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FREGE'S INFLUENCE ON WITTGENSTEIN

Reversing metaphysics via the context principle

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Gottlob Frege and Ludwig Wittgenstein (the later Wittgenstein) are often seen as polar opposites with respect to their fundamental philosophical outlooks: Frege as a paradigmatic "realist", Wittgenstein as a paradigmatic "anti-realist". This opposition is supposed to find its clearest expression with respect to mathematics: Frege is seen as the "arch-platonist", Wittgenstein as some sort of "radical anti-platonist". Furthermore, seeing them as such fits nicely with a widely shared view about their relation: the later Wittgenstein is supposed to have developed his ideas in direct opposition to Frege. The purpose of this paper is to challenge these standard assumptions. I will argue that Frege's and Wittgenstein's basic outlooks have something crucial *in common*; and I will argue that this is the result of the positive *influence* Frege had on Wittgenstein.

It would be absurd to claim that there are no important differences between Frege and Wittgenstein. Likewise, it would be absurd to claim that the later Wittgenstein was not critical of some of Frege's ideas. What, then, is the common element I see? My suggestion is that the two thinkers agree on what I call a *reversal of metaphysics* (relative to a standard kind of metaphysics attacked by both). This is not an agreement on one particular thesis or on one argument. Rather, it has to do with what is *prior* and what is *posterior* when it comes to certain fundamental explanations in metaphysics (also, relatedly, in semantics and epistemology). Furthermore, this explanatory reversal is intimately connected with Frege's *context principle*: "Only in the context of a sentence do words have meaning". As we will see, Wittgenstein takes over this principle and modifies it to: "Only in the practice of a language can a word have meaning".

The context principle has not gone unnoticed in the literature, indeed it has received a number of different interpretations. However, none of them does justice to Frege and Wittgenstein's understanding of it; and this is, it seems to me, directly connected with the standard ways of categorising Frege and Wittgenstein: as realist and anti-realist. According to the reading to be developed here, the context principle (in Frege *and* Wittgenstein) *cuts across* the usual realism-vs.-antirealism distinction. What is more, the reversal of metaphysical explanations with which the principle is tied up undermines the very *basis* for this distinction. Now, this very fact makes it also harder to explain my new perspective on Frege and Wittgenstein – since it is at odds with certain widespread (though often only implicit) assumptions in contemporary metaphysics, philosophy of language, and philosophy of mathematics. Some of these assumptions will, then, also have to be subjected to critical scrutiny in the course of this paper.

At the same time, my approach may at first glance not appear to diverge very far from the mainstream, particularly with respect to Frege. For instance, it will turn out that my Frege is a *platonist*, that is to say: he does view mathematical judgments as objective; he does hold that in the corresponding statements our number terms refer to numbers; and he does maintain that these numbers are non-mental, non-physical, self-subsistent objects. In other words, I do not want to deny at all that Frege makes such statements, not even that he means them seriously. Instead, I want to re-interpret *what he means by them*, i.e., *what kind of platonist* he is. I will argue that Frege is a *contextual platonist*, not a *metaphysical platonist* – a crucial difference which should be clear by the end of the paper. In addition, if my new reading of Frege is correct, it becomes possible to see that his platonism is less in opposition to Wittgenstein's anti-platonism than is usually assumed. I will substantiate this conclusion by re-considering Wittgenstein's position, too, with respect to both his critical and his constructive views.[1]

I. Frege and metaphysical platonism

Throughout his life Frege's main goal was to put arithmetic on a firm foundation. For him this amounted to analyzing and clarifying its logical structure, thus revealing both what it is based on and what it is about. In other words, Frege's logical investigations led him to certain epistemological and metaphysical conclusions: about the basis of arithmetic judgments and the nature of numbers (and functions). As is well-known, Frege's conclusions amount to a kind of *logicism*; that is, for him arithmetic judgments find their foundation in basic logical laws and numbers turn out to be logical entities. This much of the standard reading I do not want to

question. More problematic for me – i.e., more in need of clarification – is that Frege's position is usually also characterised as a kind of *platonism.*

Frege's platonism (vaguely and naively)

What exactly it means to say that Frege is a 'platonist' – or what could be meant by "platonism" in the first place – is a central question in this paper. I do not think there is only one possible answer. But let us first look at some *typical characterizations* of "platonism". For instance, in the *Encyclopedia of Philosophy* we can read:

> By platonism is understood the realistic view, akin to that of Plato himself, that abstract entities exist in their own right, independently of human thinking. According to this view number theory is to be regarded as the description of a realm of objective, self-subsistent mathematical objects that are timeless, non-spatial, and non-mental. Platonism conceives it to be the task of the mathematician to explore this and other realms of being. Among modern philosophers of mathematics Frege is a pre-eminent representative of platonism, distinguished by his penetrating lucidity and his intransigence.[2]
>
> (p. 529)

This passage contains the main elements of the understanding of "platonism" dominant in much of recent metaphysics, philosophy of language, and philosophy of mathematics. It focuses on the following three claims: (i) numbers and other mathematical entities are "abstract objects" which exist "in their own right"; (ii) in mathematics we "describe" these objects, i.e., we talk about them as members of a "mathematical realm" and (iii) the task of the mathematician is to "explore" this realm, i.e., to find out what is "objectively the case" in it. At the same time, an understanding of platonism which just cites these three claims (without further explication) is still very vague and naive. In fact, it will turn out to be *ambiguous*, i.e., allow for two rather different interpretations.

It is hard to deny, though, that Frege is a platonist in this vague and naive sense. He is most explicit about his views in this connection in *Foundations of Arithmetic* (1884). There he says about numbers as objects:[3]

> [S]urely the number one looks like a definite object, with properties that can be specified, for example that of remaining unchanged when multiplied by itself.
>
> (*FA*, p. II)

> But it will perhaps be objected, even if the earth is really not imaginable, it is at any rate an external thing, occupying a definite place; but where is the number 4? It is neither outside us nor within

> us. And, taking those words in their spatial sense, that is quite correct. . . . Yet the only conclusion to be drawn from that is that 4 is not a spatial object, not that it is not an object at all. Not every object has a place.
>
> (*Ibid.*, p. 72)

And about the nature of numbers and the objectivity of arithmetic:

> For number is no whit more an object of psychology or a product of mental processes than, let us say, the North Sea.
>
> (*Ibid.*, p. 34)

> But arithmetic is no more psychological than, say, astronomy. Astronomy is concerned, not with ideas of the planets, but with the planets themselves, and by the same token the objects of arithmetic are not ideas either.
>
> (*Ibid.*, p. 37)

> Even the mathematician cannot create things at will, any more than the geographer can; he too can only discover what is there and give it a name.
>
> (*Ibid.*, pp. 107–8)

Clearly all the three main ingredients of platonism mentioned above are contained in these remarks. (Note, however, that Frege does not use the term 'abstract object' for numbers; he prefers 'logical object', for reasons which will become clear later.)

Another work often cited in connection with Frege's platonism is his late article "Thoughts" (1918–19). As the title suggests, in this article Frege is mostly concerned with the nature of "thoughts", not with numbers. Thoughts in his sense, are the contents of judgments – they are what can be asserted, believed, questioned, etc. And they, too, turn out to exist as non-mental and non-physical objects; or as Frege puts it now, they exist in a special "intellectual realm" (which also contains numbers). Thus he writes:

> A third realm must be recognized. Anything belonging to this realm has it in common with ideas that it cannot be perceived by the senses, but has it in common with things that it does not need an owner so as to belong to the contents of his consciousness. Thus for example the thought we have expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no owner. It is not true only from the time when it is discovered; just as a planet, even before anyone saw it, was in interaction with other planets.
>
> (*CP*, p. 363)

Our relation to such thoughts is clarified further in a footnote: "A person sees a thing, has an idea, grasps or thinks a thought. When he grasps or thinks a thought he does not create it but only comes to stand in a certain relation to what already existed – a different relation from seeing a thing or having an idea." (*Ibid.*) For Frege the "first realm" is the universe of physical objects, existing in space-time; to it we have access through sense perception. The "second realm" is our psychological world (or worlds), i.e., each person's subjective world of ideas, feelings, and thinking processes; our access here is through direct awareness and introspection. Finally, there is a "third realm", to be contrasted with the earlier two; it contains thoughts and numbers (maybe more). This third realm is, presumably, analogous to Plato's realm of forms; thus the use of the term "platonism" (in the secondary literature, not by Frege himself).

The metaphysical platonist picture

Unfortunately, many debates about platonism – and thus also about Frege – remain content with vague, general characterizations of it, such as that quoted above. In other words, they rely on rather brief descriptions of platonism, mostly in terms of a few metaphors. Typically the following kinds of phrases are used in these debates: that "abstract objects", in particular numbers, "really exist", "independently from us", "out there"; that they are not just "created" by us, but "discovered"; that the mathematician is an "explorer", not an "inventor", etc. Such descriptions are usually followed by rejections (often mere dismissals) of "platonism" and then by proposals of alternative, "anti-platonist" positions.

Now, for my purposes it is necessary to be more careful here, i.e., to pause and ask: Which intuitions underlie these descriptions of platonism, and what exactly is implied by the corresponding metaphors? These questions lead to a first (somewhat more specific and explicit) version of platonism. Since it will be useful to have a concise way of referring to it, let me give it a name; let me call it *metaphysical platonism* ("metaphysical" used here in the pejorative sense of "hard to pin down" and "possibly incoherent"). The core of this view is a simple *picture* – a picture which is supposed to capture what we do in mathematics, in particular when we use mathematical language. More precisely, it is supposed to explain three related phenomena: the *nature* of arithmetic entities, the *meaningfulness* of arithmetic expressions, and the *objectivity* of arithmetic judgments.

Here is the metaphysical platonist picture:

Imagine two realms, namely our mathematical language, proofs, etc. on the one hand, and a world of abstract, mathematical objects on the other. These abstract objects are assumed to exist in themselves, i.e., independently of whether we think about them or not, also independently of what we do and what happens in the physical world; in other words, they exist in a separate realm. Thus on one side we have our term 'the number two', on the other the number two itself; and the latter is neither a mental nor a physical entity. Now what happens when we use a mathematical term, for example, 'the number two'? Well, we use it to refer to the number two. That is to say, somehow we establish a connection between the two sides, a reference relation. Thus, in '2 + 3 = 5', '2 is even', etc., the term '2' stands in for the number two; likewise with respect to the other expressions. Furthermore, the truth of arithmetic sentences is characterized as follows: '2 is even' is true if and only if the number two (referred to by '2') really falls under the concept "is even" (referred to by 'is even'). Or as it is often put: the sentence is true just in case it corresponds to an actual fact, that is, if and only if the mathematical realm is actually built that way. Finally, our mathematical judgments are objective insofar as they are either true or false, in the sense just explained, i.e., as measured against the initially postulated mathematical realm.

It is important to observe the following three aspects of such a picture: a) In spite of the fact that it tries to be precise and explicit, the characterization of platonism it provides is still more suggestive than definite, i.e., it still relies heavily on metaphors; b) Nevertheless the picture is presented as an *explanation*, i.e., it is supposed to do some work. In particular, certain metaphysical and semantic notions, namely "existence", "object" and "reference", are supposed to allow for an explanation of other such notions, namely "meaning", "truth" and "objectivity". c) In addition, their explanation is supposed to be completely *independent* of epistemological considerations. That is to say, questions about how we know about the postulated abstract objects are left aside initially; they are to be treated later, after we have talked about metaphysical and semantic issues. (It is, then, with respect to aspects b) and c) that we have gone beyond the "naive" view introduced above.)

Metaphysical platonism and its order of explanation

The way I have just characterized metaphysical platonism highlights questions of conceptual priority, i.e., questions about what, logically speaking, comes first and what comes later. Let me be even more explicit and manifest the *order of explanation* that is implicit in this characterization of platonism:

1) We start by assuming the *existence* of a realm of mathematical entities; one might speak here of the assumption of a "Model-in-the-Sky". This is a realm of abstract *objects*.[4]
2) Then we explain the *meaning* of mathematical expressions and thus their descriptive *use* in terms of *reference*. The reference relation used here is assumed to be some kind of "direct connection" between words and objects.
3) Next, we explain the *truth and falsity* of mathematical statements in terms of such meaning. And we explain the *objectivity* of mathematics in terms of truth/falsity.[5]

But what, then, about mathematical knowledge? This question leads to a fourth step:

4) We postulate some special form of *perception* (or an *intellectual sense*) by means of which we obtain mathematical *knowledge*. Indeed, it seems we have to do so to account for our "access" to the Model-in-the-Sky, since how else could we know about it?

Notice that an explanation along the lines of these four steps – in that order – is what is supposed to give *bite* to metaphysical platonism. That is to say, it is supposed to turn it into a philosophical *position*, as opposed to remaining an innocent picture.

What is most important about metaphysical platonism is its particular order of explanation. The three general aspects of this order to be kept in mind for later are the following: a) According to metaphysical platonism we simply take for granted notions of "existence", "object", and "reference". In other words, these are *primitive* concepts with respect to the explanation. Presumably we can understand them independently of our use of mathematical terms, our mathematical practice, etc. And this independence is essential, since it is exactly the meaningful use of our terms and the objectivity of our practice which we want to *explain*. Furthermore, b) the first two notions in the explanation, "existence" and "object", are considered to be purely *metaphysical*. And the next, "reference", is thought to be purely *semantic*. Using these three we then explain "meaning", "truth" and "objectivity"; thus, according to the point of view under discussion these latter concepts need, and are susceptible to, an explanation. c) It is only at the end that we throw in an *epistemological* notion: "knowledge". The metaphysical platonist picture suggests, thus, a clean separation of metaphysics, semantics, and epistemology. And this separation is sometimes assumed to be an advantage – it is held that by not separating metaphysics, semantics, and epistemology confusion is bound to arise.

At the same time, it is this very separation which, according to many critics, also leads metaphysical platonism into immediate trouble. Namely, it leads to the *access problem*, i.e., to the following kinds of questions: How can we know anything about the postulated mathematical realm, i.e., is there really a special mathematical perception? And if so, how should we think about it: how does it work; does everybody have it; is it always reliable; etc.? It turns out that it is not even clear how to begin answering such questions. The above exposition makes clear, I think, what the source of the problem is: we have separated our metaphysics and semantics (the notions of "existence", "object" and "reference") so thoroughly from our epistemology (the notion of "knowledge") that the very possibility of mathematical knowledge appears dubious. Put differently, we have made the *gap* between the two sides from which we started too big. As a consequence it looks impossible, almost by definition, to bridge that gap.

II. Wittgenstein and the Augustinian picture

If we look at Wittgenstein's writings on mathematics it is clear that he is opposed to the kind of platonism discussed so far. Both in *Lectures on the Foundations of Mathematics* (1939) and *Remarks on the Foundations of Mathematics* (1937–44), metaphysical platonism comes under explicit and repeated attack. In these writings most of Wittgenstein's criticisms are based on the observation that metaphysical platonism is guided by a misleading analogy between mathematics and physics. As he argues, this analogy distorts our understanding of the role of proof in mathematics; more generally, it distorts our understanding of the criteria by which mathematical propositions are "adjudicated" in usual mathematical practice. However, these arguments always remain exploratory and tentative in *Lectures* and *Remarks*. And even later Wittgenstein never manages to bring them into a completely satisfying and definite form.[6]

On the other hand, *Philosophical Investigations* (1945–49) contains a different – a subtler and more implicit – critique of platonism, one Wittgenstein also considered more definitive. This critique, or the general discussion it is part of, has implicitly guided my exposition so far. I am, of course, referring to Wittgenstein's discussion of the *Augustinian picture of language*.

The Augustinian picture of language

Wittgenstein's *Investigations* starts with a long quotation from Augustine's *Confessions*. In it Augustine describes how he learned to speak and understand language as a child, namely as follows: he observed his parents and other grown-ups naming various objects; thus he gradually learned to associate names with objects; and then, or after he had learned to pronounce the names, he used them himself to refer to the corresponding objects. This is, supposedly, all there is to learning and understanding a language. Wittgenstein comments:

> [Augustine's] words, it seems to me, give us a certain picture of the essence of human language. It is this: the individual words in language name objects – sentences are combinations of names. – In this picture of language we find the roots of the following idea:

> Every word has a meaning. This meaning is correlated with the word. It is the object for which the word stands.[7]
>
> (*PI*, §1)[7]

Later he adds:

> One thinks that learning language consists in giving names to objects. That is, to human beings, to shapes, to colors, to pains, to moods, to numbers, etc. To repeat – naming is something like attaching a label to a thing.
>
> (*Ibid.*, §26)

Note that Wittgenstein's first reaction is, thus: Augustine's remarks do not amount to a theory, i.e., a detailed, systematic position, thought through in all its details and consequences; instead they only suggest a "picture" – *the Augustinian picture of language*.

Basic for the Augustinian picture are the intuitions of *labeling* and of using the labels as *names*. Augustine conjures up these intuitions by appealing to the way children (supposedly) learn language. Similarly, the picture may make us think of language learning as an adult; all we need to learn in that case is (supposedly again) to pick up the labels of a new language, i.e., new name-tags corresponding to the ones we already know in our old language. However, it goes well beyond a view about language learning – the picture may be taken to contain an *explanation of how language works*, i.e., how it is that we manage to use words to talk about the world.

Seen as such, the Augustinian picture operates on four distinct, but related levels:

(A) The pedagogical and psychological level: On this first level the picture explains (supposedly) *language learning and understanding*; and it does so by appealing to how children pick up on word-object relations, namely by "direct observation".

(B) The semantic level: More basically, the picture explains (supposedly) the *meaning of words*. Such meaning consists simply of word-object relations; in other words, the meaning of a word just is the object it stands for.

(C) The metaphysical level: Even more fundamentally, on the first two levels it is presupposed that there is a *world that is in itself divided into objects, simply to be labeled*; that is, there is a world of "self-identifying" objects (and kinds of objects) "out there" and we merely have to put "name tags" on them so as to be able to talk about them. (In other words, we can "fit" language onto a predetermined "structure of the world".)

(D) The epistemological level: Finally, it is presupposed in steps (A), (B) and (C) that we have *direct access* to this world, i.e., that we can "know" the objects in it independently of anything else, through some kind of "direct perception".

Analyzed as such, this explanation of how language works should obviously strike us as familiar (after our discussion of metaphysical platonism above).

The Augustinian picture suggests a rather *general* point of view about language. For me what is crucial about this point of view is, again, the *order of explanation* it embodies:

1) We simply assume the *existence* of a realm of self-identifying *objects* (thus "object" and "existence" are primitive notions.) And we take tables and chairs, or people, to be paradigmatic examples.
2) The *meaning* of words is then explained in terms of some form of direct *reference* to such objects (so "reference" is primitive too.) The simple paradigm for how to establish such reference relations is pointing and labeling, as in the baptism of babies and ships.
3) Next, the *descriptive use* of our words, and the *truth/falsity* of the sentences involving them, is explained in terms of such meaning, thus in terms of reference; and the *objectivity* of our judgments is explained in terms of such truth/falsity.
4) Finally, some kind of *knowledge*, complementing steps 1)–3), is implicitly assumed or explicitly postulated. Here the paradigm is "directly observing" things, animals, and people (and thus "knowing" them), as they parade before our eyes.

As should be clear by now, most of my discussion so far has been intended to illustrate the following claim: If we apply the Augustinian picture to mathematics, we arrive exactly at a metaphysical-platonist explanation of how mathematical language is used. In other words, *the Augustinian picture is a generalization of the metaphysical platonist picture.*

One of Wittgenstein's main goals in *Philosophical Investigations* is to discredit the Augustinian picture of language (and various related ideas). And he proceeds roughly as follow: first he argues that it cannot really explain our use of language in general; then he considers the special case of psychological language in order to point out how the picture quickly leads into traditional philosophical problems there; and at the very end of the *Investigations* he adds:

> An investigation is possible in connection with mathematics which is entirely analogous to our investigation of psychology. It is just as little a *mathematical* investigation as the other is a psychological one. It will *not* contain calculations, so it is not, e.g., logistic. It might deserve the name of an investigation of the 'foundations of mathematics'.
>
> (*PI*, p. 232)

As this passage shows, Wittgenstein thinks of his investigations into psychology and into mathematics as parallel case studies. In order to understand better what is involved in this parallel – and thus what a Wittgensteinian "investigation of the foundations of mathematics" would look like – let us now consider his general critique of the Augustinian picture, as well as its application in the case of psychology.[8]

Wittgenstein's critique of the Augustinian picture

The core of Wittgenstein's general critique of the Augustinian picture is contained in the following remark:

> "We name things and then we can talk about them: can refer to them in talk." – As if what we did next were given with the mere act of naming. As if there were only one thing called "talking about a thing". Whereas we do the most various things with our sentences.
>
> (*PI*, §27)

How are we supposed to understand this cryptic passage? The main question introduced in it is the following: Suppose we have put a label (a name-tag) on a thing – can that act, in itself, determine the way in which the label is to be used as a name (i.e., the way it is to fit into sentences and the role these sentences are to play in our practices)? In other words, can it determine the *grammar* of the word (in Wittgenstein's sense)? More particularly, can such an act of labeling, in itself, determine that the word is to function as an object name, as opposed to a predicate? And if it is to function as such, how can it determine which predicates can be meaningfully applied to it? etc. Wittgenstein's suggestion is, of course: the labeling of a thing simply cannot, in itself, do all of that.

To be sure, Wittgenstein does not maintain that we, as speakers of English, are not able to point to, say, ships and name them (likewise for properties, e.g., colors). Rather, his claim is that the *mere* act of labeling does not, indeed cannot, bring the label "to life", i.e., make it into a name. In other words, the labeled thing – in itself – does not tell us how to use the label in a language; or as Wittgenstein himself puts it at one point briefly: "Don't think that you read off what you say from reality" (*Ibid.*, §292). To make a long story short, Wittgenstein's most fundamental criticism of the Augustinian picture consists, thus, in exposing an assumption implicit in it. The assumption is this: by putting a label on it, a "piece of reality" is, in itself, supposed to determine how the label is to be used as a word. Now, having made this assumption more explicit a more reasonable alternative suggests itself, namely: it is only against an elaborate *background* that naming can work, namely the background of a *language game*, i.e., a whole language and various practices connected with using it.

Early on in *Philosophical Investigations* Wittgenstein gives a simple, artificial example of a language game: that of his famous "builders" (*PI*, §2ff.). The role of this example is to highlight the centrality of *practices* behind any use of words – it is only in connection with such practices that words acquire a determinate use, and thus meaning. More particularly, Wittgenstein remarks (with respect to 'slab' and 'pillar' as used by the builders): "Now what do the words of this language *signify*? – What is supposed to show what they signify, if not the kind of use they have?" (*Ibid.*, §10). And later: "We may say: *nothing* has so far been done, when a thing has been named. It has not even *got* a name except in the language game" (*Ibid.*, §49). In other words, without the background of a language game it can hardly be clear what a word means, not even what it *names* (if it functions as a name at all), even after it has been put on something as a "label".

In the later parts of the *Investigations* Wittgenstein considers in more detail the special case of psychological language. In connection with it he uses (amongst others) exactly the same argument we have just looked at (supplemented by some considerations tailored to the peculiarities of the case).[9] Now his main target is the application of the Augustinian picture to words such as 'pain' and 'headache', also to 'thinking', 'believing', 'intending', etc. With respect to these words we encounter the same pattern of explanation as earlier: 1) Particular instances of pain, and of certain kinds of pain, are supposed to be just "there", determinate in themselves. 2) Thus we can label them, supposedly by some sort of "inner pointing"; for example, we can label one instance with 'headache'. 3) On this basis we can then talk about headaches; that is, we can refer to them and make true and false statements about them. 4) And we have access to headaches directly, since we seem to be immediately aware of them.

Notice that in this case, as in general, Wittgenstein does not deny that we can in some sense point – here point "inside" – say to a headache. However, he emphasises:

> When one says "He gave a name to his sensation" one forgets that a great deal of stage-setting in the language is presupposed if the mere act of naming is to make sense. And when we speak of someone's having given a name to his pain, what is presupposed is the existence of the grammar of the word 'pain'; it shows where the new word is stationed.
>
> (*Ibid.*, §257)

And the mistake he wants us to avoid is, once more: to suppose that there is some *primitive* kind of pointing and labeling – *prior* to our ordinary use of language, of self-identifying entities, etc. – on the basis of which we can then give an *explanation* of the meaning and use of our words, and of the truth and falsity of our sentences.

In addition, if we adopt the perspective of the Augustinian picture (i.e., if we grant its basic assumptions), a mystery about the psychological world arises immediately – like in the case of the mathematical world. In this case the problem is not so much how to think about objects, but how to think about *states* and *processes*. That is to say, the following questions arise: What is the nature of psychological state? And in which sense do psychological processes occur? As in the mathematical case, the temptation is now to rely on vague, simplistic analogies to the physical world. Thus, we may try to think of the pain of a person as being "just like" the state of a middle-sized physical object, say its temperature. We may even be tempted to assume that pain *is* a physical state, e.g., a brain state. Similarly for processes. However, this leads quickly to many further questions, since psychological states and processes also seem *different* in crucial respects from physical states and processes. But then the presumed analogy begins to dissolve again – and we are left with seemingly deep philosophical problems about psychological phenomena.

Wittgenstein is especially interested in the case of psychology because of these problems, i.e., because here reliance on the Augustinian picture leads right into some old philosophical conundrums (parallel to the case of mathematics). Most prominent amongst them are the "problem of other minds" and the "problem of the inverted spectrum". They arise as follows: What is (supposedly) special in the case of psychological language is that the meaning-giving act of "inner pointing" is completely private. That is to say, if I point inside myself and label a feeling 'pain', nobody else has access to what it is I am pointing to (it seems), since it is only me who is aware of it. Conversely, I know what 'pain' refers to only from my own case. But then, as Wittgenstein observes: "If I say of myself that it is only from my own case that I know what the word 'pain' means – must I not say the same of other people, too? And how can I generalize the *one* case so irresponsibly?" (*Ibid.*, §293). In other words, the problem is now: how do I know that another person means the same thing by 'pain' as me? It seems possible that he or she calls a pleasurable feeling "pain", doesn't it? Even worse, is it not possible that other people do not feel pain at all? Similarly, may other people's use of color terms not be different, for instance "inverted"? Wittgenstein illustrates these questions, and the corresponding philosophical puzzles, in the following graphic and memorable comparison:

> Suppose everyone had a box with something in it: we call it a "beetle". No one can ever look into anyone else's box, and every-

> one says he knows what a beetle is only by looking at *his* beetle. – Here it would be quite possible for everyone to have something different in his box. One might even imagine such a thing constantly changing. . . . The box might even be empty.
>
> (*Ibid.*)

The "beetle" here corresponds obviously to "private pain", to the "private sensation" of a color and to similar feelings or sensations.

For the purposes of this paper there is no need to go into more details with respect to these problems. But note what Wittgenstein's general comment about them is:

> How does the philosophical problem about mental processes and states . . . arise? – The first step is the one that altogether escapes notice. We talk of processes and states and leave their nature undecided! Sometime perhaps we shall know more about them – we think. But that is just what commits us to a particular way of looking at the matter. For we have a definite concept of what it means to learn to know a process better. (The decisive movement in the conjuring trick has been made, and it was the one that we thought quite innocent.)
>
> (*Ibid.*, §308)

According to my reading, the "first step", and thus the "decisive movement in the conjuring trick", is to assume the perspective of the Augustinian picture. According to Wittgenstein this perspective is fundamentally inadequate (for understanding how language works). And his critique of it applies equally in the case of psychological and of mathematical language. In fact, it applies even in the case of our everyday language for physical objects, states, and processes – since at its core there is a general, very basic argument.

III. Frege's use of the context principle

If Frege were a metaphysical platonist, all of Wittgenstein's arguments considered so far could be seen as undermining his position. And the two thinkers would be diametrically opposed, as is often assumed. Also, the access problem would be a serious challenge for Frege. However, I do not think that he is a metaphysical platonist – he is *some other kind of platonist*. In order to explain what kind, I will have to direct attention to his so-called *context principle*. Frege uses this principle both *critically* and *constructively*; I will look at each use in turn. But let us begin with the principle itself.

Frege's formulations of the context principle

Frege states the context principle explicitly at four points in *Foundations of Arithmetic*. First in the introduction:

> One must ask for the meaning of words in the context of a sentence, not in isolation.
>
> (*FA*, p. X)

Then twice in the main body of the text:

> Only in [a complete sentence] do words really have a meaning.
>
> (*Ibid.*, p. 71)

> Only in the context of a sentence do words have meaning.
>
> (*Ibid.*, p. 73)

Finally in the conclusion:

> We adopted the principle that the meaning of a word is to be explained not in isolation, but in the context of a sentence.
>
> (*Ibid.*, p. 116)

Unfortunately, Frege never elaborates much on how exactly he wants these cryptic pronouncements to be understood. Consequently it is no big surprise that different interpreters have come up with rather different interpretations. (I will discuss and criticize several of them towards the end of this section.) Given this situation, I suggest that the only way to gain more clarity about the context principle is to look at how Frege actually *uses* it. In other words, I want to look at the *context* of the *context principle* in his writings.

To begin with, notice that *Foundations of Arithmetic* is Frege's most explicitly philosophical work. Its aim is to present his main philosophical ideas (concerning arithmetic, his main object of study). In the introduction to *Foundations* Frege formulates three basic principles which are supposed to guide his whole approach. The context principle is the second of them; the first and third read as follows: (1) "One must separate sharply the psychological from the logical, the subjective from the objective" (*FA*, p. X) (3) "One must keep in mind the distinction between concept and object" (*Ibid.*). It is clear, and usually acknowledged in the literature, that these two guiding principles bear directly on Frege's platonism. They obviously have to do with his thesis that numbers are *logical objects* (since they are concerned about what is "logical" and about the notion of "object"). Curiously, the second – the context principle – has received much less attention, especially in connection with the issue of platonism. Yet, in my view this principle, if understood appropriately, is as crucial for understanding his platonism as the other two.

Frege's critical use of the context principle

In *Foundations* Frege criticizes a number of other, in his view inadequate, views. One of them, indeed in many ways his main target, is *psychologism*; more precisely, his target is psychologism *as applied to logic and arithmetic.* (Two other important targets are empiricism and formalism.) As Frege understands it, such psychologism comes in two variants; that is, its proponents are committed to one or both of the following two claims: (i) Logical and arithmetic entities, in particular numbers, are simply ideas in the minds of people; thus it is such mental entities we talk about in logic and arithmetic; (ii) logical and arithmetic laws are just psychological laws, i.e., they are laws to be studied in empirical psychology; and as such they are open to empirical verification or falsification. (These two claims have often been held together, but it is not hard to see that they are logically independent from each other.)

What, according to Frege, is wrong with these two claims? Roughly, his two main observations are: a) they misrepresent what our logical and arithmetic terms *mean*; b) they make logic and arithmetic *subjective*. These two mistakes are not unrelated; often the first leads people to the second. Now, Frege thinks that the first, the more fundamental mistake, can be avoided by paying attention to the context principle; he even claims in the introduction to *Foundations* (shortly after introducing his three guiding principles): "If [the context principle] is not observed, one is *almost forced* to take as the meaning of words mental pictures or acts of the individual mind, and so to offend against the first principle as well [i.e., the sharp separation between the psychological and the logical, the subjective and the objective]." (*FA*, p. X, my emphasis) Similarly in his conclusion: "[O]nly by adhering to [the context principle] can, as I believe, a physical view of number be avoided without slipping into a psychological view" (*Ibid.*, p. 116). But one may ask, how does ignoring the context principle lead to "taking as the meaning of words mental pictures or acts of the individual mind", thus to "slipping into a psychological view"? And how does that make arithmetic subjective?

I suggest that in these two passages Frege has the following in mind: Many proponents of psychologism look at individual expressions (say 'the number two') in order to ask, without further ado, what they could possibly mean. Looked at that way, "in isolation", "one is almost forced" to come up with something like a mental idea or a mental act as its meaning (say the mental image of two strokes or the mental act of dividing one thing into two) – since what else could fill the bill? But if one says that such an idea or act is what 'the number two' means, one has made what arithmetic is about into something mental, and thus subjective. In addition, one may then jump

to the further conclusion that arithmetic and logic are part of the study of mental processes and states, i.e., part of psychology.

Frege reacts with the following advice: Do not just look at single, individual words if you want to understand their meaning (in particular in connection with number words)! As we have seen, if one does not follow this advice one is in danger of concluding that number words stand for something mental, and thus that arithmetic is subjective. However, the basic error here does not consist in these conclusions – they are just the symptoms of a deeper error. What is really problematic, according to Frege, is the *general approach* exemplified by this kind of psychologism: to look at single, individual words; to come up with some entities, conceived of in themselves, and to connect the two directly. And the context principle is invoked to guide us away from this general approach, not just from its psychologistic application. (The principle is thus also directed against, say, certain formalists who, after looking at individual numerals, jump to the conclusion that numbers must be the numerals themselves.) That is to say, by appealing to the context principle Frege wants to cure the disease, not just the symptoms.

If this is Frege's real goal, a question suggests itself in connection with his *platonism*. Namely, is he not inconsistent, i.e., is his own platonist position not another instance of what he is opposed to in general with his context principle? This question is particularly pressing if Frege is interpreted as a metaphysical platonist – since a metaphysical platonist explanation of the meaning of number words follows exactly the same general path as that criticized in psychologism: to look at individual words, to conceive of corresponding objects in themselves, and to associate the two directly. (The only difference is that the mental entities invoked in psychologism as the meaning of number words are replaced by corresponding "abstract objects" in metaphysical platonism. Yet clearly the same violation of the context principle occurs.)

Now, should we really interpret Frege as being so obviously inconsistent? I do not think so, at least if there is an alternative. (The alternative will be to interpret him not as a metaphysical, but as another kind of platonist.) But let me dwell a bit more on Frege's uses of the context principle at this point, since they allow for interesting comparisons to Wittgenstein's views.

More anti-psychologism (in Frege and Wittgenstein)

If we compare Frege's appeal to the context principle in the context of his attack on psychologism to our earlier discussion of Wittgenstein (in Section II), two interesting connections suggest themselves. First, the psychologistic explanation of the meaning of number terms just discussed results precisely from an application of the Augustinian picture. Thus Wittgenstein's general criticism of this picture applies to this case. Second, and more strikingly, this criticism turns out to be very much in line with

Frege's attack on psychologism – in fact, in retrospect, Wittgenstein's criticism looks like a generalization of Frege's. I suggest that what we have here is not just a parallel, or a mere similarity; Wittgenstein is clearly influenced by Frege.

The main reason this influence has not found much attention so far is, I suppose, the prevalence of the usual interpretations: of Frege as a metaphysical platonist, of Wittgenstein as an anti-realist, and of their relation as that between polar opposites.[10] On the other hand, it is not hard to find evidence that Wittgenstein himself did *not* see his relation to Frege as purely antagonistic. In particular, he mentions Frege's context principle explicitly, and approvingly, in *Philosophical Investigations*. Thus he observes: "We may say: *nothing* has so far been done, when a thing has been named [by ostention]. It has not even *got* a name except in a language game" (*PI*, §49). And then he adds: "This is what Frege meant, too, when he said that a word has meaning only in the context of a sentence" (*Ibid.*). In other words, Wittgenstein takes his own criticism of the Augustinian picture to be in agreement with Frege's use of the context principle. (In a number of other passages, e.g. §10, Frege's context principle is almost as much on the surface, as we will see later.)

Somewhat more implicit, but no less striking, is a second piece of evidence. Namely, Wittgenstein's own *argumentation* in the case of psychological language (leading up to his rejection of a "private language") seems to be directly influenced by Frege's criticisms of psychologism – even with respect to some of its *details*. As an illustration take again Wittgenstein's well-known discussion of the role of private entities (or states, processes, etc.), analogous to "beetles in boxes", for explaining the meaning of words such as 'pain'. About them he says:

> The thing in the box has no place in the language game at all; not even as a *something*, for the box might even be empty. – No, one can 'divide through' by the thing in the box; it cancels out, whatever it is. That is to say: if we construe the grammar of the expression of sensation on the model of 'object' and 'designator' the object drops out of consideration as irrelevant.
>
> (*PI*, §293)

As I read this passage, Wittgenstein does not deny that human sensations or feelings, such as pain, exist (in some sense). In fact, to do so would be absurd in his eyes. Rather, his suggestion is that in the case imagined, the language game with the private beetles (used as an "object of comparison"), whatever is in the box is *irrelevant* – the beetle, be it there or not, is "not a something" *as far as an explanation of meaning goes*. Analogously in the case of ordinary words such as 'pain': if we appeal to sensations as completely private entities, then they do not, even cannot, play the explanatory role assigned to them.

Compare this with Frege's arguments against psychologism. Both Frege and his opponent are interested in explaining the meaning of arithmetic expressions, say of 'the number two'. Now suppose, along the lines of psychologism, that I associate some mental entity with this expression, for instance a certain mental picture. Can this association be used as the basis for an explanation of meaning? Frege thinks it cannot, amongst others for this reason: other people may have different ideas in their minds, these ideas may change, and for some such ideas may be completely absent – the box may contain a different beetle, its content may change, and it may even be empty. Also, like Wittgenstein, Frege is here not concerned with the *existence* of the mental; rather he calls into question the *explanatory role* of mere mental ideas with respect to the objective, public meaning of arithmetic terms.

In his later writings Frege adds further depth to his criticism of psychologism. Thus he notes in the Introduction to Volume 1 of *Basic Laws of Arithmetic* (1893):

> If every man meant something different by the name 'moon', namely one of his own ideas, much as he expresses his own pain by the cry "Ouch", then of course the psychological point of view would be justified; but an argument about the properties of the moon would be pointless: one person could perfectly well assert of his moon the opposite of what the other person, with equal right, said of his. If we could not grasp anything but what was within our own selves, then a conflict of opinion, a mutual understanding would be impossible, because a common ground would be lacking, and no idea in the psychological sense can afford us such a ground. There would be no logic to be appointed arbiter in the conflict of opinions.
>
> (*BL*, p. 17)

In this passage Frege focuses again on a view according to which the meaning of our words are just private mental ideas. Here he emphasizes what such a view entails: it makes all our judgments subjective, i.e., agreements and disagreements turn out to be impossible; there does not remain any "common ground" (any common understanding) from which to arbitrate disputes. (Remember that ideas are subjective, and thus completely private, according to the view under discussion.) For Frege this conclusion is clearly unacceptable. In particular, it amounts to a *reductio ad absurdum* of such psychologism with respect to mathematics – agreements and disagreements are clearly possible in mathematics. His basic diagnosis of what has gone wrong is that our use of words has been misunderstood.

Again, Wittgenstein clearly agrees with this diagnosis. In fact, in his own attack on the Augustinian picture, especially as applied to psychological words such as 'pain', he pushes Frege's line of thought even further. Thus he asks (amongst others): if the Augustinian picture gave us the right explanation concerning the meaning of words such as 'pain', could we even agree or disagree with *ourselves*? Even that seems problematic. What is at issue here is this: have we (along the lines of the Augustinian picture) been provided with enough of a criterion for judging applications of such words to be correct or incorrect even when applied to our *own* mental processes and states?

Frege's constructive use of the context principle

So far I have discussed what Frege means by "asking for the meaning of a word in isolation"; and I have clarified his (and Wittgenstein's) arguments against doing so. That is to say, I have discussed his *critical* use of the context principle. But what about the other side of the coin, "to explain the meaning of words as they are used in the context of sentences"? In other words, what does it mean for Frege to follow the context principle in a *constructive* way? In order to answer this question it is useful again to first look at *Foundations of Arithmetic*, the work in which the principle is mentioned explicitly.

In *Foundations* Frege's central concern is to clarify our understanding of the notion of "number". An important part of this is to give an account of what number terms "mean". If we go beyond Frege's criticisms of inadequate views (as discussed above), his own positive account consists of two main parts: (i) He analyses our ordinary use of arithmetic terms, including their use in informal arithmetic; thus he says: "It should throw some light on the matter to consider number in the context of a judgment which brings out its basic use" (*FA*, p. 59). (ii) He gives an outline for a formal and rigorous reconstruction of arithmetic, within his logicist framework. Now, two of Frege's direct invocations of the context principle occur in this second connection, i.e., his logicist reconstruction. To quote the first more fully: "Only in [a complete sentence] do words really have a meaning. . . . It is enough if the sentence as a whole has a sense; it is this that confers on its parts also their content" (*Ibid.*, p. 71). And the second, also quoted more fully, reads: "Only in the context of a sentence do words have meaning. Thus our concern becomes this: to explain the sense of a sentence in which a number word occurs" (*Ibid.*, p. 73). Later in *Basic Laws of Arithmetic*, Frege continues with the second part of his account; that is, he makes his "explanation of the sense of arithmetic sentences", and thus of "the content of their parts", more complete and systematic. The corresponding reconstruction of arithmetic will turn out to be most important for us in the end.

However, we should consider the first part of the account, too, since Frege's reconstruction will be guided by it.

Frege makes several related observations about our "basic use" of number terms. For example, in our everyday sentences we often use number terms with the definite article, as in "the number two". We also say things like, "the number two is even"; that is, we often use 'the number two' in subject position, complemented by a predicate term, here 'is even'. Furthermore, even if we sometimes seem to use number terms in purely adjectival form, as in "there are nine planets in the solar system", such sentences can be transformed into a form so that the number terms appear again only in subject position, as in "the number of planets in the solar system = the number nine". Facts such as these show that number terms function in many ways like terms such as 'the Moon', 'the black chair in my apartment', etc. – in post-Fregean terminology: they are both used as "singular terms"; in his own terminology: both 'the Moon' and 'the number nine' are used as *object names*.

The following is interesting to note in this connection: With respect to some of these observations (e.g., the first two above) Frege just stays on the "surface" of language, i.e., he follows ordinary grammar. But with respect to others he goes "deeper". Thus his claim that the adjectival use of numerical terms is reducible to a substantival use involves moving from mere observation about ordinary grammar to some further, deeper analysis, namely an analysis of the "logic" of our terms, as used in ordinary sentences. A central result of this analysis is: "The content of a statement of number is an assertion about a concept" (*FA*, p. 59). A simple illustration, one that should also make this result plausible in itself, is the following: the statement "there are nine planets in the solar system" contains an assertion about the concept "planets in the solar system", namely the assertion that nine objects "fall under" it, i.e., are subsumed by it. It is such results about "content" in which Frege is really interested; that is, he is interested in *logical content*, as revealed by *logical analysis*.

Frege's sensitivity to ordinary language in *Foundations* shows again a striking similarity to what Wittgenstein does in his *Investigations*. In fact, Frege's logical analysis, in the sense just described, concerns exactly the "grammar" of words in Wittgenstein's sense; and Frege analyzes both their "surface grammar" and their "depth grammar" (cf. *PI*, §664). Once more this does not look like a fortuitous parallel to me; it is another case of Frege's direct influence on Wittgenstein. On the other hand, there certainly remain important differences between the two in this connection. In particular, analyzing the ordinary use of arithmetic terms is for Frege only the first step towards a systematic, logicist *reconstruction* of arithmetic. In other words, his real goal is a *scientific* one; in his own words: "Now our concern here is to determine a concept of number usable for the purposes of science."

(*FA*, p. 69). In contrast, the later Wittgenstein does not have any comparable scientific goal; he aims only at clearing up certain kinds of *philosophical confusion*.

In his criticism of J. S. Mill's views in *Foundations*, Frege clarifies further what exactly his aim is: he wants a scientific reconstruction of "pure", as opposed to "applied", mathematics. He accuses Mill of mixing up the two: "Mill always confuses the applications that can be made of an arithmetic proposition, which often are physical and do presuppose observed facts, with the pure mathematical proposition itself" (*FA*, p. 13). As it turns out, Mill always focuses on the relation of arithmetic operations (such as addition) to physical operations (such as the combination and arrangement of pebbles). Relatedly, Mill thinks that there is a role for empirical observations in connection with justifying arithmetic results. In sharp contrast, Frege's main concern is with statements like "2 + 3 = 5" or "There are infinitely many prime numbers" seen as parts of pure mathematics. And he denies that empirical observations can play any role in justifying such statements.

Frege's general aim is, thus, a scientific reconstruction of pure arithmetic. I suggested above that this reconstruction is guided by his analysis of ordinary linguistic usage, especially the usage of number terms in informal arithmetic judgments. But what exactly does this guidance amount to? Frege's analysis of ordinary language and informal arithmetic has led him to the conclusion that number terms are usually used as "object names". That is to say, they are, in a sense important to logic, used *like* 'the Moon'. Suppose, then, we want to build that conclusion into a rigorous, systematic reconstruction of arithmetic. This leads to the question: how do we "define the sense of a sentence in which a number words occurs" given that it is supposed to function as an object name? More particularly, Frege asks himself the following question: "[F]or every object there is one type of sentences which must have a sense, namely recognition-statements, which in the case of numbers are called identities. . . . The concern, therefore, [is] this: to fix the sense of a numerical identity. . . ." (*FA*, p. 116) In other words, the question is: how should we analyze and treat equations, i.e., sentences in which number terms occur on both sides of '=' (the equality sign)?

As a first attempt to answer this last question Frege considers the use of "contextual definitions" of a certain form (see *FA*, p. 73 ff.). Such definitions were used in the geometry of Frege's time, e.g., the definition of "direction of a line" in terms of the notion of "parallelism".[11] In this example the sense of a sentence such as 'the direction of line *a* = the direction of line *b*' is defined in terms of the sense of 'line *a* is parallel to line *b*'. Could we proceed analogously in arithmetic, now using the notion of "equinumerosity" (1-1-mappability)? That is to say, what about defining the sense of sentences such as 'the number of *F*s = the number of *G*s', in terms of '*F* is equinumerous to *G*' (where '*F*' and '*G*' are "concept names")? In the middle

parts of *Foundations* Frege first defends such contextual definitions with respect to several apparent general problems. However, in the end he rejects them, i.e., he thinks they are inadequate for his purposes. One reason for this rejection is that this whole method is not encompassing enough; it only allows us to treat sentences of one particular *kind*. What Frege needs is a method for defining the sense of *all* relevant kinds of sentences.[12]

Frege's next step is to develop such a more encompassing method. Basically it consists of an extended attempt to *reduce arithmetic to logic*. This amounts to the following: (i) all arithmetic notions (and all less basic logical notions) are defined in terms of more basic logical notions; (ii) the most basic (undefinable) logical notions are determined by means of logical laws. In the second half of *Foundations* Frege first indicates informally how to do (i) (at least for some central arithmetic notions). In *Basic Laws of Arithmetic* he is then more rigorous and systematic with respect to both (i) and (ii): he formulates precise, formal definitions for all the terms used; and he specifies explicitly the basic logical laws needed in his system. An example (and a crucial part of such a reduction of arithmetic to logic) is to define the meaning of number words in terms of the notion of "extension of a concept". Frege stipulates: 0 (the number zero) is identical with the extension of the concept "equinumerous with 'x is not identical with itself'"; similarly, 1 (the number one) is identical with the extension of the concept "equinumerous with 'x is identical with 0'"; etc.

At this point it may appear that the context principle does not play a big role any more, i.e., with respect to Frege's reconstruction of arithmetic. If so, it would not be crucial to his constructive project in the end, contrary to what I have suggested. (One may even be tempted to wonder: did he implicitly reject it at this point, together with the "contextual definitions" mentioned?) On the other hand, we have already seen that Frege repeats the context principle explicitly – and approvingly – in his summary, i.e., at the end of *Foundations*. More importantly, a remaining question is: how is the meaning of, say, "extension of the concept 'x is not identical with itself'" determined according to Frege? At one point in *Foundations* he says merely: "I assume that it is known what the extension of a concept is" (*FA*, p. 80, fn.; cf. also p. 117). But then he adds, briefly but significantly: "How we think of [extensions of concepts] emerges clearly from the basic assertions we make about them" (*FA*, p. 80). It seems to me that this passage is once again a direct appeal to the context principle, now explicitly at the basic level of extensions. Thus, in Frege's mature system the principle still plays a crucial role. It now applies to extension terms; it guides us in understanding their meaning. Furthermore, the principle then applies still to number terms, too – indirectly, via explicit definitions of numbers in terms of extensions. Notice also that in *Basic Laws* the "basic assertions" by means of which the meaning of extension terms is determined are his basic logical axioms (including his infamous Axiom V).

Conclusions about the context principle and Frege's platonism

I want to end my discussion in this section with some general conclusions about the context principle and its use in Frege's writings. These conclusions will allow me to identify and criticize, at least briefly, some of the ways in which the context principle has been misunderstood in the literature. They will also form the basis for my subsequent reinterpretation of Frege's platonism.

My most basic observation is the following: Frege's various appeals to the context principle, both in his criticisms of other views and in his own constructive project, occur in the context of explaining the meaning of *various* arithmetic expressions, including that of number words. Consequently, the principle is clearly not *just* meant, as one may suspect at first, to explain the meaning of "syncategorematic" expressions (in Russell's sense, e.g., the 'dx' in Calculus). It is also not *just* meant to explain the meaning of concept words (and thus the nature of concepts), as has been suggested in the literature.[13] Instead, Frege applies the context principle *both* to concept names and to object names as they occur in arithmetic. He even adds: "This observation [that psychologism can be avoided following the context principle] is suitable, it seems to me, to throw light on quite a number of difficult concepts, among them that of the infinitesimal, *and its scope is not restricted to mathematics either*." (*FA*, pp. 71–2; my emphasis) Thus Frege's principle is meant to apply quite *generally*, even beyond mathematics.

Next, Frege's context principle is not, as it may be tempting to think, meant as a defense of "contextual definitions" (in the sense mentioned earlier, e.g., the cited definition of "direction of a line"). In particular, it does not tell us to look at only *one kind* of sentences (e.g., only identity-statements) if we want to explain the meaning of a word. Rather, we have to look at *all kinds* of sentences in which the word in question occurs (or can occur). A related and even more basic observation is the following: the context principle does not tell us to look at *only one sentence* in which a word occurs. It is easy to be misled into this view if one focuses merely on Frege's explicit formulations of the principle in *Foundations*. Admittedly, most of them are in the form: "Don't study the meaning of a word in isolation, but in *a* sentence!" Nevertheless, if we look at Frege's actual *use* of the context principle, in particular in his systematic reconstruction of arithmetic, it is hard to deny the following: he appeals to it within the analysis of a whole *system* of judgments, and thus of sentences. The relevant "context" must, therefore, be more than one sentence.

This last suggestion – that we have to understand "context" more broadly than as a single sentence – is both controversial and crucial for my interpretation of Frege. Let me therefore dwell on it a bit more. A good illustration, one that should help to prove my general point, is provided by Frege's analysis of the distinction between "concept terms" and "object terms", together

with his argument that number terms are object terms. For Frege, both the distinction and the argument are based on a systematic analysis of the logical relations that hold between all our judgments. But if so, then they are based on an analysis of a whole system of sentences, namely all the sentences we can use to make these judgments. How exactly does Frege make his distinction? And how does his argument proceed? Roughly, there are four steps. (Notice that in each step we have to look at all the sentences in which the relevant terms occur.)

(i) In *Begriffsschrift* Frege develops his new logic, including a new analysis of quantification. Within it we encounter a basic difference between two kinds of terms: first-order versus second-order (and higher-order) terms. This difference is constituted by facts about which inferences are counted as correct and which not.[14] (ii) Next, it turns out that in the applications of logic to ordinary language object terms, say 'the Moon' and 'Gottlob Frege', occur as first-order expressions, while concept terms, say 'is blue' and 'is a logician', occur as second-order expressions. (iii) In *Foundations* and in *Basic Laws of Arithmetic* Frege shows (or tries to show) that his new logic can be used for a systematic reconstruction of all arithmetic judgments and inferences; and in this reconstruction number terms, say '7', occur as first-order expressions, while concept terms, say 'is prime', occur as second-order expressions. In other words, number terms turn out to function, logically speaking, like ordinary object terms. Finally, there is one more step, often overlooked. (iv) According to Frege what he has given us is not just a possible reconstruction, rather it is the *right* one (or so he thinks). But if all of this is the case, number terms really *are* object terms, i.e., Frege has revealed their "true natures".[15]

Frege's thesis that number terms are object terms is not just an isolated syntactical or logical point. Rather, it is intimately connected with his thesis that *numbers are objects*. This brings me to my final observation about Frege's context principle (and it leads us back to the question of how to understand his platonism). Based on paying attention to the actual use of the context principle in *Foundations* and *Basic Laws* one can see, I suggest, that it is ultimately a *metaphysical principle* for him – not *just* a semantic principle, also not *just* an epistemological principle, as has been claimed in the literature.[16] The context principle guides Frege's answer to the question what numbers are, or what their nature is (namely logical objects); and that I take to be a paradigmatic metaphysical issue.

Going beyond the context principle now, Frege's argument for the thesis that numbers are logical objects exemplifies the *explanatory reversal* which is at the core of this paper. Let me summarize the argument again in order to highlight its crucial features. It consists of four main steps: 1) Frege studies the use of number words in ordinary sentences, i.e., in ordinary language and informal mathematics; that is, he studies how they function in our usual judgments and inferences. He concludes, amongst others, that ordinary number words play the role of object terms, not of concept terms. 2) This conclusion, together with his new logic, guide Frege in his rigorous, systematic reconstruction of arithmetic. His initial conclusion about number words finds, then, a reflection within this reconstruction: in it number terms play again the role of object terms, not that of concept terms (or function terms); more precisely, they function as first-order object names. 3) Frege grounds his reconstruction in explicit definitions and basic laws; these definitions and laws allow (supposedly) to derive all the usual arithmetic propositions. In this context the meaning of number terms is defined in terms of the meaning of certain extension terms; and the meaning of extension terms is, in the last respect, determined by means of the basic logical laws. 4) Finally, based on some additional considerations, Frege argues that his is, in some sense, the right reconstruction. If so, it captures the "true nature" of numbers.

It is not my intention to defend Frege's whole account as contained in these four steps. In fact, I think there are serious problems with it, especially with respect to 3) and 4) (and thus with Frege's logicism). As to 3): The most basic problem is, of course, that Frege's system turns out to be inconsistent, as Russell's antinomy shows. And Gödel's Incompleteness Theorem for arithmetic introduces deep additional complications. As to 4): We know now, after the work of Russell, Zermelo, and others, that alternative reductions of arithmetic are possible (reductions to type theory, to set theory, etc.). Comparing Frege's reduction to these it is hard to see why it should be superior, i.e., the "right" reduction (even supposing it were consistent). Also, and more fundamentally, it is hard to see why *any* of these reductions should capture the "true nature" of numbers. In fact, all reductionist accounts assign additional, non-arithmetic properties to numbers; and these properties seem inappropriate in an answer to the question what the nature of numbers is.[17]

Yet, I think that in one respect Frege's approach is still very interesting, even attractive: its *order of explanation*. To repeat, Frege's aim is to explain the meaning of number words in a rigorous way. He notices that in order to give such an explanation we need to look carefully at all the sentences, or all the judgments, in which number terms are used. Consequently he first analyzes and then reconstructs all arithmetic judgments in a systematic way. As a result of this reconstruction the sense of all arithmetic sentences is determined, in the following sense: (i) It is specified, in an explicit and perspicuous way, which roles the various kinds of expressions are allowed to play in arithmetic sentences – i.e., the *logico-syntactic use* of all relevant expressions is fixed. (ii) It is determined, in a systematic and objective way, how arithmetic sentences follow from the basic axioms of the system (if they do), thus when they are true and when false – i.e., the *truth-grounds* of all relevant sentences are fixed. But if the sense of all arithmetic sentences is determined

in this way, then (remember the context principle) the meaning (the "sense" and the "reference") of number words occurring in them is explained, too – (i) and (ii) together constitute this explanation.[18]

If this is right, what follows for Frege's platonism? It is simply the following: We have determined that number terms play the role of object terms in arithmetic sentences; in addition, these sentences are objectively true or false depending on whether they or their negations follow from our basic axioms; and all the axioms needed are logical axioms. Thus numbers are logical objects, *since that is what it means to be a logical object.* Notice, furthermore, that along these lines the *objectivity of arithmetic* is not explained via the appeal to a simply postulated realm of abstract objects. Rather, it is the basic logical axioms (together with definitions and rules of inference) which give arithmetic judgments their objectivity. And this is exactly what Frege says in passages such as the following:

> My explanation [of number] lifts the matter onto a new plane; it is no longer a question of what is subjectively possible, but of what is *objectively definite*. For in fact, that one proposition *follows* from certain others is something *objective*. . . .
>
> (*FA*, p. 93; my emphasis)

The position we have arrived at is far from metaphysical platonism. I propose to call this new position "contextual platonism", since it is guided by the context principle. My suggestion is, thus, that *Frege is a contextual, not a metaphysical platonist.*

I am well aware that this reinterpretation of Frege flies in the face of conventional wisdom, i.e., the received views about him.[19] For that reason it is probably necessary for me to expound it further (in particular the remark about objectivity just made). Let me then devote one more section of this paper to it, i.e., to a further clarification and defense of my interpretation – and also of contextual platonism itself. (After that I will come back to Wittgenstein, specifically to his adaptation of the context principle.)

IV. Frege's contextual platonism

Let me sum up again the main results of Section III: If we examine Frege's critical and constructive use of the context principle carefully, a new interpretation of his platonism suggests itself. He turns out to be a *contextual platonist*, not a metaphysical platonist. The core of contextual platonism is a new explanation of the meaning of number terms and the nature of numbers, one that is characterized by its *reversed order* (relative to metaphysical platonism) – platonism is turned "upside-down" (or rather "downside-up"). Since my account of this reversal may have gone by rather quickly, I now want to explain it more. I will proceed as follows:

First, I will highlight the exact sense in which contextual platonism reverses the order of explanation inherent in metaphysical platonism. Second, I will show how all of Frege's platonist remarks, in *Foundations* and in his later writings, can be understood along contextual lines.

Frege's reversed, contextual platonism

As I interpret Frege, his platonism starts with a look at our arithmetic sentences, seen as used in a whole system of arithmetic judgments. For him this does not mean just to accept this system uncritically. Rather, he analyzes it, by reflecting on our ordinary usage of terms and by applying his new logical tools. This leads him to a systematic reconstruction of arithmetic consisting of the following three steps (compare Section III): (i) All arithmetic notions are reduced to logical notions, in particular the notion of "number" to that of "extension"; (ii) the logical notions, in turn, are determined in a system of logical judgments, a system grounded in fundamental logical laws (i.e., fundamental axioms and rules of inference); and (iii) these logical laws find a justification in "reason". Seen as a whole, this reconstruction is what determines the *sense* and the *truth value* of all arithmetic *sentences*, in a rigorous, systematic way. And remember: "it is enough if a sentence taken as a whole has a sense; it is this that confers on its parts also their content" (*FA*, p. 71). Thus, the *sense* and the *reference* of arithmetic terms, including *number words*, is also fully determined in Frege's reconstruction.

What, then, is the *nature of numbers*? Well, the meaning of numbers terms has been defined in terms of the meaning of extension terms. And extension terms are used as object names, in objectively true or false statements. But that means that extensions, as the referents of extension terms, are objects. Thus numbers, as the referents of number terms, are *objects*, too. More precisely: number terms refer to *logical* objects, i.e., objects whose identity is completely determined by logical laws. Finally, all of this also clarifies what the nature of arithmetic *knowledge* is. Such knowledge amounts to knowledge of our explicit definitions, our basic logical laws, and what follows from both. (In *Foundations* and *Basic Laws* one of Frege's main goals is to establish, in a rigorous and systematic way, exactly this kind of knowledge.)

What I have just recounted is how a contextual platonist, i.e., someone guided by the context principle, explains the meaning of number words, the nature of numbers, and the status of arithmetic knowledge. Let me make the *conceptual order* underlying this kind of explanation even more explicit – and thus its difference to the explanation we saw earlier. In contextual platonism we proceed as follows:

1) We start from our *logical laws*, i.e., our basic logical axioms and rules of inference. (They are fundamental for thought in general). The ultimate justification of these laws is understood in terms of the notion of

"reason". Relatedly, our *knowledge* of these laws is based on "reason" (ultimately at least).
2) We then explain the *truth/falsity*, and thus also the *objectivity*, of arithmetic sentences in terms of whether they follow from our logical laws or not. In particular, we determine the truth/falsity and the objectivity of *existence* claims along these lines. And all of this is done within the framework of a rigorously reconstructed system of arithmetic.
3) Next, a reflection on our systematic reconstruction shows that number words are *used* as object names in arithmetic sentences, not as concept names. But this means that they *refer to objects* and thus that numbers *are objects*. More precisely, numbers reveal themselves to be *logical* objects, since the truth/falsity of the arithmetic sentences containing them turn out to be completely determined by logical laws.
4) At the same time, our reconstruction, being a logicist reduction, shows in what *arithmetic knowledge* consists. It turns out to be just logical knowledge, namely: knowledge of the basic logical axioms, of logical definitions, and of what follows from both.

Recall at this point how one proceeds in metaphysical platonism: One starts from primitive notions of "object", "existence", and "reference"; one then uses these to explain "truth" and "objectivity"; and at the end one adds a primitive notion of "mathematical perception". In contextual platonism, in contrast, the notions of "object", "existence", and "reference" are not primitive; they are explained notions, as is that of "arithmetic knowledge". What is primitive, instead, are the notions of "logical law" and "reason". This is the sense in which contextual platonism proceeds in *reversed order* relative to metaphysical platonism.

Two aspects of the *context principle*, as used by a contextual platonist, also stand out now. First, this principle is built right into this explanation, namely on level three. As such, it is a central part of explaining what the notions of "object", "existence", etc. amount to in the case of arithmetic. But that means the context principle is part of a *metaphysical* explanation. Second, this piece of metaphysics is intimately tied up with *semantic* and *epistemological* considerations. Along the lines of contextual platonism questions about what it means for a term to have reference (semantics), questions about the nature of its referent (metaphysics), and questions about how we know about both (epistemology) are all connected – they all find answers when we look at two things: a) the *logico-syntactic function* of the term, i.e. the way it fits into all relevant sentences; b) the *truth-ground* of judgments made by means of these sentences, i.e., their ultimate justification. In the case of our main example: number terms are used as object terms in objectively true/false arithmetic judgments, thus numbers are objects; and our arithmetic judgments find their ultimate justification in logical laws (supposedly), thus numbers are logical objects.

Making sense of Frege's platonist remarks

At this point I anticipate the following doubt: Is contextual platonism, with its reversed order of explanation, really Frege's position? More particularly, can one understand all of his "platonist" remarks within this framework? Those interpreters who insist on reading Frege as a metaphysical platonist will most likely assume that one cannot do that.[20] Let me now show in detail that one can do so, indeed that it is not hard. I also want to discuss some further passages from Frege's writings – passages which make perfect sense if he is read as a contextual platonist, but not if he is read as a metaphysical platonist.

Recall Frege's most strikingly "platonist" claims (as quoted in Section I): that numbers are "definite, self-subsistent objects", even if they are "not spatial [or temporal] objects"; that "number is no whit more an object of psychology or a product of mental processes than, let us say, the North Sea"; that "arithmetic is no more psychological than, say, astronomy"; that "the mathematician cannot create things at will"; and that we must recognize a "third realm", distinct from the realms of physical objects and of mental ideas. Now, the first thing to note about these Fregean claims is that most of them occur in the context of his criticism of psychologism. Thus their primary function is negative: to signal that his own position is different from psychologism. But can we also make sense of them in a more positive way, in particular along contextual platonist lines? Let me begin with the claim that numbers are "definite, self-subsistent objects".

According to contextual platonism there is an important difference between objects and concepts. It is explained via the different logico-syntactic uses of object terms and concept terms. And Frege has shown that number terms are used as objects terms, not as concept terms. In addition, he has pointed out that we use these terms in objectively true or false sentences. But then it is perfectly legitimate to conclude that numbers are "objects" – as I said, that is what it means to be an object according to contextual platonism. Next, numbers are "definite" in the following sense: we have precise, objective laws that determine the truth value of the sentences in which number words occur. Furthermore, one way to understand the "self-subsistence" of numbers is also related. Namely, traditionally "properties" have been said to be *not* self-subsistent – presumably they only "subsist" in the objects which have them. But numbers are objects, not properties, as we have just seen; in that sense they are then self-subsistent.

For textual evidence that this is exactly how Frege often thinks about "self-subsistence" consider the following passage: "The self-subsistence which I am claiming for numbers is not to be taken to mean that a number word signifies something when removed from the context of a proposition, *but only to preclude the use of such words as predicates or attributes, which appreciably alters their meaning*." (*FA*, p. 72; my emphasis) But there is also another way in which "self-subsistence" is understood by Frege, namely in

the sense of "mind-independence". Consider in this connection a mental entity, say one of the ideas (in the psychological sense) entertained by me right now. This idea depends in its existence on my mind; that is, if my mental activity ceased, or if I thought of something else for a while, the idea itself would cease to exist. As Frege sometimes puts it, the idea needs a "bearer", it can only exist "in someone's mind" – and as such it is *not* self-subsistent. But, to Frege, numbers are not like that; in particular, they are not mental entities.

Remember here also that Frege's context principle is supposed to apply rather widely ("its scope is not restricted to mathematics", *FA*, p. 72). As I propose to interpret the principle, it applies thus even to expressions for physical entities. Consequently, if pressed Frege would explain the sense in which, say, chairs are "definite objects" in exact parallel to the mathematical case: by appealing to the logico-syntactic function of the corresponding words and the objective truth grounds of the corresponding sentences. And chairs, like numbers, are also "self-subsistent": that is, they, too, are not properties, but objects, and they do not exist "in people's minds", but in the physical world. To put the latter point more positively: the ultimate justification of existence claims concerning chairs does not depend on what is true of the mental; likewise for existence claims about numbers.

I hasten to add this is not to say that there are no important differences between these two cases for Frege. Both the logico-syntactic function of physical and arithmetic object words and the truth grounds for the corresponding sentences are not *exactly* the same. With respect to truth grounds, or ultimate justification, the situation is roughly as follows: The truth-values of statements about physical objects are objectively determined by empirical facts and physical laws, to be ascertained by observation, scientific induction, and related considerations. On the other hand, the truth-values of statements about numbers are determined by arithmetic definitions and arithmetic laws (which, according to Frege's reconstruction, can be reduced to logical definitions and laws). Thus both are independent of what is true of the mental – but in different ways. For textual evidence note, once more, the following passage from *Foundations*, now quoted in its entirety:

> But, it will perhaps be objected, even if the earth is really not imaginable, it is at any rate an external thing, occupying a definite place; but where is the number 4? It is neither outside us nor within us. And, taking those words in their spatial sense, that is quite correct. To give spatial co-ordinates for the number 4 *makes no sense*; but the only conclusion to be drawn from that is that 4 is not a spatial object, not that it is not an object at all. Not every object has a place.
>
> (*FA*, p. 72; my emphasis)

Frege acknowledges here that number words and words for physical objects differ in this respect: while it is perfectly normal to use predicates involving spatio-temporal location in connection with "the earth", it "makes no sense" to do so in connection with number words. It is exactly in this sense that numbers are "not spatial (or temporal) objects" for Frege. At the same time they are still objects – Frege's notion of "object" is broad enough to cover not just physical objects, but also logical, even (mind-dependent) mental objects.[21]

Such a distinction between physical, mental, and logical objects corresponds exactly to Frege's *three realms*. The "first realm" contains physical objects, the "second realm" contains mental entities; and then there is a "third realm", a realm containing numbers (amongst others). Frege introduces the term "third realm" in his "Thoughts'. In this essay he is, as the title suggests, mainly concerned about certain other "inhabitants" of the third realm, namely *thoughts*. For him, thoughts are those things that are expressed by our sentences and contained in our judgments. And as in the case of numbers, the expressions with which we refer to such thoughts function as object names in objectively true or false judgments. Thus thoughts, too, are "objects" and "self-subsistent". (But are they equally "determinate"? From a contextual platonist perspective this leads to the question: what exactly determines their properties and relations – maybe also some logical laws? Unfortunately Frege never gives us a rigorous answer to that question.)

What about Frege's claims that numbers are "not the object of psychology", that they are "not the product of mental processes", and that "the mathematician cannot create things at will"? I suggest that in order to make sense of these remarks we have, once more, to look at the truth grounds of arithmetic judgments. In particular, we have to consider the truth grounds for arithmetic existence-claims. From a contextual platonist perspective the question to ask is: how are claims about the existence of numbers ultimately justified? Frege answers that they are justified insofar as they follow from basic logical laws (and definitions) – and these logical laws are neither psychological nor physical laws. But then it follows that numbers are neither "the product of mental processes" nor "created" (or, for that matter, "destroyed") by mathematicians. In fact, it does again not even make sense to say that numbers "come into existence" or "go out of existence"; they just exist (supposing that the corresponding existence claims in fact follow from our basic logical laws).

Of course it is true that we, as human beings, have come up with particular *formulations* of our logical laws at some point in space and time. Similarly, one may perhaps be able to locate our (implicit or explicit) *adherence* to these laws in space-time. And it makes sense to ask at what point someone has *shown* for the first time that a certain existence claim follows from our basic laws. But these are all just observations about us as human beings (or

maybe about mathematics as a human practice); as such they have a mere psychological character (perhaps also sociological or historical). Crucially, they are *not* observations about numbers *per se* – since they are not about the *validity* of our basic laws and about what *follows* from them. To confuse these two sides is, as we may say, a *category mistake*; in Frege's terms, it is to confuse "being true" with "taking to be true". This also clarifies why numbers, just like the Moon and Julius Caesar, are not "objects of psychology" (while mental ideas and thought processes concerning each of them are).

I conclude, altogether, that there remains not much of a mystery with respect to interpreting Frege's platonist remarks, i.e., his claims that numbers "exist", in a "separate realm", as "self-subsistent, determinate objects", etc. Likewise, there remains not much of a mystery in connection with his corresponding claims about arithmetic "thoughts" or "truths", say the Pythagorean theorem: they, too, "exist" in the "third realm" etc. That is to say, both kinds of claims can be understood from the perspective of contextual platonism, not just from that of metaphysical platonism.

Let me now quickly turn the tables. I want to present four pieces of textual evidence which seem to me to speak directly *against* the *usual interpretations* of Frege as a metaphysical platonist. Two of them we have already encountered, the other two are new. First, consider again the following passage about "self-subsistence" from *Foundations*: "The self-subsistence which I am claiming for numbers is not to be taken to mean that a number word signifies something outside of the context of a proposition, but only to preclude the use of such words as predicates or attributes, which appreciably alters their meaning." (*FA*, p. 72) As we have seen, this passage is clear on a contextual platonist reading. But, I submit now, it seems very hard to make sense of it from the point of view of metaphysical platonism. In particular, how are we to understand its last part along those lines?

Second, we have also already encountered the following comment about the non-spatial nature of numbers: "But where is the number 4? It is neither outside us nor within us. And, taking those words in their spatial sense, that is quite correct. To give spatial-coordinates for the number 4 makes no sense" (*Ibid.*, p. 72). According to my interpretation, Frege presents a logico-syntactic observation in this passage, i.e., an observation about which predicates it makes sense to apply to numbers. And how else could we understand it, in particular along metaphysical platonist lines?

Third, and in addition to what we have seen so far, there is the following general remark about "independence" in *Foundations*: "[F]or what are things independently of reason? To answer that would be as much as to judge without judging, or to wash the fur without wetting it" (*Ibid.*, p. 36). But in connection with arithmetic objects we can now observe the following: it is metaphysical platonists (as well as proponents of psychologism) who try to "wash the fur without wetting it"; since they try to explain what numbers are without considering the basic judgments we make about them. And Frege is explicitly opposed to any such attempt.

Finally, consider the following passage about "arithmetic knowledge": "In arithmetic we are not concerned with objects which we come to know as something alien from without through the medium of the senses, but with objects given directly to reason and, as its nearest kin, utterly transparent to it" (*Ibid.*, p. 115). Frege denies here the existence of any special problem about "access" to arithmetic objects. Furthermore, his appeal to reason in this passage sounds fundamentally different from postulating an extra "intellectual sense", i.e., a special kind of mathematical "perception" (analogous to our usual senses). Both observations speak again directly against a metaphysical platonist reading of his position. On the other hand, if one interprets him as a contextual platonist these things all make sense.[22]

V. Wittgenstein's extension of the context principle

Wittgenstein is directly opposed to metaphysical platonism, as we saw earlier. And he is not exactly a contextual platonist either, as we will see now. At the same time, he is fundamentally sympathetic to Frege's *context principle*. In Section II we already discussed Wittgenstein's critical use of that principle. More particularly, we studied the way in which his criticism of the Augustinian picture is an extension of Frege's criticism of certain psychologistic views. But Wittgenstein, like Frege, also uses the context principle in a more constructive way; and in doing so he clarifies it further and extends its range of application. I now want to consider this constructive use – in its general form and with respect to the particular cases of psychology and mathematics. This will lead us to Wittgenstein's notions of *grammar* and *criterion*; and it will lead us to my thesis that he can be seen as a *grammatical realist* (at least with respect to psychology).

The context principle and the notion of grammar

With respect to Wittgenstein my focus in this paper is on his later writings, in particular on *Philosophical Investigations*. However, the context principle comes up already in earlier texts, at a number of places; in fact, positive considerations of Frege's principle occur in almost all of Wittgenstein's writings, from the *Tractatus* all the way to *Last Writings*. It is interesting to compare his most striking reiterations and reformulations of the principle.

Recall, first, Frege's two main formulations of the context principle:

> One must ask for the meaning of words in the context of sentences, not in isolation.
>
> (*FA*, p. X; similarly on p. 116)

> Only in the context of a sentence do words have meaning.
>
> (*ibid.*, p. 73; also on p. 71)

Now consider the following sequence of remarks in Wittgenstein's writings:

> Only in the context of the sentence does a name have meaning.
>
> (*TLP*, 3.3; 1918)

> A name has meaning, a sentence has sense, in the calculus to which it belongs.
>
> (*PG*, p. 63; 1932–34)

> Only in the practice of a language can a word have meaning.
>
> (*RFM*, VI, 41; 1943–44)

> Only in the stream of life do words have their meaning.
>
> (*LW*, *Vol. I*, §913; 1948–49)

Clearly these formulations are not only direct echoes of Frege, they are also variations on a theme. The theme – the context principle – gets developed further and further by Wittgenstein. In particular, his views about the relevant context expand: from "sentence" over "calculus" and the "practice of a language" (or "language games") to the "stream of life".

Briefly, what does change and what remain constant in this development? Most fundamentally, throughout his writings Wittgenstein keeps insisting that we have to look at the *use* of words when explaining their meaning. Thus he says already in the *Tractatus*: "In order to recognize a symbol by its sign we must observe its meaningful use" (*TLP*, 3.326). "If a sign has *no use*, then it is meaningless. . . . (If everything behaves as if a sign had meaning, then it does have meaning.)" (*Ibid.*, 3.328). And what we have to pay attention to with respect to this use, according to the *Tractatus*, is what it reveals about the *logical form* of an expression. As Wittgenstein puts it: "A sign determines a logical form only together with its logico-syntactic use" (*Ibid.*, 3.327). But why is it so important to consider the logical form of a word? It is because this form reflects the logical form of the object to which the word, as a name, refers. That is to say, the logical form shows what *kind* of an object it is for which the name stands. If we look back to Frege, this thesis should sound familiar. In particular, remember that for Frege the fundamental difference in kind between objects and concepts shows itself in the different logico-syntactic uses of object names and concept names.[23]

Moving ahead to Wittgenstein's later writings, in particular to *Philosophical Investigations*, the same basic focus on the use of words occurs, now in the following form: "Let the use of words teach you their meaning" (*PI*, p. 220) Or (as we have already seen): "Now what do the words of this language *signify*? – What is supposed to show what they signify, if not the kind of use they have?" (*Ibid.*, §10). Once again, the use of a word reflects the identity of its referent (if it has one). Now, at this point Wittgenstein also insists on another point, namely: we cannot *derive* this use of words from some direct, primitive connection to the world. Thus he says: "Don't always think that you read off your words from the facts; that you map these into words according to rules" (*Ibid.*, §292). For Wittgenstein – like for Frege – it is rather the other way around: the use of a word is *prior*, relative to its referent, which is *posterior*, with respect to an explanation of what the word means. What that amounts to is this: Wittgenstein's writings, from early to late, exhibit the same basic *reversal in the order of explanation* as Frege's (now seen relative to the Augustinian picture in general); and this reversal is again guided by the *context principle*.

However, not everything stays the same when we go from the early to the later Wittgenstein; in fact, there are some significant changes. Most importantly, in Wittgenstein's writings after the *Tractatus*, the notion of "logical form" is replaced by that of "grammar". Thus he remarks in the *Investigations*: "Grammar tells what kind of object something is" (*PI*, §373). This transition – from logical form to grammar – is at the core of one of Wittgenstein's central moves in his later writings, namely: away from looking at language as a system structured in terms of a crystalline logical "scaffolding" and used to "picture" the world; towards looking at it as a "language game", i.e., as intimately tied up with various practices and deeply embedded in the "stream of life".

Wittgenstein's transition goes through at least two stages. In the first stage, from his early writings (including the *Tractatus*) to those of the early 1930s (in particular *Philosophical Grammar*), the Tractarian conception of language is replaced by that of language as a *calculus*. Correspondingly, the notion of "logical form" is replaced by that of "role in the calculus" (*PG*, p. 63) – a first meaning of "grammar". In the second stage, through the late 30s and up to Wittgenstein's mature writings in the 1940s (in particular *Philosophical Investigations*), he shifts from looking at language as a calculus to looking at it as a *language game*. Thus, the notion of "role in the calculus" is replaced with that of "role in the language game" – a second, more mature meaning of "grammar". The reason for this second shift is Wittgenstein's recognition that language does not function mechanically; that is, words do not "apply themselves", according to some hidden rules.[24]

Like Frege, Wittgenstein always sees the context principle as a *general* principle. But unlike Frege, he works out its general application in considerable detail; that is, he applies the maxim "grammar tells what kind of object something is" explicitly to a number of different cases (including that of words for ordinary physical objects and people). In *Philosophical Investigations* one case finds Wittgenstein's special attention: *psychological* language.

In Section II we considered some of his critical remarks in connection with it. Now I want to add a brief review of his constructive remarks, i.e., those in which he starts to investigate the grammar of words such as 'pain', 'understanding', etc. This review will also illustrate further his general notion of "grammar" and it will lead us to his notion of "criterion", as well as to my thesis that he is a "grammatical realist".

The notion of criterion and grammatical realism

As is well known, Wittgenstein rejects "inner pointing" as the sole basic for the meaning of a word such as 'pain'. More precisely, he denies that the meaning of this word is based on some simple act of inner ostention. Likewise, he denies that the word 'understanding' obtains its meaning simply by being attached to a process in the mind or brain. We may say, then, that Wittgenstein is not an "empiricist" with respect to psychological language – someone who thinks that one can just *read off* the meaning of psychological words from the brain or from the mind. But he is also not an "anti-realist", in several senses – not in the sense of denying the *existence* (or occurrence) of pain and understanding; not in the sense of *reducing* such pain and understanding to mere *physical* states or processes; and also not in the sense of reducing them to mere categories of *behavior*. Instead, he says in *Remarks* (somewhat cryptically): "Not empiricism and yet realism in philosophy, that is the hardest" (*RFM*, VI, 23.).[25] On the basis of remarks such as this I suggest that Wittgenstein is a *grammatical realist*, in particular with respect to psychology. In other words, for him cases of pain and understanding are "real", i.e., they do exist or occur – but their reality (and nature) cannot be understood independently from the grammar of 'pain' and 'understanding'.

How exactly does Wittgenstein think about the *grammar* of such words? Two basic and complementary observations need to be made here (at least). First, our usual *logico-syntactic use* of 'understands' and 'is in pain' is *similar* in important ways to the use of, say, 'is five feet tall'. In particular, in both cases these expressions can be preceded by names, definite descriptions of people, or pronouns. Also, in both cases the resulting expressions can be used in sentences by means of which we make *objective judgments*. It is such similarities which allow us to say that understanding and pain are "states" or "processes". (Wittgenstein also notes that it does not make sense to attribute understanding or pain to mind-less objects. Put the other way around, the attribution of such states or processes, or at least the possibility of such attributions, are prerequisites for attributing a mental life to someone or something. In that sense understanding and pain are "mental".) Wittgenstein's second basic observation in this connection is the following: States or processes such as "understanding" and "pain" are also *different* in important respects from, say, "being five feet tall". In particular, there are differences with respect to the kinds of *criteria* used for evaluating the correctness of the corresponding judgments. Crucially, in the case of "understanding" and "pain", unlike the case of "being five feet tall", these criteria include observations about behaviour.[26]

For my purposes in this paper one thing is crucial about Wittgenstein's discussion of psychological language. Namely, he always focuses on two aspects of the grammar of words: (i) logico-syntactic facts, i.e., facts about how these words fit into sentences; (ii) criterial facts, i.e., facts about how the truth and falsity of sentences containing them is determined. But if this is so, a close parallel to Frege's approach to arithmetic reveals itself; compare: (i) Frege's logico-syntactic distinction between object names and concept names, and his related observation that number terms function as object terms; (ii) his focus on the truth grounds of arithmetic statements, as reconstructed on the basis of logical definitions and laws. Consequently, Wittgenstein's *grammatical realism* in the case of psychology parallels Frege's *contextual platonism* in the case of arithmetic.

Given my suggestion that Wittgenstein is a "grammatical realist", another possible misunderstanding should be prevented at this point. Namely, according to me Wittgenstein is *not*, as may be falsely inferred, siding with "realism", as that notion is widely used today in the literature; *nor* is he on the side of "anti-realism". In fact, from the point of view developed here the usual opposition of "realism versus anti-realism" reveals itself as rather *misleading* (and to call Wittgenstein a "grammatical realist" is intended to point in a different direction, one orthogonal to the realism-vs.-antirealism dichotomy). Notice in this connection what Wittgenstein says about the case of mathematics in his *Lectures*:

> [We get into] queer trouble: one asks such a thing as what mathematics is about – and someone replies that it is about numbers. Then someone comes along and says that it is not about numbers but about numerals; for numbers seem very mysterious things. And then it seems that mathematical propositions are about scratches on the blackboard. That must seem ridiculous even to those who hold it.
>
> (*LFM*, p. 112)

As I read this passage, Wittgenstein distances himself in it *both* from metaphysical platonism *and* from its opposites, e.g., from simple kinds of formalism. At other points he discusses such formalist views also under the name of 'finitism' (thereby misusing the latter term somewhat); and he compares finitism to "behaviorism" in psychology. Thus he writes in his *Remarks*: "Finitism and behaviorism are quite similar trends. Both say, but surely, all we have here is. . . . Both deny the existence of something, both with a view

to escaping from a confusion" (*RFM*, II, 61). Similarly in his *Lectures*, now in a more critical and categorical tone:

> Hence we want to see the absurdities both of what the finitists say and of what their opponents say – just as we want in philosophy to see the absurdities both of what the behaviorists say and of what their opponents say. Finitism and behaviorism are as alike as two eggs. The same absurdities, and the same kind of answer. Both sides of such disputes are based on a particular kind of misunderstanding – which arises from gazing at a form of words and forgetting to ask yourself what's done with it, or from gazing into your own soul to see if two expressions have the same meaning, and such things.
>
> (*LFM*, p. 111)

Wittgenstein's position is, thus: Both "finitism" and "behaviourism" are attempts to "escape from a confusion". Namely, finitism attempts to avoid metaphysical platonism (and thus the "access problem" and questions about the "mysterious nature" of numbers); behaviorism attempts to avoid certain forms of psychologism and dualism (with their corresponding problems about "privacy" and questions about the "mysterious nature" of mental states). But both finitism and behaviorism are themselves still based on a fundamental "misunderstanding" – in fact, as much so as the views to which they are opposed.

The source of this misunderstanding is, in Wittgenstein's view, that most philosophers involved in these debates (formalism *versus* platonism, behaviorism *versus* dualism, etc.) have been careless with respect to the way they think about the meaning of words; in particular they have neglected to look carefully at "what's done with words". Consequently, they have gotten stuck in simplistic dogmas. As Wittgenstein puts it in the *Investigations* (now very generally): "For this is what disputes between Idealists, Solipsists and Realists look like. The ones attack the normal form of expression as if they were attacking a statement; the others defend it, as if they were stating facts recognized by every reasonable human being." (*PI*, §402) And he proposes the following way out: "What we have to do is to accept the everyday language game and to note false accounts of the matter as false" (*Ibid.*, p. 200). One thing I have done in this paper is to look in some detail at Wittgenstein's (and Frege's) criticisms of "false accounts" of the matter, especially in the cases of our psychological and mathematical language games. But I have also pointed out some positive results. In other words, Wittgenstein's (and Frege's) investigations into the grammar of words go beyond just "accepting the everyday language games" – they help us to understand these language games better (at least with respect to some of their philosophical aspects).

Remaining differences between Frege and Wittgenstein

So far I have argued that Wittgenstein's philosophical perspective agrees with Frege's in a fundamental way, namely with respect to the context principle and the corresponding reversal of metaphysics. In this sense the simple opposition between Frege, the "arch-platonist", and Wittgenstein, the radical "anti-realist", turns out to be inadequate and misleading. Nevertheless, clearly Wittgenstein does not agree with Frege on *everything*. In order to avoid the misunderstanding that this is what I am suggesting, let me add a few remarks about remaining *differences* between them. I will mention two in particular: (i) In his later writings Wittgenstein calls into question the usefulness and coherence of any strong notion of *meaning* – including Frege's notion of "sense" or "thought"; (ii) he proposes a view of logic and arithmetic as mere collections of *techniques* – not, like Frege, as systems of truths.

First, to "meaning"; already in the *Blue Book* (1934–35) Wittgenstein remarks critically:

> Frege ridiculed the formalist conception of mathematics by saying that the formalists confused the unimportant thing, the sign, with the important, the meaning. Surely, one wishes to say, mathematics does not treat of dashes on a bit of paper. Frege's idea could be expressed thus: the propositions of mathematics, if they were just complexes of dashes, would be dead and utterly uninteresting, whereas they obviously have a kind of life. And the same, of course, could be said of any proposition: Without a sense, or without the thought, a proposition would be an utterly dead and trivial thing. And further it seems clear that no adding of inorganic signs can make the proposition live. And the conclusion which one draws from this is that what must be added to the dead sign in order to make a live proposition is something immaterial, with properties different from all mere signs. But if we had to name anything which is the life of the sign, we should have to say that it was its *use*.
>
> (*BB*, p. 4)

He clarifies what he is opposed to as follows: "The mistake we are liable to make could be expressed thus: We are looking for the use of the sign, but we look for it as though it were an object *co-existing* with the sign" (*Ibid.*, p. 5). And he adds: "As a part of the system of language, one may say, the sentence has life. But one is tempted to imagine that which gives the sentence life as something in an occult sphere, accompanying the sentence" (*Ibid.*). On the surface, what Wittgenstein rejects here is any postulation of an "occult sphere" of "meanings" (parallel to the realm of "abstract

objects" in metaphysical platonism). On closer inspection it becomes clear that what he is primarily concerned about is the postulation of a notion of "meaning" (or "sense", "thought") according to which the meaning of words is *independent* from, or *prior* to, their use.

A second remaining disagreement between Wittgenstein and Frege concerns the question whether logic and arithmetic should be seen as collections of *propositions* or not. Frege defends the view that they should, most explicitly in his article "Thoughts". In his terminology, the question is whether logic and arithmetic are "sciences", i.e., bodies of substantive "truths"; and his answer is affirmative. Wittgenstein proposes instead: logic and arithmetic are mere collections of *techniques*. Crucially, such techniques cannot be said to be either true or false; they are just more or less useful (in a variety of ways). Relatedly, he sometimes likens mathematical statements to *rules* or *imperatives*; as such they say: do this, do that! In *Remarks on the Foundations of Mathematics* he gives a number of simple examples, amongst them "$25 \cdot 25 = 625$". He analyzes this sentence as saying the following: If you want to determine that you have 625 objects, arrange them in groups of 25 objects and count these groups, up to 25! Thus, Wittgenstein analyzes the arithmetic sentence as expressing a technique, rule, or imperative for what to do in *practical applications*.

Such an analysis corresponds to Wittgenstein's general idea that our understanding of mathematical terms, or of words in general, is ultimately tied to how we use them in simple, everyday applications. It is along these lines that he says early on in the *Investigations*:

> Now think of the following use of language: I send someone shopping. I give him a slip marked "five red apples". He takes the slip to the shopkeeper, who opens the drawer marked "apples"; then he looks up the word "red" in a table and finds a color sample opposite it; then he says the series of cardinal numbers – I assume that he knows them by heart – up to the word "five" and for each number he takes an apple of the same color as the sample out of the drawer. – It is in this and similar ways that one operates with words. – "But how does he know where and how he is to look up the word 'red' and what he is to do with the word 'five'?" – Well, I assume that he *acts* as I have described. Explanations come to an end somewhere. – But what is the meaning of the word "five"? – No such thing was in question here, only how the word "five" is used.
>
> (*PI*, §1)

Here the "meaning", or better the use, of the word 'five' is put in the context of a practical procedure: that of counting from one to five while correlating numbers with objects. Notice that there are certain immediate advantages to such an analysis. Most importantly, it suggests a new understanding of the *a priori* character of arithmetic (or at least of simple, applied arithmetic). Namely, if we ask why arithmetic statements are not subject to empirical verification or falsification, Wittgenstein can answer: it is because they express rules, and rules are not true or false – just more or less useful.

However, this interesting suggestion – to be found mostly in *Remarks on the Foundations of Mathematics* and in other writings from the 1930s – is never fully worked out in Wittgenstein's later writings. Thus it is not clear just how seriously he takes it in the end. Also, he never explicitly addresses the question whether an analysis along these lines can be carried through for *all* of mathematics, i.e., beyond simple, applied arithmetic – and it seems rather problematic that it can. Overall, it seems to me that an approach which treats pure mathematics (including arithmetic) as consisting of a body of propositions has a better chance of being applicable and illuminating. Thus Frege's "propositional" point of view seems more appropriate in these cases than Wittgenstein's "imperative" one after all.

Final observations about the relation between Frege and Wittgenstein

As just explained, Wittgenstein is opposed to certain aspects of Frege's general views about *meaning*. In particular, he objects to Fregean "senses" or "thoughts". In addition, he entertains the idea that logical and arithmetic sentences express techniques, rules, or imperatives, an idea which is undeniably opposed to Frege's *platonism* – even if Frege is interpreted as a contextual platonist. Nevertheless, with respect to this second issue Wittgenstein's opposition to Frege is not as complete as it might appear now. This brings me back to two final observations about *similarities* with respect to their basic outlooks.

Let us assume, first, that Wittgenstein completely endorses an analysis of mathematics as consisting merely of techniques, not of truths. Then he is clearly not a contextual platonist (much less a metaphysical platonist). Nevertheless, he does not disagree with Frege on a *fundamental* level – at bottom both of them study how logical and arithmetic words are *used* in sentences. Their disagreement concerns merely the question *how* to think about our use of logical and arithmetic *sentences*. In their respective answers Wittgenstein concentrates on simple applications of logic and arithmetic and he analyses them in terms of his notion of "technique"; Frege, on the other hand, aims at a systematic reconstruction of pure logic and arithmetic, by means of his new logical tools.

But, second, it is not so clear that Wittgenstein completely and finally endorses the view of mathematics just attributed to him; or it is not so clear how far he means it to apply. Undoubtedly he plays with it as a general idea

for a while, in particular in his *Remarks on the Foundations of Mathematics*. And in *Philosophical Investigations* he maintains at least that there is something right about it in the context of simple, applied mathematics (such as shopping in a grocery store). At the same time, he now explores other, differing ideas, too. Consider for example the following very general remark from the *Investigations*: "Think how many different kinds of things are called 'description': description of a body's position by means of its co-ordinates; description of a facial expression; description of a sensation of touch; or of a mood" (*PI*, §24). Now, what about the following suggestion: why can we not say that sentences in pure arithmetic, say, form a *special class* of "descriptions"? The difference between it and other classes would not be hard to explain along Wittgensteinian lines: we just have to look at how the truth of various kinds of descriptions is to be *determined* – by means of mathematical proofs, by mean of empirical observations, by mean of introspection, etc. Keeping in mind such *criterial* differences, we could, then, maintain that arithmetic contains "descriptive" truths after all, couldn't we?

As it turns out, Wittgenstein's later writings on mathematics, in particular *Lectures* and *Remarks*, do contain a number of specific passages which go exactly in this direction. Thus, in some of them Wittgenstein recognizes, even emphasizes, the importance of *proof* in mathematics. For instance, he remarks: "The proof is part of the surroundings of the [mathematical] sentence" (*RFM*, VII, 70). And: "The proof belongs to the background of the sentence, to the system in which the sentence has an effect" (*Ibid.*, 74). In passages such as these Wittgenstein, very much like Frege, points to our whole system of mathematical judgments in connection with understanding mathematical expressions. And he points out that what holds this system together is proof (and calculation).

Sometimes Wittgenstein makes the same point about proof also in a more concrete way, in connection with particular examples (from pure mathematics). Thus he remarks about theorems concerning roots of equations in analysis: "If the proposition 'A quadratic equation has two roots' stood alone, it would be as meaningless as '$25 \cdot 25 = 625$' would if it stood alone outside any system of multiplication – although it is English and it looks all right." (*LFM*, p. 155) And even in the case of Russell's system of logic, usually an object of attack, he admits: "The symbols '$(x) \cdot \phi x$' and '$(\exists x) \cdot \phi x$' are certainly useful in mathematics, so long as one is acquainted with the technique of proofs for the existence or non-existence to which Russell's signs refer *here*" (*RFM*, V, 13.). Note that Wittgenstein connects the *existence* of numbers (and of other mathematical objects) with "techniques of proof". For him, the meaning of "existence" in mathematics has, thus, to do with how we *prove* existence claims. This idea is, I think, exactly in line with Frege's contextual approach; indeed, it makes it more explicit and clarifies it further.

To be sure, according to Wittgenstein one has to be careful when talking about "existence" in the case of mathematics – carelessness may lead to the misunderstanding that one is promoting metaphysical platonism. Because of this danger, he writes:

> It looks like obscurantism to say that . . . mathematics does not treat of signs, or that pain is not a form of behavior. But only because people believe that one is asserting the existence of an intangible, i.e. shadowy, object side by side with what we all can grasp. Whereas we are only pointing to different modes of employment of words.
> (*RFM*, III, 76.)

It is exactly in order not to lead people back into such "obscurantism" that Wittgenstein often stresses the differences between various kinds of descriptions, in particular between those used in mathematics, psychology, and physics. But he also admits: "I will have to stress the differences between things, where ordinarily the similarities are stressed, though this, too, can lead to misunderstandings" (*LFM*, p. 15). In the end we should realize the following: There are *important similarities and important differences* between describing the fact that a table is four feet long, describing the fact that my headache has gone away, and describing the fact that 17 is prime. Furthermore, Frege usually stresses the similarities, while Wittgenstein usually stresses the differences; and from the point of view developed in this paper these are *two sides of the same coin.*

Finally a brief observation about the personal relation between Frege and Wittgenstein: It is well known that Wittgenstein, throughout his life, showed great respect for Frege. Thus, in the *Tractatus* (1918) he talks about his debt to "Frege's great works"; in *Culture and Value* (1931) he lists Frege as one of his main influences; and in *Zettel* (1945–49) he says: "The style of my sentences is extraordinarily strongly influenced by Frege. And if I wanted to, I could establish this influence where at first sight no one would see it". – If I am right, Frege's influence on Wittgenstein is not just one of style; it is quite *substantive*. Maybe Wittgenstein's awareness of that fact also explains his continued respect? [27]

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Notes

1 With respect to my whole perspective (on Frege, Wittgenstein, and platonism) I have been guided by Tait 1986a. My interpretation of Frege has, in addition, been influenced by Dummett 1978a, Ricketts 1986, and the first few chapters of Diamond 1991. Similarly, my interpretation of Wittgenstein has also been influenced by Tait 1986b and by Steve Gerrard's dissertation *Wittgenstein in Transition. The Philosophy of Mathematics* (University of Chicago, 1986), published in part as Gerrard (1991).

2 See Barker 1967.

3 I will quote from Frege's writings using the following (standard) abbreviations: "*BL*" for *The Basic Laws of Arithmetic* (Frege 1967); "*CP*" for *Collected Papers* (1984); "*FA*" for *Foundations of Arithmetic* (1968) and "*PW*" for *Posthumous Writings* (1979). Notice, however, that in a number of cases I have found it necessary to *amend the usual translations.*

4 It is usually assumed to contain functions, too; for simplicity's sake I ignore them here.

5 What is suggested is, thus, an "adjudication" of our judgments by the assumed "Model-in-the-Sky". The term, "Model-in-the-Sky", is from Tait 1986a; for a further discussion of such "adjudication", see Gerrard 1991.

6 All of this is examined in more detail in Chapter 4 of my dissertation *Frege, Wittgenstein, and Platonism in Mathematics*. For a discussion of how Wittgenstein's arguments change from the 1930s to the 1940s, compare also again Gerrard 1991.

7 I will quote from Wittgenstein's writings using the following (standard) abbreviations: "*BB*" for *The Blue and Brown Books* (Wittgenstein 1960); "*CV*" for *Culture and Value* (1980); "*LFM*" for *Lectures on the Foundations of Mathematics* (1975); "*LWPP*" for *Last Writings on the Philosophy of Psychology* (1982); "*PG*" for *Philosophical Grammar* (1974); "*PI*" for *Philosophical Investigations* (1958); "*RFM*" for *Remarks of the Foundations of Mathematics* (1978); "*TLP*" for *Tractatus Logico-Philosophicus* (1961) and "*Z*" for *Zettel* (1970). Notice, once more, that I have occasionally found it necessary to *amend the standard translations.*

8 Gordon Baker and Peter Hacker have also pointed out that Wittgenstein's criticism of the Augustinian picture is meant to apply both to the case of psychological and mathematical language (they speak of "two fruits upon one tree"); *cf.* Baker and Hacker 1985. I agree with many parts of their interpretation of Wittgenstein, but strongly disagree with their reading of Frege (see Sections III and IV).

9 W. W. Tait makes this observation in Tait 1986b.

10 See in particular Dummett 1973 and Dummett 1978b; *cf.* also Baker and Hacker 1980, especially chs 4 and 8 and Baker and Hacker 1984. I should note that both Dummett and Baker and Hacker bring up a number of the individual points I make in this paper – but they do not put the pieces of the puzzle together in the right way.

11 For some interesting historical remarks, compare here Wilson 1992 and Wilson 2005.

12 For instance, the method cannot be used with respect to defining the sense of 'the number 9 = Julius Caesar' (the so-called "Julius Caesar problem"). It also cannot be used for sentences such as '17 is prime' (a sentence of pure arithmetic).

13 For references see Dummett 1981. Dummett himself agrees that these narrow interpretations of the context principle are inadequate. Compare also Milne 1986. Milne mentions another simple, indeed simplistic, proposal for how to interpret Frege's context principle: that it just has to do with "disambiguation", i.e., with distinguishing the meanings of ambiguous words in different sentences (e.g., of a word such as 'bank', which can mean either "a place to keep money" or "the slope of a river"). Any real attention to Frege's use of the context principle in *Foundations of Arithmetic* makes quickly clear, I think, that this is not what it is about.

14 Some examples are: from "for all x: $F(x)$" it is correct to infer "$F(a)$"; likewise, from "for all X: $X(a)$" it is correct to infer "$G(a)$"; but, from "for all x: $F(x)$" it is *not* correct to infer "$F(G)$".

15 Compare Frege's article "Function and Concept" (*CP*, pp. 137–56); there he says about the distinction between *first-order* and *second-order functions*: "It is founded deep in the nature of things" (p. 156).

16 For a merely epistemological interpretation of the context principle, see Haaparanta 1985. For a merely semantic interpretation (within the framework of Donald Davidson's views), see Wallace 1977. For a (more implicit) semantic and epistemological but not metaphysical interpretation, see Burge 1986 and Burge 1992. Closest to my own interpretation of Frege's context principle comes Dummett 1978a, at a few points also Dummett 1991. However, when it comes to drawing conclusions with respect to Frege's *platonism* (its interpretation and its appraisal), I disagree with Dummett in crucial respects. Similarly, Crispin Wright, in Wright 1983, comes close to Dummett's, and thus to my, interpretation in certain respects (and he develops a more sympathetic interpretation of Frege's platonism than Dummett).

17 Michael Dummett discusses some of Frege's reasons for preferring his construction of the natural numbers over those of others (e.g., Dedekind's) in Dummett 1991 (see in particular Chapter 23). For a critique of both Frege and Dummett on these issues, compare Tait 1997.

18 Exactly the same holds for function words in arithmetic; note in this connection what Frege says in "Function and Concept": "The first time where a scientific expression appears with a *clear-cut meaning* is where it is required for the statement of *law*. This case arose as regards functions upon the discovery of higher Analysis. Here for the first time it was a matter of setting forth *laws* holding for functions in general." (*CP*, pp. 137–38; my emphasis)

19 This remark needs some qualification: with respect to my general approach towards Frege, I see myself in agreement with certain ideas expressed in Diamond 1991, Ricketts 1986, Tait 1986a and Weiner 1990, to some degree also Wright 1983 and Dummett 1978a (compare earlier footnotes). However, I do not think that these ideas have attained the status of "conventional wisdom"; on the contrary, interpreting Frege as a metaphysical platonist seems to me to be still the most widespread and dominant approach. Furthermore, my particular interpretation of the *context principle*, and the *central role* it plays for Frege, seems still unique in the literature.

20 See Burge 1992 for a recent, sophisticated reading of Frege as a metaphysical platonist. Characteristically his interpretation is based on a collection of "platonist" remarks from Frege's writings (all of which can be interpreted along my lines, as I am about to argue.)

21 Frege uses the term 'wirklich' to distinguish physical from logical objects. Thus the earth, unlike the number 4, is "wirklich" in the following senses: (i) it can be located in space-time; (ii) it interacts causally with other physical objects;

and, more particularly, (iii) it produces effects on our senses, i.e., it can be seen, touched, etc. (Note that "existent", as "wirklich" is sometimes translated, is very misleading, since numbers certainly do exist for Frege but they are not "wirklich".)

22 So as not to be misunderstood: The access problem, as described by me in Section I, concerns the possibility of gaining access to a postulated *realm of mathematical objects* (a Model-in-the-Sky). I would claim that Frege never takes *this* problem seriously; in the passage cited he even denies it directly. It is, of course, true that Frege wonders about our ability to "grasp" *thoughts* (compare especially "Thoughts"). Still, "grasping" a thought (i.e., understanding it) seems rather different from "perceiving" a number (in some quasi-sensual way). In addition, even with respect to our ability to grasp thoughts, Frege never entertains any fundamental *skeptical* doubts.

23 Of course it is true that in the *Tractatus* Wittgenstein rejects Frege's distinction between objects and concepts; in a Tractarian world there are only objects (with different logical forms). Nevertheless, the two thinkers agree, I think, with respect to their *fundamental perspective*: both focus on what the use of words in sentences shows us, since both follow the context principle.

24 For more on Wittgenstein's general shift from a "calculus conception" to a "language-game conception" of language, see again Gerrard 1991.

25 Wittgenstein makes this remark in connection with mathematics. However, if I am right that his treatments of mathematics and psychology are parallel, it applies equally to psychology. For an interesting discussion of this passage, compare Cora Diamond's "The Realistic Spirit", in Diamond 1991, pp. 39–72. She interprets it in a broader way, but I think our two interpretations are compatible.

26 For more on "criteria", including their difference to necessary and sufficient conditions, compare Baker and Hacker 1980. Notice that with his use of the notion of "criterion" Wittgenstein clearly goes *beyond* Frege. In fact, I think that here we have reached an important point where the two thinkers begin to *differ* in their views – Wittgenstein's notion of "criterion" leads to a kind of *anti-reductionism* (in particular in the case of psychology) that is quite foreign to Frege's general reductionist tendencies (as exhibited mostly in the case of arithmetic). It would be worth exploring this difference further, but I cannot do so in this paper.

27 See Wittgenstein 1961, p. 3; Wittgenstein 1980, p. 19 and Wittgenstein 1970, §712. I have since documented Wittgenstein's great respect, indeed admiration, for Frege further, in Reck 2002.

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