.[26] Thus he clearly accepts this aspect as well. In fact, (iii) is intimately tied up with Carnap's general goal of analyzing "the total system of knowledge," "the total language of science," and with his emphasis on "the unity of science" (Carnap 1963, pp. 12, 32, etc.). That goal, in turn, is also what motivates (v): the requirement that mathematics be systematically connected to, especially, the empirical sciences. Consequently, (v) is again something Carnap is committed to throughout his career.

The most interesting aspect for our purposes is (iv), however: that according to logicism all terms, including all mathematical terms, are to be given a *definite meaning* within the basic system. This is usually taken to mean that, e.g., arithmetic terms such as "1," "2," "3," . . . are to be defined *explicitly*, as in the works of Frege and Russell. "Implicit definition" of mathematical terms by means of the Peano axioms for arithmetic, Hilbert's axioms for geometry, and so on are rejected as inadequate and inherently problematic. Now, it is exactly such "implicit definitions," or better the corresponding formal axiomatic approach to the theories involved, that are at the center of attention in Carnap's logic project from the 1920s. This focus is very clear in *Untersuchungen zur Allgemeinen Axiomatik*, but it can also be found in *Abriss der Logistik* and other published writings from this period.

The earliest published text by Carnap with this focus is the article "Eigentliche und Uneigentliche Begriffe" (Carnap 1927). At the core of this article is Carnap's distinction between explicitly defined mathematical terms and notions, such as what he calls the "Russell numbers," and only implicitly defined terms and notions, such as the "Peano numbers." Concerning the latter, Carnap talks about "improper" or "inauthentic concepts" [uneigentliche Begriffe], in contrast to "proper" or "authentic concepts" [eigentliche Begriffe]. He also makes clear that the notion of "implicit definition" is really somewhat misleading. What introducing the Peano Axioms does is not so much to "define implicitly" the meaning of "1," "2," "3," . . ., as to define explicitly a corresponding *higher-order concept*: that of a "progression" ("natural number system," "w-sequence"); similarly in the case of geometry. Carnap reaffirms this point later in *Abriss der Logistik* (Carnap 1929a, pp. 71–72) and elsewhere. And it is, of course, exactly the way in which Frege responded to Hilbert's axioms of geometry in his lectures on logic, in particular in the 1914 class "Logik in der Mathematik" attended by Carnap.

24. For explicit endorsement of logicism by Carnap, in addition to the quotations given below, see especially (Carnap 1930c) and (Carnap 1931a); compare also the general discussion in (Bohnert 1975).

25. One difficult question concerning the axioms of infinity and choice is whether they are tautologies, or analytic truths, as logical truths are supposed to be for Carnap. I have to leave considerations of this interesting question aside here; compare (Goldfarb 1996) for an informative discussion.

26. This changes after 1930–31, especially in Carnap's *Logische Syntax der Sprache*, when the one logical system is replaced by a plurality of such systems; compare (Goldfarb 1996).

But this is not all. Carnap's main line of thought in this connection is that, at least in the case of a *complete* system of axioms, more is accomplished than just introducing a higher-level concept. Let us take "completeness" here primarily in the sense of "monomorphy," or categoricity. To repeat, in the case of a categorical axiom system, such as the (higher-order) Peano axioms, all models are isomorphic; or as Carnap puts it occasionally, they all "have the same structure" (Carnap 2000, p. 102, my translation; cf. also Carnap 1930a, p. 305). Applying his two central "theorems" from *Allgemeine Axiomatik*, Carnap infers that such an axiom system is then also "non-forkable" and "decidable" (in his sense of those terms). Putting aside the difficulties with Carnap's definitions and proofs, what is the significance of these results according to him? It is this: What a system of axioms with these properties defines is in an important sense *definite* or *completely determinate*. Actually, the sense of determinateness is two-fold: First, such a system of axioms defines "one structure" (Carnap 2000, p. 127). Second, the law of bivalence holds for any sentence *j*, in the sense that either it or its negation follows semantically from the system of axioms (although not syntactically, as we now know from Gödel's Theorems). And why are these results important for Carnap? Because the corresponding "implicit definitions" are, then, justified and acceptable after all.[27]

The general conclusion I want to draw from this observation about Carnap's project is the following: His attitude towards a formal axiomatic approach to fields such as arithmetic and geometry is not exactly "logicist," at least not in the usual sense of logicism in which it includes *all five* aspects mentioned above – including (iv), understood in the strong sense that all "implicit definitions" are to be rejected as inadequate and fundamentally problematic. At the same time, this does not mean that Carnap rejects attempts at a logicist construction of, say, the natural numbers along the lines of Russell. What he seems to be after, instead, is to study arithmetic *both* genetically *and* axiomatically, i.e., to study both the "Russell numbers" and the "Peano numbers."[28]

Assuming this is correct, what is *its* significance? Should we say that Carnap was not a logicist after all, in spite of all his explicit uses and endorsements of this label? My answer to that question is the following: In connection with Carnap's work, "logicism" is a very slippery term. In particular, one can use it in a wide and in a narrow sense. Used in the narrow sense, it consists of points (i) to (v) above, where (iv) is understood as implying the rejection of implicit definitions. This amounts, of course, exactly to the kind of logicism advocated by Frege and Russell (in Frege's case as applied to arithmetic, in Russell's case more generally). The wide sense of logicism, in contrast, has at its core the following more general thesis: Logic and mathematics can and should be developed together, in one system, and in such a way that it becomes evident that mathematics, like logic, is "analytic." Here, to establish that mathematics is analytic may be attempted by showing that all mathematical concepts are definable explicitly in logical terms, along the lines of logicism in the narrow sense. But it doesn't have to be done that way. Other approaches may be admitted as well, such as the "implicit definitions" given by complete systems of axioms and studied by Carnap in his *Allgemeine Axiomatik*.

27. For a different interpretation of "Eigentliche und Uneigentliche Begriffe" see (Goldfarb 1996). In it, Carnap's paper is interpreted primarily as a reaction against Moritz Schlick's use of "implicit definitions"; the precise content and the further goals of *Allgemeine Axiomatik* are not taken into account.

It is fair to say, I think, that Carnap's formulations are often ambiguous in terms of which sense of logicism he has in mind, narrow or wide (perhaps intentionally so because of some of the remaining problems in a Fregean and Russellian approach). There is also a question as to whether the second, wide view still deserves the title "logicism." Be that as it may, I take it to be uncontroversial that Carnap, throughout his career, was an adherent of logicism in the wide sense. What our discussion of his logic project from the 1920s reveals, however, is this: To interpret him as a logicist in the *narrow sense* is problematic, in spite of the strong influence Frege and Russell had on him. Instead, the best way to describe the situation is probably this: From early on in his career, Carnap was attached to logicism, as defended by Frege, Russell, and Ramsey. But he also quickly became attracted to certain aspects of formalism, as he encountered them in Hilbert's writings, in conversations with Behmann, and in Fraenkel's general discussion of the axiomatic method. Carnap's logic project from the 1920s consists, then, of an attempt to *synthesize* logicism and formalism, to get the best of both worlds.[29] Having that aim – trying to mediate between seemingly irreconcilable philosophical positions by focusing on the technical, positive insights on both sides – would certainly be in the spirit of Carnap more generally.

Acknowledgments

I would like to thank Steve Awodey and André Carus for getting me interested in Carnap's work on logic from the 1920s; the Pittsburgh Archives for Scientific

28. How exactly Carnap conceives of the relation of "Russell numbers" and "Peano numbers" is never clarified completely, I think. One problem is that the usual constructions of the Russell numbers within type theory presuppose the axiom of infinity, an axiom whose exact status remains in question; cf. footnote 25.

29. This was not lost on some members of the Vienna Circle. Thus, when Carnap presented his project to the Circle in the late 1920s Herbert Feigl described it (mockingly) as an attempt to "Hilbertize *Principia Mathematica*"; see (Bonk & Mosterin), p. 24. Note here that I am talking about Carnap's views before *Logische Syntax der Sprache*. For the latter, a corresponding thesis has been defended in (Oberdan 1993).

Philosophy, especially Gerald Heverly, for making available relevant material from Carnap's Nachlaß; Gottfried Gabriel and Carsten Klein for inviting me to the conference *Rudolf Carnap: From Jena to L.A.* (September 2001); Michael Friedman, Thomas Ricketts, and W.W. Tait for encouraging my research on Carnap and related topics; Sally A. Ness for her editorial suggestions and support overall; and, last but not least, Anna Lucile Reck for allowing me to write this paper during the first few weeks of her life by sleeping peacefully in my study for hours.

Literature Cited

Awodey, S. & Carus, A.W. 2001. "Carnap, Completeness, and Categoricity: The *Gabelbarkeitssatz* of 1928" *Erkenntnis* 54, pp. 145–72.
Awodey, S. & Reck, E. 2002a. "Completeness and Categoricity, Part I: 19th Century Axiomatics to 20th Century Metalogic" *History and Philosophy of Logic* 23, No. 1, pp. 1–30.
——. 2002b. "Completeness and Categoricity, Part II: 20th Century Metalogic to 21st Century Semantics" *History and Philosophy of Logic* 23, No. 2, pp. 77–94.
Behmann, H. 1922. "Beiträge zur Algebra der Logik, insbesondere zum Entscheidungsproblem" *Mathematische Annalen* 86, pp. 163–29.
——. 1927. *Mathematik und Logik*, Volume 71 of Mathematische-physikalische Bibliothek, Leipzig.
Bohnert, H. 1975. "Carnap's Logicism", in *Rudolf Carnap: Logical Empiricist*, J. Hintikka, ed., Dordrecht: Reidel, pp. 183–216.
Bonk, T. & Mosterin, J. 2000. "Editors' Introduction", in (Carnap 2000), pp. 1–54.
Carnap, R. 1921. Letter to Russell, Nov. 17, 1921, ASP/RC 102-68-34.
——. 1922a. *Der Raum. Ein Beitrag zur Wissenschaftslehre.* Kant-Studien, Ergänzungshefte 56, Berlin: Reuther & Reichard.
——. 1922b. Letter to Russell, June 13, 1922, ASP/RC 102-68-33.
——. 1922c. Letter to Russell, July 29, 1922, ASP/RC 102-68-31.
——. 1922d. Letter to Russell, September 29, 1922, ASP/RC 102-68-32.
——. 1923a. Letter to Russell, February 20, 1923, ASP/RC 102-68-30.
——. 1923b. Letter to Russell, September 9, 1923,ASP/RC 102-68-23.
——. 1926a. Letter to Russell, February 27, 1926, ASP/RC 102-68-21.
——. 1926b. Letter to Russell, October 15, 1926,ASP/RC 102-68-28.
——. 1927. "Eigentliche und Uneigentliche Begriffe", Symposion 1, pp. 355–74.
——. 1928a. *Scheinprobleme in der Philosophie.* Berlin: Weltkreis Verlag.

——. 1928b. *Der logische Aufbau der Welt.* Berlin: Weltkreis Verlag.
——. 1928c. *Untersuchungen zur Allgemeinen Axiomatik.* manuscript, Archives for Scientific Philosophy, University of Pittsburgh; cf. (Carnap 2000).
——. 1929a. *Abriss der Logistik.* Vienna: Springer.
——. 1929b. Review of A. Fraenkel, *Einleitung in die Mengenlehre*, 3rd ed., *Kant-Studien* 34, pp. 428–29.
——. 1929c. Letter to Russell, April 29, 1929, ASP/RC 102-68-19.
——. 1930a. "Bericht über Untersuchungen zur allgemeinen Axiomatik" *Erkenntnis* 1, pp. 303–10.
——. 1930b. "Die alte und die neue Logik" *Erkenntnis* 1, pp. 12–26.
——. 1930c. "Die Mathematik als Zweig der Logik" *Blätter für deutsche Philosophie* 4, pp. 298–310.
——. 1931a. "Die logizistische Grundlegung der Mathematik" *Erkenntnis* 2, pp. 91–105.
——. 1931b. Review of A.N. Whitehead & B. Russell, *Principia Mathematica*, 2nd ed., *Erkenntnis* 2, pp. 73–75.
——. 1932. "Überwindung der Metaphysik durch logische Analyse der Sprache" *Erkenntnis* 2, pp. 219–41.
——. 1934. *Logische Syntax der Sprache.* Vienna: Springer.
——. 1936-37. "Testability and Meaning" *Philosophy of Science* 3, pp. 419–71, and 4, pp. 1–40.
——. 1939. *Foundations of Logic and Mathematics.* Chicago: University of Chicago Press.
——. 1942. *Introduction to Semantics.* Cambridge, MA: Harvard University Press.
——. 1943. *Formalization of Logic.* Cambridge, MA: Harvard University Press.
——. 1947. *Meaning and Necessity.* Chicago: University of Chicago Press.
——. 1950a. *Logical Foundations of Probability.* Chicago: University of Chicago Press.
——. 1950b. "Empiricism, Semantics, and Ontology" *Revue Internationale de Philosophie* 4, pp. 20–40.
——. 1954. *Einführung in die symbolische Logik.* Vienna: Springer.
——. 1963. "Intellectual Autobiography" in (Schilpp 1963a), pp. 3–84.
——. 1966. *Philosophical Foundations of Physics.* New York: Basic Books.
——. 2000. *Untersuchungen zur Allgemeinen Axiomatik.* T. Bonk & J. Mosterin, eds., Darmstadt: Wissenschaftliche Buchgesellschaft. cf. (Carnap 1928c).
Carnap, R. & Bachmann, F. 1936. "Über Extremalaxiome" *Erkenntnis* 6, 166–88; Engl. trans. "On Extremal Axioms" *History and Philosophy of Logic* 2, H. Bohnert, trans., pp. 67–85.
Church, A. 1940. "A Formulation of the Simple Theory of Types" *Journal of Symbolic Logic* 5, pp. 56–68.

——. 1956. *Introduction to Mathematical Logic*. Vol. 1, Princeton: Princeton University Press.
Fraenkel, A. 1928. *Einleitung in die Mengenlehre*. 3rd revised ed., Berlin: Springer. (1st ed. 1919, 2nd ed. 1923.)
——. 1967. *Lebenskreise. Aus den Erinnerungen eines jüdischen Mathematikers*. Stuttgart:Deutsche Verlags-Anstalt.
Frege, G. 1879. *Begriffsschrift. Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*. Halle: Nebert.
——. 1884. *Die Grundlagen der Arithmetik*. Breslau: Koebner.
——. 1893/1903. *Grundgesetze der Arithmetik*. Volumes 1–2, Jena: Pohle.
——. 1914. "Logik in der Mathematik" in *Nachgelassene Schriften*, H. Hermes et al., eds., Hamburg: Meiner, pp. 219–70.
——. 1996. "Vorlesungen über Begriffsschrift" *History and Philosophy of Logic* 17, G. Gabriel, ed., pp. 1–48.
——. forthcoming. *Frege's Lectures on Logic, 1911–1914, Carnap's Jena Notes*. E. Reck & S. Awodey, ed. and trans., Chicago: Open Court.
Friedman, M. 1999. "Tolerance and Analyticity in Carnap's Philosophy of Mathematics" in *Reconsidering Logical Positivism*, Cambridge: Cambridge University Press, pp. 198–233.
Gödel, K. 1931. "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I" *Monatshefte für Mathematik und Physik*; reprinted, with Engl. trans., in *Kurt Gödel: Collected Works*, Vol. 1, S. Feferman et al., eds., New York: Oxford University Press, pp. 144–95.
Goldfarb, W. 1979. "Logic in the Twenties: the Nature of the Quantifier" *Journal of Symbolic Logic* 44, pp. 351–68.
——. 1996. "The Philosophy of Mathematics in Early Positivism" in *Origins of Logical Empiricism*, R. Giere & A. Richardson, eds., Minneapolis: University of Minnesota Press, pp. 213–30.
——. 2001. "Frege's Conception of Logic" in *Future Pasts: The Analytic Tradition in Twentieth-Century Philosophy*, J. Floyd & S. Shieh, eds., New York: Oxford University Press, pp. 25–41.
Grattan-Guinness, I. 1979. "On Russell's Logicism and its Influence, 1910–1930" *Proceedings of the 3rd International Wittgenstein Symposium*, Vienna: Hölder-Pichler-Tempsky, pp. 275–80.
Hahn, H., Neurath, O. & Carnap, R. 1929. *Wissenschaftliche Weltauffassung: Der Wiener Kreis*. Vienna: Arthur Wolf Verlag.
Hilbert, D. 1899. *Grundlagen der Geometrie*. 1st ed., Leipzig: Teubner.
——. 1918. "Axiomatisches Denken" *Mathematische Annalen* 78, pp. 405–56; reprinted in *Gesammelte Abhandlungen*, Vol. 3, Berlin: Springer, pp. 146–56.
Hilbert, D. & Ackermann, P. 1928. *Grundzüge der theoretischen Logik*. 1st ed., Berlin: Springer.

Hintikka, J. 1988. "On the Development of the Model-Theoretic Viewpoint in Logical Theory" *Synthese* 77, pp. 1–36.
——. 1991. "Carnap, the Universality of Language, and Extremal Axioms" *Erkenntnis* 35, pp. 325–36.
——. 1992. "Carnap's Work in the Foundations of Logic and Mathematics in a Historical Perspective" *Synthese* 93, pp. 167–89.
Lindenbaum, A. & Tarski, A. 1935. "Über die Beschränktheit der Ausdrucksmittel deduktiver Theorien" *Ergebnisse eines Mathematischen Kolloquiums*, fascicule 7, 1934–35; Engl. trans. "On the Limitations of the Means of Expression of Deductive Theories" in *Alfred Tarski: Logic, Semantics, Metamathematics,* J. Corcoran, ed., Indianapolis: Hackett, pp. 384–92.
Mancosu, P. 1999. "Between Russell and Hilbert: Behmann on the Foundations of Mathematics" *Bulletin of Symbolic Logic* 5, pp. 303–30.
Oberdan, T. 1993. "The Synthesis of Logicism and Formalism in Carnap's *Logical Syntax of Language*" in *Scientific Philosophy: Origins and Developments*, F. Stadler, ed., Dordrecht: Kluwer, pp. 157–68.
Ramsey, F. 1925. "The Foundations of Mathematics" *Proceedings of the London Mathematical Society* 25, pp. 338–84; reprinted in *Philosophical Papers*, D.H. Mellor, ed., Cambridge: Cambridge University Press, 1990, pp. 164–224.
Ricketts, T. 1986. "Objectivity and Objecthood: Frege's Metaphysics of Judgment" in *Frege Synthesized*, L. Haaparanta & J. Hintikka, eds., Dordrecht: Reidel, pp. 65–95.
Russell, B. 1903. *The Principles of Mathematics.* Cambridge:Cambridge University Press. Reprinted by Norton, New York, 1937.
——. 1914. Our Knowledge of the External World. LaSalle, IL: Open Court.
——. 1922. Notes for Rudolf Carnap "Principia Mathematica," ASP/RC 111-01-01A.
Schilpp, P., ed. 1963a. *The Philosophy of Rudolf Carnap*. LaSalle, IL, Open Court.
Sieg, W. 1999. "Hilbert's Programs: 1917–1922" *Bulletin of Symbolic Logic* 5, pp. 1–44.
Tappenden, J. 1997. "Metatheory and Mathematical Practice in Frege" *Philosophical Topics* 25, pp. 213–64.
Thiel, C. 1993. "Carnap und die Wissenschaftliche Philosophie auf der Erlanger Tagung 1923" in *Wien-Berlin-Prag: Der Aufstieg der wissenschaftlichen Philosophie*, R. Haller & F. Stadler, eds., Vienna: Hölder-Pichler-Tempsky, pp. 175–88.
van Heijenoort, J. 1967. "Logic as Calculus and Logic as Language" *Synthese* 17, pp. 324–30.
Weyl, H. 1926. *Philosophie der Mathematik und Naturwissenschaft*. 1st ed., München: Oldenbourgh.

Whitehead, A.N. & Russell, B. 1910–13. *Principia Mathematica*. Volumes 1-3, Cambridge: Cambridge University Press.
——. 1925. *Principia Mathematica*. Vol. 1, 2nd revised ed., Cambridge: Cambridge University Press.