

Frege on Truth, Judgment, and Objectivity

Erich H. Reck, University of California at Riverside, November 2006

SUMMARY:

In Frege's writings, the notions of truth, judgment, and objectivity are all prominent and important. This paper explores the close connections between them, together with their ties to further cognate notions, such as those of thought, assertion, inference, logical law, and reason. It is argued that, according to Frege, these notions can only be understood properly together, in their inter-relations. Along the way, interpretations of some especially cryptic Fregean remarks, about objectivity, laws of truth, and reason, are offered, and seemingly opposed "realist" and "idealist" strands in his position reconciled.

I

In Frege's writings, the three notions mentioned in the title of this paper—truth, judgment, and objectivity—are all prominent and important. They are also closely related to each other, as is made explicit at various places. In "On *Sinn* and *Bedeutung*", Frege relates the first two as follows: "Judgments can be regarded as advances from a thought to a truth value" (Frege 1997, p. 159); at other places, including the late article "Thought", he also characterizes judging as "the acknowledgement of the truth of a thought" (ibid., p. 329). Relating the second and third notions, he remarks in *The Foundations of Arithmetic*: "What is objective ... is what is subject to laws, what can be conceived and judged, what is expressible in words" (Frege 1994, p. 35).

As these initial quotations already indicate, it is not just truth, judgment, and objectivity that are connected for Frege, but also several other notions, including those of thought, law, conceivability, and expressibility; and among the relevant laws, those of logic are especially important. Thus, in *Foundations* Frege advises us "always to separate sharply the psychological from the logical, the subjective from the objective" (Frege 1994, p. x); and in "Thought" he adds: "I assign to logic the task of discovering the laws of truth. ... The meaning of the word 'true' is spelled out in the laws of truth." (Frege 1997, p. 326) Occasionally the notion of reason is thrown into the mix as well:

It is in this way that I understand objective to mean what is independent of our sensations, intuitions, and imagination, and of all construction of mental pictures out of memories of earlier sensations, but not what is independent of reason. For what are things independent of reason? To answer that would be as much as to judge without judging, or to wash the fur without wetting it (Frege 1994, p. 36).

The purpose of this paper is to explain, or better to elucidate, all of these notions as understood by Frege, including providing interpretations of the passages just quoted, cryptic as some of them are.¹ It will become apparent that, according to Frege, a proper elucidation of them will require exactly paying close attention to their inter-relations. Crucial in this connection, since centrally related to all the others, is the notion of judgment. Its centrality has been pointed out before, in particular by Thomas Ricketts.² My discussion will build on some of Ricketts' insights and arguments, not just concerning the notion of judgment, but also those of truth and objectivity. On that basis, I will attempt to clarify further some relatively neglected aspects of Frege's position, including especially cryptic remarks about objectivity, laws of truth, and reason.

II

It will help to begin by reminding ourselves, briefly, of the broader context in which Frege brings up the notions mentioned. Throughout his work, Frege's main interest is in the foundations of mathematics, and especially in the foundations of arithmetic. What he intends to investigate in this connection, clearly and in depth, are the following issues: What are the fundamental concepts and principles of arithmetic; what does their ultimate justification consist in; and thus, what does arithmetic's content and objectivity amount to? Considering such questions leads straight away to Frege's logicism, his claim that arithmetic is reducible to logic. To establish that claim, he develops a new logic, much more powerful than what was available previously; and he attempts to reconstruct arithmetic within that framework. Along the way, he reflects on the nature of logic, both to combat widespread misconceptions concerning it and to defend his novel logical system.

The main kind of misconceptions combated by Frege consists of a group of psychologistic views about mathematics and logic, or more generally of empiricist and naturalistic views, which he finds in the literature of his time.³ Frege's initial attack on

¹ This paper complements Reck (1997) and (2000/2005); see also Reck (forthcoming). With respects to my considerable debts to other Frege scholars, especially Thomas Ricketts, see the following footnotes.

² See Ricketts (1986) and (1996), as well as the summary of Ricketts' interpretation in Kremer (2000). A similar emphasis on judgment in connection with Frege can be found in Sullivan (2005).

³ Frege's opposition to such views was first emphasized and put into historical context in Sluga (1980).

such views occurs in *The Foundations of Arithmetic*, at first focused on the case of mathematics. The attack gets extended and sharpened in subsequent writings, from *Basic Laws of Arithmetic* to the late article "Thought", now with the focus more on logic. Overall, Frege locates the core of the problem in a confused understanding of the nature of judgment, an understanding that, as he tries to make evident, is incoherent and ultimately self-refuting.⁴

We have arrived at the main negative reason why the notion of judgment is central for Frege. Before we can get clear about the foundations of logic and mathematics, what needs to be rooted out first, according to him, are various misguided accounts of the nature of judgment, as they can be found in the literature of his time. Typically, such accounts involve psychologistic appeals to "sensation, intuition, imagination, construction of mental pictures out of memories and earlier sensations" (as quoted above). More generally, they appeal to naturalistically conceived and empirically accessible aspects of cognition, i.e., general features of thinking conceived of as a mental occurrence or even as a process in the brain.

In itself it is not illegitimate, of course, to study thinking in a naturalistic way. Frege's point is that doing so tends to mislead in connection with what he is ultimately concerned with—the foundations of logic and mathematics, thus their content and objectivity—especially if it is done in the forms criticized by him. It is precisely in order to counteract such tendencies that Frege admonishes us, as well as himself, "always to separate sharply the psychological from the logical, the subjective from the objective" (as quoted above). Now, if we are not to conceive of the foundations of logic and mathematics in such problematic terms, how are we to do so instead? Answering that question is the main positive challenge for Frege.

III

Below I will elaborate on how Frege's response to the challenge just mentioned leads him to his sustained reflections on close relations between the notions of judgment, truth, objectivity, assertion, inference, logical law, etc. But before doing that, let me

⁴ In Ricketts' words, Frege opposes "a confused admixture of psychology and logic", rooted in "a naturalistic empiricist view of cognition", that "collapses into subjective idealism" (Ricketts 1986, p. 121).

bring up another issue, one that is unavoidable in this connection. I am referring to Frege's alleged platonism, often conceived of precisely as an explanation of the content and the objectivity of logic and mathematics. Turning to this issue first will be useful both to make clear the inherent limitations of such an explanation and to prepare my later discussion of its relation to an alternative.

What philosophers usually mean by "platonism", in our context, is a basic realism concerning abstract objects. In Frege's writings, two kinds of abstract objects come up centrally: classes (or value ranges more generally), with numbers conceived of as a special case; and thoughts (or senses more generally). In *The Foundations of Arithmetic*, it is the nature of numbers that takes center stage. Against psychologistic attempts to conceive of numbers as mental entities or, more generally, to account for the content of arithmetic by appeal to mental states or processes, Frege insists that numbers are not "psychological and subjective", but "logical and objective" (as quoted above). He also characterizes numbers as "independent" and "self-subsistent objects" (Frege 1994, p. 72), both in the sense of being different from concepts and of not needing a bearer for their existence, as mental phenomena do. As such, they are the things referred to, and attributed properties to, in objectively true or false arithmetic statements.⁵

In Frege's later writings, from *Basic Laws of Arithmetic* to "Thought", his reaction against psychologistic trends typically involves an appeal to the objective status of thoughts, conceived of as the contents of judgments, including logical and mathematical judgments. In order to distance himself from problematic psychologistic views in this connection—which, in his view, render such contents hopelessly subjective—Frege goes so far as talking about a "third realm" in which objective thoughts reside (Frege 1997, p. 337). This third realm is presented as parallel to the "first realm", the spatio-temporal world of physical objects and processes, and to the "second realm", the world, or worlds, of mental pictures, subjective ideas, etc., as occurring in individual people's minds.⁶

⁵ See Reck (1997) and (2000/2005) for extensive quotations.

⁶ See again Reck (1997) and (2000/2005) for further discussion. While Frege does not say so explicitly, I assume that numbers, classes, and value-ranges in general, plus all other abstract entities countenanced by him (truth values, concepts, functions in general), are also inhabitants of the "third realm". An alternative interpretation of Frege's "three realms" talk, suggested to me by Danielle Macbeth, would be the following: Take the first realm to contain all objective entities to which we can refer (physical as well as abstract objects, also functions, including concepts); the second realm to contain everything subjective; and the

Both Frege's characterization of numbers as "independent, self-subsisting objects" and his location of thoughts in a "third realm" feed into interpretations of him as a strong and perhaps crude platonist—a "metaphysical platonist", as I propose to call it. Along those lines, it is assumed that Frege's response to psychologistic views is founded on a primarily and essentially ontological construal of the subjective-objective distinction, and more specifically, on the postulation of abstract entities in an object-based metaphysics. That postulation is then taken to form the basis of an explanation of the content and objectivity of logical and mathematical judgment.⁷ It is also often seen as immediately problematic as a position in itself. The main problem is familiar: How could we ever have access to a realm of abstract objects, as postulated here, especially if any causal contact to them is ruled out, as it seems to be by Frege?

IV

What is crucial with respect to a metaphysical-platonist interpretation of Frege, as just sketched, is the following: We start with the postulation of certain abstract entities, conceived of independently of how we make judgments. It is such entities that are first and foremost seen as objective (entities both in the first and third realms, while those in the second realm are subjective). This is, in a subsequent step, supposed to lead to an explanation of the objectivity of corresponding judgments. A tempting way to arrive at the latter explanation may be in terms of the notion of correspondence, in the sense of adjudicating arithmetic statements against facts involving the initially postulated abstract entities. In any case, the objectivity of judgments, including arithmetic and logical judgments, is treated as secondary—as derivable, or explainable, in terms of more basic notions.

Several formulations Frege uses, especially in the context of expressing his strong opposition to psychologistic views, seem indeed to support an interpretation of him as a

third realm to contain thoughts and other senses, understood as objective entities distinct from both objects and functions (as "modes of presentation", i.e., ways in which entities in the other two realms can be given to us). While the question which of these two interpretations is correct seems significant for understanding Frege's notion of sense, I do not see that it affects the discussion in the present paper much.

⁷ I owe the phrases "ontological construal of the objective-subjective distinction" and "object-based metaphysics" to Ricketts (1986). Compare Reck (1997) and (2000/2005) in which a further clarification of the position is attempted, by focusing on the order of explanation inherent in "metaphysical platonism".

metaphysical platonist. However, there are some immediate problems with such an interpretation. Besides saddling him, rather uncharitably, with a heavy-handed and objectionable view, these problems arise from what he says about three of our main notions: truth, objectivity, and reason. Concerning each, Frege makes remarks that appear to be in direct conflict with a metaphysical-platonist interpretation; or more cautiously, if we assume it, the remarks appear to lead to an internal tension in his views.

We just saw that, if we take the notion of objectivity to apply primarily to entities, then an explanation of the objectivity of corresponding judgments needs to be added. And what may suggest itself in connection with the latter—especially if the postulation of numbers, classes, etc. is the starting point—is an appeal to correspondence. In fact, if we take such an appeal seriously, the notion of truth may seem to be explainable at the same time, in the form of a correspondence account of objective truth (whatever the more specific details). But if so, then a first interpretive problem is the following: As is well known, Frege explicitly rejects an explanation of truth as correspondence at various points in his writings. Even stronger, in "Thought" he rejects any attempt to reduce truth to other, more basic notions. His argument is, in a nutshell, that all such attempts will be circular; while trying to explain truth, they will already presuppose it.⁸

Now, perhaps there is no conflict, after all, between Frege's explicit rejection of a correspondence account of truth and a metaphysical-platonist reading of him; since perhaps an explanation of the objectivity of judgments, even along platonist or realist lines, need not be tied to such a correspondence. But in that case, an alternative explanation seems called for, especially in the case of logical and mathematical judgments. What could such an explanation look like? Perhaps we need to start with the postulation of thoughts as abstract objects, rather than with that of numbers and classes? (More on that idea below.) Then again, if we look at what Frege himself says about objectivity, in connection with logic and mathematics as well as more generally, it seems different from anything one would expect along metaphysical-platonist lines.

As quoted initially, Frege characterizes what is objective as "what is subject to

⁸ See the discussion of Frege's "regress argument" in Ricketts (1986) and (1995). In the present paper, I put aside Ricketts' stronger claim that Frege's views about truth rule out any kind of metalogic. Compare Tappenden (1997) for criticism and an alternative perspective.

laws, what can be conceived and judged, what is expressible in words" (Frege 1994, p. 35). In passages such as these, he connects the notion of objectivity neither with that of correspondence nor with any other object-based notions. Instead, the notions of law, judgment, conceivability, and expressibility are brought up as crucial. A little later in *Foundations*, the following passage can be found:

My explanation [of number in terms of logic] lifts the matter onto a new plane; it is no longer a question of what is subjectively possible, but of what is objectively definite. For in fact, that one proposition follows from certain others is something objective." (ibid., p. 93)

Here again, what is objective is not explained in terms of what one would expect from a metaphysical-platonist perspective. Rather, objectivity is connected with the notion of logical consequence or inference, thus with judgments according to logical laws.

The conflict or tension that should be apparent now, between Frege's remarks about truth and objectivity, on the one hand, and a metaphysical-platonist interpretation, on the other, becomes even more pronounced if we bring in some related remarks about reason. Remember this passage from *Foundations* (quoted early on):

It is in this way that I understand objective to mean what is independent of our sensations, intuitions, and imagination, and of all constructions of mental pictures out of memories of earlier sensations, but not what is independent of reason. For what are things independent of reason? To answer that would be as much as to judge without judging, or to wash the fur without wetting it. (ibid., p. 36)

Later on in the same work, Frege comes back to the notion of reason briefly, as follows: "In arithmetic we are not concerned with objects which we come to know as something alien from without through the medium of the senses, but with objects given directly to our reason and, as its nearest kin, utterly transparent to it". He adds: "And yet, or rather for that very reason, these objects are not subjective fantasies. There is nothing more objective than the laws of arithmetic (ibid., p. 115). Now, neither of these two passages is immediately transparent; both call for interpretation. But it should be noted that Frege connects objectivity again with the notions of judgment and law in them; and all three are presented as closely tied to the notion of reason.

V

As argued so far, from the perspective of a metaphysical-platonist reading of Frege the following should puzzle us: his rejection of any reductive account of truth, including a correspondence account; remarks in which he associates the notion of

objectivity closely, not with object-based notions, but with those of judgment and law; and corresponding remarks about reason. Once more, perhaps it is possible to argue that, as a platonist or realist, one does not have to subscribe to a correspondence account of truth. Still, why does Frege relate objectivity so closely to judgment and law? Even more, what about his remarks on reason, which seem to fly in the face of a metaphysical-platonist reading—should we simply put them aside, as rhetorical flourishes that are not to be taken seriously, or perhaps as an early aberration in *Foundations*, never repeated in Frege's later, more mature writings? Well, not if there is a plausible alternative.

Instead of interpreting Frege as a metaphysical platonist—as someone who starts with the postulation of abstract entities, in an object-based metaphysics—the alternative is to attribute a judgment-based metaphysics to him.⁹ What that means is, first of all, to recognize the notion of judgment as central and primary, not the notion of object. Second, it amounts to taking more seriously than so far the close relationships between the notions of judgment, truth, and objectivity, as well as their ties to the notions of thought, assertion, inference, logic, and reason. In fact, crucial for this alternative interpretation of Frege is the claim that these notions can only be clarified, or elucidated, together, as opposed to being reduced to more primitive notions.

First again to truth. In the article "Thought", right after the passage in which Frege presents his argument that truth cannot be explained in terms of any more basic notion, he remarks: "So it seems likely that the content of the word 'true' is *sui generis* and indefinable" (Frege 1997, p. 327). This does not mean, however, that we cannot say anything further about truth. Frege himself goes on to talk about its relation to the notion of judgment; more specifically, this is one of the places where he talks about judging as "the acknowledgment of the truth of a thought" (ibid., p. 329). Looking at the latter remark out of context, it may be mistaken as a Fregean reductive explanation, or definition, of judgment. Doing so would presuppose that the notion of truth can be understood prior to, or independently of, the notion of judgment. In contrast, one may see Frege's remark, not as a definition or reduction, but as an elucidation, i.e., as a

⁹ The phrase "judgment-based metaphysics" is again from Ricketts (1986). In Reck (1997), I talk about "contextual platonism" as opposed to "metaphysical platonism"; see also again Reck (2000/2005).

clarification in which the two notions are related to each other.¹⁰

Let me clarify further the notion of elucidation as it is employed here. It is presupposed in such an elucidation that we already have some understanding of the notions involved, even if only a partial or implicit understanding, perhaps also an understanding that is easily misrepresented. What the elucidation provides, then, is an articulation of that understanding, by relating the relevant notions to each other in an explicit, particular, and hopefully illuminating way. As such, it involves a kind of circle—not a vicious circle, but a hermeneutic one—which distinguishes it from more linear explanations, reductions, or definitions. This can be illustrated by means of the case just mentioned. Actually, the elucidation here involves not just truth and judgment, but also a third notion: thought. Concerning all three notions, what we are told has three aspects or "directions": First, a thought, in Frege's sense, is (to be understood as) that which we acknowledge as true in a judgment. Second, a judgment is (to be understood as) the acknowledgement of the truth of a thought. Third, truth is (to be understood as) what we acknowledge about a thought in a judgment. Taken together, these three statements form an elucidatory circle, or here a triangle.

Saying that truth is what we acknowledge about a thought in a judgment may still seem obscure. The following additional elucidations, in terms of further cognate notions, may help: Truth is the aim, or goal, of a judgment, while a thought is its content. More specifically, we can distinguish the act of judging, which has truth as its goal, from its particular content, the thought. We can also distinguish between thinking, as the mere grasping of a thought, and judging that the thought is true. And we can recognize assertion as the explicit manifestation or expression of a judgment (verbally, in writing, etc.). In Frege's own words, again from "Thought":

We can distinguish:

- (1) the grasp of a thought — thinking,
- (2) the acknowledgement of the truth of a thought — the act of judgment,
- (3) the manifestation of this judgment — assertion.

(Frege 1997, p. 329)

¹⁰ Ricketts, whom I again follow here, call Frege's remark "equally elucidatory of judgment and truth" (Ricketts 1986, p. 131). It should be clear that elucidation is here not meant in a Carnapian sense.

VI

In the previous section, I started to take seriously the passages in which Frege relates the notions of truth, judgment, thought, and assertion closely to each other. We still need to do the same with several further notions, especially those of inference, logic, and logical law, before then coming back, in the next section, to those of reason and objectivity. The ways in which inference, logic, and logical law are related to the notions discussed so far come to the fore when we recognize that thoughts, as the contents of judgments, often stand in logical relations to each other. In particular, two thoughts can exclude each other; one thought can imply another; and a thought can be a generalization of another. This much is, once again, understood implicitly when we understand the corresponding thoughts. What logic allows us to do is to make that understanding more explicit, by using the notions of negation, conditional, and universal quantification—exactly the three basic notions in Frege's logical system—and by formulating general laws concerning the latter.¹¹

The notion of logic at work here is that of a field that deals with inference. But how exactly should the latter notion be understood? Staying in line with our earlier suggestions concerning Frege, we can conceive of inference as a kind of judgment. What we do in an inference is this: We judge one thought to be true on the basis of judging other thoughts to be true.¹² Logical laws—as made explicit in Frege's logical writings, starting with *Begriffsschrift*—are then the general principles governing this activity, i.e., this kind of judging. Insofar as logical laws are laws of judgment and insofar as truth is the goal of judgment, a close connection between logic and truth manifests itself. And it is precisely such considerations that lead Frege to saying: logical laws are "laws of truth"; and even: "the meaning of 'true' is spelled out in the laws of truth" (as quoted above).

While the general ties between logic and truth should now be apparent, these last two quotations from Frege still need further clarification. We noted earlier that for him "the content of the word 'true' is *sui generis* and indefinable". If so, then we cannot take

¹¹ Compare the discussion of the explicative role of logic in Brandom (1994). I assume here a standard interpretation of Frege's logic, including his conception of quantification. For an interesting alternative, see (Macbeth 2005). Along Macbeth's lines, some of my remarks about Frege's views on inference, logic, and logical laws would have to be reformulated. But my overall point seems unaffected.

¹² Here I again follow Ricketts closely; see especially Ricketts (1986, pp. 135 ff.).

logical laws to provide any definition of truth. Still, these laws—as alleged "laws of truth"—make explicit something crucial. What is it they make explicit? Again, truth is what we aim at in judging and inferring; or put slightly differently, truth is their norm. Now, insofar as we cannot define truth—insofar as we cannot reduce it to anything more basic, anything on which we have an independent handle—we cannot give "external standards" for truth. What we can do, instead, is to articulate "internal standards" for judgment and inference, i.e., basic principles concerning them, namely the logical laws; and that makes these laws internal standards for truth as well.

Let me reformulate this last point slightly, thus illuminating another aspect of Frege's views. While logical laws do have a descriptive content for Frege—as he conceives of them, they are themselves truths—they also play a normative role. Namely, they prescribe how we are to judge, or how we are to aim at truth. Moreover, unlike the laws of special sciences such as geometry or mechanics, they do so in a general way. In the posthumously published piece "Logic", Frege puts it this way:

How must I think in order to reach the goal, truth? We expect logic to give us the answer to this question, but we do not demand of it that it should go into what is peculiar to each branch of knowledge and its subject matter. On the contrary, the task we assign logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject matter. ... Consequently, we can also say: logic is the science of the most general laws of truth. (Frege 1997, p. 128)

To paraphrase, any scientific law provides a standard for judging correctly, as it demands that we judge in accordance with it. What is peculiar about logical laws is that they provide the most general such standards; and this is the sense in which they, especially or exclusively, deserve the title "laws of truth".

To round off this discussion of the inter-relations between truth, judgment, logic, and inference, let me come back to a notion already treated briefly above: assertion. Recall that what we do in an assertion is to make a judgment manifest. Now, this statement should again be seen as elucidatory of both judgment and assertion. That is to say, we can understand an assertion as the making manifest of a judgment; but we can also understand a judgment as what can be made manifest in an assertion. Put slightly differently, we understand what a judgment is insofar as, or to the extent to which, we understand what an assertion is. We also understand what truth is insofar as, or to the extent to which, we understand what a judgment is, as we saw above. Combining these two points, we get a direct connection between assertion and truth. Moreover, the laws of

logic, by being laws of judgment, are laws of truth. But then we can conceive of the laws of logic as laws of assertion as well. Indeed, occasionally Frege goes so far as saying: "[W]hat logic is really concerned with is not carried in the word 'true' at all but in the assertoric force with which a sentence is uttered" (Frege 1997, p. 323).

VII

At this point, we are in a good position to return to Frege's striking, but rather cryptic remarks about objectivity and reason. Remember especially this passage:

It is in this way that I understand objective to mean what is independent of our sensations, intuitions, and imagination, and of all construction of mental pictures out of memories of earlier sensations, but not what is independent of reason. For what are things independent of reason? To answer that would be as much as to judge without judging, or to wash the fur without wetting it. (Frege 1994, p. 36)

We noted early on why Frege distances himself from an appeal to "sensation, intuition, and imagination, ...", namely to oppose psychologistic views. Logic and arithmetic should not be seen as dependent on what the "psychologistic logicians" appeal to, since that would, in particular, undermine their objectivity, and thus misrepresent them in a fundamental way. Saying this much explains the first half of the passage. But what about the rest—why does Frege accept, indeed emphasize, that objectivity, including the objectivity of logic and arithmetic, is not independent of "reason"?

Given our discussion so far, Frege's reference to judging in the last sentence of the passage can point us in the right direction. One more conceptual relationship needs to be elucidated, however: that between reason and judgment. Frege himself is not very helpful in this connection, as he is largely silent on the issue. But consider the following: What is it we do in reasoning, i.e., what kind of activity is it? Well, both according to traditional logic and according to Frege, in reasoning we apply concepts (for Frege, also higher-level concepts) to make determinations in judgments and inferences. Indeed, reasoning amounts to doing these things in a systematic, law-governed way. What, then, is reason? It is the competence for reasoning, i.e., the normative ability for applying concepts in judgments and inferences, in a systematic, law-governed way.¹³

¹³ Using Kantian terminology one could talk about the corresponding "faculty", understood precisely as the normative ability for doing these things. The ability is normative in the sense that it involves doing things correctly or incorrectly relative to a certain goal, in this case truth.

Returning to another of our initial quotations, not just the connection between reason and judgment, but also those between reason and objectivity and between judgment and objectivity fall into place now. Remember, once again, that for Frege "what is objective ... is what is subject to laws, what can be conceived and judged, what is expressible in words" (Frege 1994, p. 35). While his rejection of "sensation, intuition, and imagination ...", in the earlier passage, amounts to characterizing objectivity negatively, what we have here is Frege's main way of characterizing it positively. The part about expressibility can be seen as a reference to the notion of assertion, itself closely tied to that of judgment; and the part about conceivability and judgeability, together with the reference to law-governedness, can be seen as references to reasoning, as just explained. Note that, along such lines, it is exactly right to say that being objective does not amount to being "independent of reason"; and to assume otherwise would be an attempt to "judge without judging, or to wash the fur without wetting it". This is so because objectivity amounts to the possibility of making judgment ("what can be judged"), in terms of concepts ("what can be conceived"), as expressible in assertions ("what is expressible in words"), and in a law-governed way ("what is subject to law").

I have tried to illuminate how Frege ties the notion of objectivity primarily to that of judgment (and some closely related notions), not to that of object.¹⁴ Our earlier slogan that Frege's is a judgment-based metaphysics, not an object-based metaphysics, should thus have become clearer. Yet, how are we to think about the objectivity of various entities, including numbers, along such lines? Frege certainly wants to say that numbers, too, are objective, not just arithmetic judgments; similarly for physical objects. Along my lines, the suggestion is this: Here we are dealing with a secondary and derivative use of the notion of objectivity. That is to say, the objectivity of entities is to be understood in terms of the objectivity of the corresponding judgments. And again, the latter has to do with the systematic law-governedness of those judgments, within the general framework provided by logical laws.

This fits together well with the following (already quoted) Fregean remark:

¹⁴ As Ricketts puts this point suggestively: "[Frege] begins philosophizing from a conception of objectivity that is internal to judgment making" (Ricketts 1995, p. 140). In Sullivan (2005) a related kind of "internalism" is attributed to Frege, although the author disagrees with Ricketts' discussion of truth.

My explanation [of number in terms of logic] lifts the matter onto a new plane; it is no longer a question of what is subjectively possible, but of what is objectively definite. For in fact, that one proposition follows from certain others is something objective. (Frege 1994, p. 93)

Note, in addition, that what is special about numbers and arithmetic is this: In their case all we need to appeal to for ensuring "objective definiteness" is logical laws, while in the case of other sciences (geometry, mechanics, etc.) further laws play a role as well (basic laws of those fields). To highlight what is crucial about the interpretation we have arrived at, the following slogan may help: "Don't ask what numbers are except by asking how we reason about them!" And as Frege tries to establish, we reason about them purely logically. I take this to be the core of Frege's logicism.¹⁵

VIII

Central to the interpretation suggested by me is the claim that, according to Frege, objectivity is to be understood primarily in connection with the notion of judgment, not in connection with the notion of object. Similarly, truth is to be conceived of as closely related to judgment (and some cognate notions), not in an independent way, especially not in a reductive way. In the past few sections, the focus was on establishing the following: If we accept such claims, we can make sense of several of Frege's cryptic and otherwise puzzling remarks about objectivity; likewise for his even more cryptic remarks about reason. Now let me add some further reflections on the interpretation overall.

A main remaining issue concerns the sense, if there is any, in which we are left with a "platonist" or "realist" position. Indeed it may appear, at this point, that what is attributed to Frege is an idealist position, as opposed to a realist one. More specifically, my emphasis on Frege's remarks about reason, coupled with making the notion of judgment central, may prompt such a challenge. After all, aren't these exactly the notions at the core of traditional idealist views, including Kant's transcendental idealism? At some level, I want to say that this challenge works with a distinction between "realism" and "idealism" that is too simple and unreflective, i.e., in serious need of clarification and refinement if it is to advance the discussion. But to make such a response plausible at all, we first need to get clearer about the proposed interpretation in one crucial respect.

¹⁵ See again Reck (1997) and (2000/2005) for further discussion; compare also Tait (1986) which influenced me strongly, in this and other respects.

For that purpose, let me distinguish four possible reactions to my focus on the notions of judgment, reason, etc. in connection with Frege. First, someone may question, and ultimately reject, that this gets at a significant aspect of Frege's position at all, insisting instead on a metaphysical-platonist or realist reading of him, based on his remarks about "self-subsistent objects", the "third realm", etc. Second, someone may acknowledge that the discussion has brought to the fore a significant idealist strand in Frege's thinking, but then argued that what this really shows is that there is a tension in his views, perhaps even an incoherency, as the remarks about "self-subsistence" etc. obviously pull in a realist direction.¹⁶ Third, one may go to the other extreme and deny the significance of the platonist- or realist-sounding remarks in Frege, by either seriously deflating or essentially explaining them away, so that all we are left with is the idealist side. Fourth and finally, one may attempt to reconcile the "two strands" in Frege's writings, arguing that, if understood properly, the "realist" and the "idealist" sides are not opposed to each other, so that neither needs to be explained away.

It may appear that it is the third of these options that has been adopted in the present paper. But that would be a misunderstanding; it is actually the fourth option—probably the hardest one to make plausible, both as an interpretation of Frege and as a position in itself—that is intended. As this is an important point, let me put aside options one and two from now on.¹⁷ Instead I want to address the question of how, or in which sense, my proposal amounts to adopting option four above, not option three. That is to say, I want to make clearer how Frege's remarks about judgment, reason, etc., as interpreted in the previous sections, can be seen as reconcilable with his platonist- or realist-sounding remarks, in such a way that the latter are not simply explained away.

IX

So far, two aspects of Frege's remarks about objectivity have been emphasized. On the negative side, these remarks are meant as an emphatic rejection of, in particular,

¹⁶ I was confronted with the second response at the conference where this paper was originally presented. Thanks to Oswaldo Chateaubriand and Michael Beaney for pressing me on this issue. The first response seems fairly widespread, especially among critics of Frege; compare again Reck (1997) and (2000/2005).

¹⁷ Like Reck (1997) and (2000/2005), this paper is not meant to provide decisive refutations of options one and two. However, the more interpretive option four can be made plausible, the more this will speak against both of them, as well as against option three; or so I hope.

psychologistic views about logic and arithmetic. On the positive side, objectivity is to be understood primarily in connection with the notion of judgment, not the notion of object. Insisting on the latter does, no doubt, lead to an interpretation—some will want to say a re-interpretation—of certain Fregean claims about mathematical objects, especially numbers. In particular, it leads to making sense of the objectivity of numbers by way of the objectivity of arithmetic judgments.¹⁸ Yet this move is not meant to deflate completely, or to reveal as having no content, Frege's corresponding platonist- or realist-sounding remarks. Instead, they are seen as amounting to something substantive and interesting. It is just that what they amount to—what the significance is of saying that numbers are "independent", "self-subsistent objects" etc.—is not taken to be self-evident, but in need of interpretation, indeed an interpretation that allows for a defense of them.

If this is the intention, is it really what we have arrived at? As a first step in responding to this question, note the following: If explaining the objectivity of arithmetic objects in terms of the objectivity of corresponding judgments were coupled with certain views about the latter—e.g., with a psychologistic understanding of such judgments or, perhaps, with an account that bases them ultimately on conventions (as suggested by some twentieth-century philosophers)—this would indeed lead to a rather deflationary reading of Frege. But we do not need to adopt such views about arithmetic judgments. In fact, it is clear that Frege himself rejects them, explicitly those of a psychologistic character, more implicitly also those of a conventionalist character. Turning this line of thought around: If we want to retain real substance to Frege's platonist- or realist-sounding statements about numbers, along the lines suggested in this paper, we better not start with views about arithmetic judgments that are too "thin".

Frege's logicist project has the goal of reducing arithmetic to logic. In particular, arithmetic judgments are to be reduced to logical judgments. More than that, everything is to be grounded in basic logical laws in the end (and corresponding definitions). As noted earlier, for Frege the laws of logic are different from, say, the laws of geometry or of mechanics in one important respect: they guide inquiry more generally. At the same

¹⁸ To avoid a possible misunderstanding, it is not denied here that the notion of objectivity as applied to numbers has an "ontological" sense for Frege. What is denied, instead, is that an object-based explanation lies at its bottom; rather, judgment-based notions take priority. Compare again Ricketts (1986), Reck (1997), and Reck (2000/2005), especially the appeal to explanatory priority in the latter two.

time, they share with geometric and mechanical laws that they are supposed to be truths. Furthermore, they are truths investigated and established systematically in a science—the science of logic. The latter point is crucial now. Remember, once again:

My explanation [of number in terms of logic] lifts the matter onto a new plane; it is no longer a question of what is subjectively possible, but of what is objectively definite. For in fact, that one proposition follows from certain others is something objective. (Frege 1994, p. 93)

The suggestion is this, then: It is by keeping in mind, and taking seriously, that for Frege arithmetic judgments are grounded in logical laws and, in addition, that those laws are established in a systematic science—logic, conceived of as such—that we can avoid working with an understanding of such judgments that is not "thick" enough.

This suggestion can be reinforced by coming back to a Fregean view already mentioned, although only briefly: the view that logical laws are truths. At various points in his writings, Frege emphasizes that "being true" should not be confused with "being held as true". As he puts it early on in *Basic Laws of Arithmetic*:

Being true is quite different from being held as true, whether by one, or by many, or by all, and is in no way to be reduced to it. There is no contradiction in something being true which is held by everyone as false. (Frege 1997, p. 202)

For Frege, this point explicitly applies to logic. Thus, even what is taken to be a logical falsehood, "by one, many, or all", may actually be true; and conversely, what is taken to be a logical truth at some point in time may later reveal itself as false (or at least as not true).¹⁹ But then genuine logical laws, as well as all truths that follow from them, are clearly different in status from conventions, not to speak of mere subjective convictions.

X

What the previous section has made explicit, but what was already implicit earlier, is that for present purposes a lot hangs on Frege's views about logical laws and their status. In particular, if we want to retain a sense that Frege's platonist- or realist-sounding remarks amount to something substantive, as opposed to just explaining them away, we have to take seriously his claim that logical laws are truths, indeed truths established in a systematic science. Can anything further be said in this connection?

There is a temptation, at this very point, to attribute to Frege once more a version

¹⁹ Compare Frege's reaction to Russell's antinomy. Frege took it to show that Basic Law V, which he and others had taken to be true, is not true after all (but without truth value, as some apparent names in it lack reference). Indeed, Frege had already made room for such a possibility earlier; see Frege (1997, p. 195).

of metaphysical platonism, now not by starting with numbers and other mathematical objects, but by focusing on logical laws and the objectivity of thoughts. To see how this fits into our earlier discussion, let us go back to the notion of reason, as appealed to by Frege. Given the conclusions we just reached, reason—our capacity for systematic reasoning—cannot be based on a merely subjective or conventional stipulation of laws, especially logical laws. A different understanding of its foundations is needed. But what is available as an alternative? In search of one, we may be tempted to think of reason as grounded in some kind of grasp of a logical structure "out there", in particular a structure built into the "third realm" of thoughts. In other words, don't we have to assume, on pain of falling back into subjectivism or conventionalism, that in reasoning we let ourselves be informed by an abstract, independent structure of thoughts; and how could the latter be understood except in a metaphysical-platonist sense?²⁰

Should such a position be attributed to Frege, though? I think it shouldn't, for several reasons. Most crucially, this position involves again a notion of objectivity that is object-based, not judgment-based. The relevant objects are now thoughts, together with the system of logical relations between them, while in our earlier discussion it was numbers and other mathematical objects; but their objective existence is again simply postulated, as something basic and primitive. Yet is it really possible to make sense of objective, interrelated thoughts prior to and independently of the notions of judgment, assertion, logical law, reasoning, etc.? Also, if thoughts are conceived of in this way it seems problematic, once more, that we have access to them. And if we read Frege along such lines, his "idealist" remarks have to be explained away again; or it has to be acknowledged that there is a tension in his views, perhaps even an inconsistency.

Someone may respond that these are all Frege's problems, not ours. That is to say, perhaps such a position is really what his platonist- and realist-sounding remarks amount to. It is just that the position is hard to make sense of, even harder to defend, and

²⁰ See Burge (1992, p. 645, fn. 16): "Frege sees the whole logical structure, not just objects, in a Platonic fashion." Compare also Hart (1992), in which a reading of Frege is sketched that is similar to mine, exactly up to the point where a "robustly platonic" position on logic and reason is attributed to Frege. Hart is dismissive of Frege in this respect, while Burge endorses his "Platonic" perspective. Concerning the latter—and as suggested to me by Michael Beaney—I should add that in more recent work Burge seems to play down that aspect and move closer to the reading of Frege suggested in the present paper; see the introduction to Burge (2005). Compare also footnote 23 below.

seemingly in conflict with other remarks he makes. Then again, interpretive charity usually demands of us to resist such a conclusion, at least if there is an alternative.

XI

There is a dilemma at this point that can be described as follows: It seems that more needs to be said about Frege's views on the status of logical laws, and with them about his understanding of the notions of objectivity, reason, etc., if we want to substantiate and further defend the line of interpretation suggested above. In doing so, we don't want to fall into various kinds of subjectivism, including a crude form of idealism, the versions of psychologism explicitly rejected by Frege, and the simple conventionalism mentioned above. We also don't want to fall into metaphysical platonism or realism, neither concerning numbers, nor concerning thoughts and the structure of logical relationships between them. It is like a high wire act. How can we avoid falling down on either side? What does staying on the wire even amount to?

A subtle response to this dilemma is contained in Thomas Ricketts' work. As he writes: "From the perspective Frege acquires in starting from judgments and their contents, the distinction between objective and subjective exhibited in our linguistic practice needs no securing and admits of no deeper explanation" (Ricketts 1986, p. 72). To prevent a possible misunderstanding right away, I do not interpret Ricketts as saying in such passages that, for Frege, objectivity simply amounts, or can be reduced, to some aspects of our linguistic practices, where the latter are seen as basically conventional. Otherwise we would be back to a kind of subjectivism already rejected. Rather, the idea is this: What we have reached when we think of objectivity as grounded in logical laws, and what is as such built into our linguistic practices, is itself "rock bottom".

In connection with commenting on "the sources of Frege's conception of objectivity and logic" (ibid., p. 68), Ricketts elaborates on this point as follows: "Frege's primary given is our awareness of obvious implications and contradictions" (ibid., p. 73). In other words, it is such implications and contradictions that are basic and primitive. There are two things we can do in connection with them: First, we can formulate logical laws in which we make them explicit. In doing so, we provide "laws of truth", in the sense of internal standards for truth, judgment, assertion, etc. Second, we can relate the

notions of truth, judgment, objectivity, though, assertion, logical law, and reason explicitly to each other, as we did above. But that is all—these notions can only be elucidated in terms of inter-relating them, as opposed to reducing them to something else. Likewise, the logical laws cannot be grounded in anything deeper. And why is that? As Frege indicates in the case of truth, any attempt at a further explanation or grounding would already presuppose what it is supposed to provide.

As should be obvious by now, I am sympathetic to this general perspective on Frege. However, I think that Ricketts' remarks, as considered so far, are not entirely satisfactory, or not quite the final word. For one thing, simply calling something basic and primitive seems unsatisfactory. One may wonder, then, whether the resulting position is really less mysterious than the metaphysical-platonist views criticized above. Also, is it really impossible to say anything further in this connection? Maybe we cannot "dig deeper"; but perhaps we can elaborate things further in some other ways, moving more "laterally"? Actually, Frege himself has a few further remarks to offer concerning the justification of logical laws. While these remarks are again rather cryptic, we should consider them (as Ricketts does, too).

At a couple of points in his writings, including *The Foundations of Arithmetic*, Frege simply states that logical laws "neither need nor admit of proof" (Frege 1994, p. 4). In *Basic Laws of Arithmetic*, he is slightly more explicit:

The question why and with what right we acknowledge a law of logic to be true, logic can answer only by reducing it to another law of logic. Where that is not possible, logic can give no answer. (Frege 1997, p. 204)

Frege goes on to explain that any appeal to what we are forced to "by our nature and external circumstances" in connection with logical laws would merely concern their "being held as true", not their "being true"; it thus wouldn't, indeed it couldn't, account for "with what right we acknowledge [them] to be true" (ibid.). That is to say, no such psychological or naturalistic considerations can provide a justification for the logical laws, in the sense Frege is concerned with. And as the passage above indicates, within logic only reductive justification is available, which doesn't help either when we reach the bottom level of such reductions, namely basic logical laws.

Does this mean that no justification of basic logical laws is possible at all? More generally, is any further inquiry concerning which logical laws to adopt impossible? Let

us not jump to conclusions here. The following two points seem clear enough: First, Frege rejects problematic psychologistic or more broadly naturalist appeals in this context. Second, in the case of basic logical laws any further justification, if there was one, could not proceed reductively. Now, is that the end of the matter? Ricketts' remarks, as quoted above, suggest so. He seems to conclude that the second point here, about the limits of reductive justification, rules out inferential justification of basic logical laws in general; and as he adds: "Frege has almost nothing to say about non-inferential justification" (Ricketts 1986, p. 74). I want to close this paper by challenging Ricketts' conclusion or, put more positively, by suggesting an alternative involving inferential, but non-reductive justification.

XII

My concluding suggestion starts not from anything specific Frege writes about this issue, as he indeed says little, not only about non-inferential justification, but also about justification in the relevant non-reductive sense.²¹ Instead, it starts from what he does more generally, or from what is implicit in his procedure. What follows is thus somewhat speculative, as I am aware. Consider again Frege's overall project. His goal is to reduce arithmetic to logic; and to do so he introduces a new logical system—first in *Begriffsschrift* and then, in a revise and expanded form, in *Basic Laws of Arithmetic*—so as to explore what is derivable in that system. Now, the fact that Frege modifies his logical system from *Begriffsschrift* to *Basic Laws* is already noteworthy. Beyond that, observe what happens after he is informed of Russell's antinomy: He fundamentally questions, and ultimately rejects, one of his logical laws, Basic Law V. Moreover, this is done because of inferential considerations, namely the realization that, against the background of the other laws, a contradiction can be inferred from it. But if so, aren't we dealing with a case of justification for a basic logical law that goes beyond what was considered in the previous section, both in terms of rejected naturalistic appeals and

²¹ I am putting aside Frege's remarks in the Introduction to *Basic Laws* about a "logical madman", i.e., someone who rejects a basic logical law. I hope to be able to pick up on this issue—in connection with Ludwig Wittgenstein's related discussions in his later works—in a future publication, but I have to leave it aside here for lack of space. (Ricketts has some interesting comments about those remarks.) As pointed out to me by Dirk Greimann, there are also some little known remarks in "Compound Thoughts" on how and why we accept logical laws (Frege 1977, pp. 55-77). Those, too, I have to leave aside here.

reductive inferences? To be sure, it is a case of negative justification; still.

If this is correct, the question arises what a similar kind of positive justification might amount to, if there can be one at all. Even in that respect, there are some hints in Frege. At points he seems to suggest, more implicitly than explicitly, that the successful completion of his whole project would have constituted such a positive case—the working out of a comprehensive logical system, together with a systematic, conclusive demonstration that arithmetic can be derived (from suitable definitions) within this system, and perhaps also other parts of mathematics. This leads to an extension and strengthening of my interpretation so far. Part of it was that, for Frege, reasoning involves the systematic, law-governed use of concepts in judgments and inferences. What is emphasized now is the systematic aspect, as exemplified in scientific inquiry, including logical inquiry. And the additional observation is that, for Frege, such inquiry involves more than the piece-by-piece derivation of one thought from another. It also brings with it more global, holistic desiderata, including the following: the commitment to coherence and consistency; the goal of working out of our earlier, pre-scientific understanding in a systematic form, as refined in successive accounts; and corresponding virtues such as fruitfulness, both with respect to basic laws and definitions.²²

What I am suggesting, then, is not that basic logical laws can be justified in some reductive or extrinsic sense, such as an appeal to a logical structure "out there" or to what we are forced to "by our nature and external circumstances". Rather, a further kind of non-reductive, intrinsic justification seems possible, both in a negative and a positive form. Indeed, Frege's *de facto* commitment to such justification can be seen as another aspect of his appeal to reason, as manifested in systematic, scientific inquiry.²³ How far this kind of justification can lead us, especially in the positive direction, is another question. The history of logic and the foundations of mathematics in the twentieth century indicates that no quick solutions to all relevant questions can be expected, in

²² Concerning the issue of fruitfulness, see Tappenden (1995). With respect to the importance of the notion of system in Frege, or of systematic scientific inquiry, I am indebted to conversations with Norma Goethe; compare Goethe (2006). Here the present paper also connects directly with Reck (forthcoming).

²³ Compare the rich discussion of Frege's (largely implicit) views about rational inquiry in Burge (1998), parts of which seem compatible with my suggestion. However, the compatible parts would have to be divorced from the metaphysical-platonist aspect; see fn. 20. Similarly for Jeshion (2001), although the specific account of "self-evidence" presented in it seems psychologistic at bottom, thus less congenial.

some cases perhaps no definite solutions at all. Nevertheless, this aspect of rational inquiry should not be ignored, including for understanding Frege's views.²⁴

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²⁴ I am grateful to Dirk Greimann for organizing the conference *Frege on Truth*, Federal University of Santa Maria, Brazil, December 2005; to my audience at that conference, including Oswaldo Chateaubriand, Norma Goethe, Marco Ruffino, and Hans Sluga, for feedback on a first version of this paper; and to Michael Beaney, Dirk Greimann, and Danielle Macbeth for comments on later drafts. As I did not follow all of their suggestions, the remaining weaknesses and mistakes should, as usual, be attributed to me.

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